Air-fuel ratio fuzzy controller handling delay: Comparison with a PI/Smith

Thomas Laurain, Jimmy Lauber
LAMIH (UMR 8201)
Univ. Valenciennes, CNRS
Valenciennes, France
tlaurain@univ-valenciennes.fr

Zsófia Lendek
Department of Automation
Technical University of Cluj-Napoca
Cluj-Napoca, Romania
zsofia.lendek@aut.utcluj.ro

Reinaldo Palhares
Dept. of Electronics Engineering
Federal University of Minas Gerais
Belo Horizonte, Brazil
rpalhares@ufmg.br

***Abstract*—This paper is dedicated to the control of the air-fuel ratio in a gasoline engine. The use of the crank-angle domain handles the variable transport delay, which is the main challenge when controlling air-fuel ratio. The Takagi-Sugeno methodology is considered to deal with the nonlinearities in the model. The control law is obtained using an extended state that includes the delayed variables. Simulations confirm the feasibility of the method and highlight the efficiency of the proposed air-fuel ratio controller.**

Keywords—Gasoline engine; air-fuel ratio; injection timing control; Takagi-Sugeno model and control; variable transport delay; engine test bench

I. Introduction

Due to the increasing number of cars, dealing with the exhaust gases is a crucial issue. Car industries are constrained by norms that are becoming tougher and tougher. That is why the currently implemented static look-up tables that are currently implemented in the electronic control units should be replaced by a closed-loop controller.

In a gasoline engine, the transformation of the exhaust gases is realized by the catalytic converter which works optimally when air and fuel are in stoichiometric proportions. The air-fuel ratio (AFR) represents the ratio between the amount of air and the amount of fuel that have been injected in the cylinder during the intake phase. The amount of air is responsible for the torque production and is controlled by a separate module, either the driver pedal request, or the idle speed controller. The amount of fuel has to be controlled automatically.

The AFR is measured by the lambda sensor (or UEGO sensor). The main issue when controlling the AFR is the variable transport delay due to the position of the lambda sensor. If the delay could be included in the controller design, the performance of the controller would be improved. That is why this paper aims to present an original way to deal with the variable transport delay which makes it possible to include it in the controller design.

Usually in the literature, the variable transport delay has been considered as fixed [1], obtained by approximation [2], [3] or by mapping [4]. Variations have also been proposed as PI + Smith controllers as in [5]–[7]. Moreover, the Smith predictor structure combined with a PID controller has been adapted for time-varying delays [8]. Therefore, in this paper as a benchmark, a PI/PID controller structure has been investigated. Since this controller cannot deal with variable transport delay in a proper way, a Smith predictor is added assuming that the delay is known.

The PI+Smith solution presents the inconvenience that it cannot deal with nonlinear systems, which is the case of most air-fuel ratio models that can be found in the literature. In this paper, the Takagi-Sugeno (TS) representation is used to handle the nonlinearities [9]. TS models can provide an exact representation of the nonlinear system and have been used in the literature for several applications, including other parts of the engine like the air path [10], biomechanics applications [11] or networked systems [12]. An original transformation to the crank-angle domain allows fixing the variable transport delay [13] which becomes equal to a number of samples. However, the methodology to design the controller presented in [13] is too conservative so it is not possible to include an integral action for reference tracking and external disturbance rejection. This paper proposes a less conservative methodology to systematically design controllers for nonlinear systems with variable transport delays.

This paper is divided as follow: Section II introduces preliminaries including notations and the engine test bench whose measured signals are used for identification and simulation results. Section III presents the air-fuel ratio model, the transformation to the crank-angle domain and the Takagi-Sugeno representation. Section IV details the design of the controller. Section V presents the simulation setup as well as the results and a comparison with a PI+Smith controller. Finally, Section VI offers conclusions and perspectives to this work.

II. Preliminaries

A. Experimental setup

For the experimental data, the engine test bench located at LAMIH, Valenciennes is used. It is composed of a Renault engine D4FT 1.2L with turbocharger. It is equipped with commercial sensors and actuators. The Electronic Control Unit (ECU) is programmable but commercial-like, e.g. it has low power and low memory. Figure 1 depicts a photo of the considered engine equipment (@Alexis Chazière).

Fig. 1. Engine test bench of the LAMIH, Valenciennes, France

B. Notations

Let us consider a discrete-time nonlinear state space representation that is affine in control:

$$x(k+1) = A(x)\cdot x(k) + B(x)\cdot u(k) \tag{1}$$

The associated Takagi-Sugeno representation is:

$$x(k+1) = \sum_{i=1}^{r} h_i(k)\left(A_i \cdot x(k) + B_i \cdot u(k)\right) \tag{2}$$

where r is the number of rules, A_i and B_i are the local linear models and $h_i(k)$ the membership functions with $h_i(k) > 0$ and $\sum_{i=1}^{r} h_i(k) = 1$. In what follows, A_z denotes $\sum_{i=1}^{r} h_i(k)\cdot A_i$ and:

$$x(k+1) = A_z \cdot x(k) + B_z \cdot u(k) \tag{3}$$

Similarly, define: $\sum_{i=1}^{r} h_i(k-1)\cdot A_i = A_{z-}$, $\sum_{i=1}^{r} h_i(k+1)\cdot A_i = A_{z+}$

III. Air-Fuel Ratio Model

A. Air-fuel ratio dynamics

In order to design a controller to maintain the air-fuel ratio in stoichiometric proportions, a model is needed. As mentioned in the introduction, the main challenge is the variable transport delay that appears due to the location of the lambda sensor. The delay is time varying since it depends on the engine speed. Based on experimental trials, it can be expressed as a function depending on a fixed part and the inverse of the engine speed, i.e.

$$\delta(t) = \frac{\theta_{fix}}{6\cdot n(t)} \tag{4}$$

where $\delta(t)$ is the variable transport delay (sec), $n(t)$ is the engine speed (rpm) and θ_{fix} is a fixed part (deg). It is commonly considered in the literature [14], [15] that this fixed part is equal to two engine turns, i.e. $\theta_{fix} = 720°$.

The air-fuel ratio dynamics can be modelled as a first order system where the dynamics of the lambda sensors depend on inputs that are delayed $\delta(t)$ [13]:

$$\dot{\lambda}(t) = -\frac{K_2(n(t))}{\tau}\lambda(t) + \frac{K_1(n(t))}{\lambda_s \tau K_{inj}} \frac{\dot{m}_{air}(t-\delta(t))}{t_{inj}(t-\delta(t)) - t_0} \tag{5}$$

where $\lambda(t)$ is the air-fuel ratio, so the output. $t_{inj}(t)$ is the control input and represents the injection timing, i.e. the duration of injection after the start-of-injection point (defined by maps). $\dot{m}_{air}(t)$ is an external input considered as a disturbance since it comes from either the driver throttle control or the idle speed control loop. The variable transport delay $\delta(t)$ can be expressed as (4). τ is the sensor constant time, K_{inj} is the injector gain such that $K_{inj}\cdot t_{inj}(t)$ represents the amount of air. λ_s is the stoichiometric coefficient and is given by $\lambda_s = 14.67$. t_0 is the injector deadtime. $K_1(n(t))$ and $K_2(n(t))$ are polynomial expressions depending on the engine speed, such that:

$$\begin{cases} K_1(n(t)) = a_0 + a_1 n(t) + a_2 n^2(t) \\ K_2(n(t)) = b_0 + b_1 n(t) + b_2 n^2(t) \end{cases} \tag{6}$$

where $\{a_0, \dots, a_2, b_0, \dots, b_2\}$ are constant parameters. These parameters, as well as τ, K_{inj} and t_0, have been identified [13] with the engine test bench located at Valenciennes: $\tau = 1.944$, $K_{inj} = 0.2027$, $t_0 = -0.001327$, $a_0 = 9.059$, $a_1 = -0.0188$, $a_2 = 1.0565\times10^{-5}$, $b_0 = -1.3482$, $b_1 = 0.002564$ and $b_2 = 3.68\times10^{-7}$.

B. Crank-angle domain

The next step is to convert the continuous-time model into a particular discrete-time domain whose sampling period depends on the crankshaft angle $\theta(t)$ [16]. Conversion from the continuous-time variable $\dot{\lambda}(t)$ to the recursive law in the crank-angle domain is given by [17]–[19]:

$$\lambda^{\theta}(k+1) = \lambda^{\theta}(k) + \frac{T_s^{\theta}}{6n(k)} \frac{d\lambda^{\theta}(k)}{dt} \tag{7}$$

where k is the sample index according to the sampling value T_s (chosen appropriately to avoid loss of information) and the superscript θ denotes a variable expressed in the crank-angle domain. Thanks to this transformation, it is possible to obtain a model whose sampling period is implicitly time-varying and always adapted to the engine speed (it avoids over-sampling in idle speed conditions). In the case of the air-fuel ratio, it has another advantage: the delay becomes constant.

Indeed, by considering a sampling value of $T_s = 180$, the variable transport delay of two engine turns becomes equal to four samples in the crank-angle domain [13]. By applying the recursive law (7) to the continuous-time model (5), the following air-fuel ratio model in the crank-angle domain is obtained:

$$\lambda^{\theta+} = \lambda^{\theta} + \frac{T_s^{\theta}}{6\tau n}\left(-K_2\lambda + \frac{K_1}{\lambda_s K_{inj}} \frac{\dot{m}_{air}(k-4)}{t_{inj}(k-4)-t_0}\right) \qquad (8)$$

In what follows, when possible, the index k is omitted. λ^{+} denotes $\lambda(k+1)$.

C. Takagi-Sugeno model

Since the model (8) involves nonlinearities, they have to be managed. Instead of using linearization, the sector nonlinearity methodology is used to obtain an exact representation of the nonlinear system (8). The method results in a nonlinear combination of several linear local models, triggered together with nonlinear membership functions $h_i(k)$. These membership functions verify the property of convex sum, i.e.

$$h_i \in [0,1] \quad \sum_{i=1}^{r} h_i = 1 \qquad (9)$$

The interested reader can find all the details in [9]. The TS model is composed of 2 nonlinearities: $NL_1(k) = \frac{T_s^{\theta} K_2(k)}{6\tau n(k)}$ and $NL_2(k) = \frac{T_s^{\theta} K_1(k)\dot{m}_{air}(k-4)}{6\times 14.67\tau K_{inj} n(k)}$. It is a SISO system, i.e. the state vector is a scalar $x^{\theta}(k) = \lambda^{\theta}(k)$. The following transformation is considered to obtain a model affine in control: $u^{\theta}(k) = \frac{1}{t_{inj}(k)-t_0}$. As in [13] and with the notation introduced in (3), it is possible to write the TS model of the nonlinear system (8):

$$x^{\theta}(k+1) = A_z \cdot x^{\theta}(k) + B_z \cdot u^{\theta}(k-4) \qquad (10)$$

where $A_z = 1 - NL_1(k)$, $B_z = NL_2(k)$ and the local models:

$$\begin{matrix} A_1 = 0.98 & A_2 = 0.95 & A_3 = A_1 & A_4 = A_2 \\ B_1 = 5.7\times 10^{-5} & B_2 = B_1 & B_3 = 2.7\times 10^{-4} & B_4 = B_3 \end{matrix} \qquad (11)$$

IV. Controller Design

A. Integral action

Now that the exact TS representation (10) has been obtained, it is possible to design a controller by using the Lyapunov direct method to assess the stability of the whole closed-loop system. The advantage is that what is proved on the TS model is also true on the nonlinear system (8).

Since the aim of controlling the fuel injection is to maintain the air-fuel ratio in stoichiometric proportions, i.e. tracking the reference $\lambda_{ref} = 1$, and to reject external disturbances such as a change on the air mass inside the cylinder $\dot{m}_{air}(k)$, an integral action has to be included. Indeed, if the air-fuel ratio can reach the reference in static conditions thanks to an additional gain as in [13], an integral action can compensate for modelling errors during the transient phase as well as estimation errors that could appear in the identification process.

Therefore, the state vector $x^{\theta}(k)$ is augmented with the integral of the error $e(k) = \lambda_{ref} - \lambda^{\theta}(k)$, so the augmented state stands for $\bar{x}^{\theta}(k) = \begin{bmatrix} x^{\theta}(k) & e(k) \end{bmatrix}^T$ and the matrices in the TS model (10) are also augmented:

$$\bar{x}^{\theta}(k+1) = \bar{A}_z^{\theta} \cdot \bar{x}^{\theta}(k) + \bar{B}_z^{\theta} \cdot u^{\theta}(k-4) \qquad (12)$$

In what follows, the model (12) is considered.

B. Handling the delay

The LMI conditions for a PDC controller with the augmented state in [13] leads to an unfeasible problem. Because these conditions are too conservative, this paper proposes another methodology in order to reduce the conservativeness. The main contribution of this paper is to deal with the delay by including it in the state vector as a dynamical extension. Let us consider the new states $\{v_1(k), \ldots, v_4(k)\}$ such that:

$$\begin{cases} \bar{x}^{\theta}(k+1) = \bar{A}_z^{\theta} \cdot \bar{x}^{\theta}(k) + \bar{B}_z^{\theta} \cdot v_1(k) \\ v_1(k+1) = v_2(k) = u_{\lambda}^{\theta}(k-3) \\ \vdots \\ v_4(k+1) = u^{\theta}(k) \end{cases} \qquad (13)$$

Thanks to such a transformation, the delay no longer appears on the control input but it is included in the state vector $\breve{x}^{\theta}(k) = \begin{bmatrix} \bar{x}^{\theta}(k) & u^{\theta}(k-4) & \cdots & u^{\theta}(k-1) & e(k) \end{bmatrix}^T$. The state space representation (12) becomes:

$$\breve{x}^{\theta}(k+1) = \breve{A}_z^{\theta} \cdot \breve{x}^{\theta}(k) + \breve{B}_z^{\theta} \cdot u^{\theta}(k) \qquad (14)$$

Now, thanks to the simple structure of (14), the control law can be designed.

C. Controller design

Since the main objective is real-time implementation and to include industrial constraints, the control law is chosen as a linear state-feedback:

$$u^{\theta}(k) = -F^{\theta} \cdot \breve{x}^{\theta}(k) \tag{15}$$

The closed-loop system is:

$$\breve{x}^{\theta}(k+1) = \breve{A}_z^{\theta} \cdot \breve{x}^{\theta}(k) - \breve{B}_z^{\theta} \cdot F^{\theta} \cdot \breve{x}^{\theta}(k) \tag{16}$$

and its stability is assessed by the direct Lyapunov method with the following quadratic Lyapunov function:

$$V(k) = \breve{x}^{\theta}(k)^T \cdot P \cdot \breve{x}^{\theta}(k) \tag{17}$$

Since the delay no longer appears in the input signal, LMI conditions are easily obtained by adapting results from [20]. The closed-loop system is stable if the following quantity:

$$\Upsilon_i = \begin{bmatrix} \beta \cdot X & (*) \\ \breve{A}_i^{\theta} \cdot X - \breve{B}_i^{\theta} \cdot F^{\theta} & X \end{bmatrix} \tag{18}$$

with $X = P^{-1}$ verifies $\Upsilon_i > 0, \forall i \in \{1, ..., r\}$ for a given β, where $\beta = 1 - \alpha^2$ is the decay-rate such that the Lyapunov function should verify $V^+ < -\alpha^2 V$.

V. Simulation Results

In order to obtain a more realistic simulation without adding any artificial noise, the model is fed with measured signals from the engine test bench, such as the engine speed $n(t)$ or the amount of air $\dot{m}_{air}(t)$. To assess the efficiency of the proposed controller, a PI controller is designed to make a comparison:

$$u_{PI}(t) = K_p \cdot \left(1 + \frac{1}{T_i \cdot p}\right) \tag{19}$$

with $K_p = -3000$ and $T_i = 0.25$. The PI is tuned to obtain an empirical trade-off between performance and oscillations. A Smith predictor structure adapted to time-varying delays is added so that the comparison between the two controllers is fair. By computing the LMI conditions with a decay-rate of $\beta = 0.985$, the following control gains are obtained:

$$F^{\theta} = \begin{bmatrix} 2675 & 0.43 & 0.41 & 0.39 & 0.39 & -4340 \end{bmatrix} \tag{20}$$

The control law (15) with the gains (20) is implemented in simulation to control the nonlinear continuous-time model (5). Three scenarios are designed. The first one assesses the reference tracking ability of both controllers at idle speed conditions. The control of the throttle, so the amount of air, is realized by a non-PDC controller designed as in [21] and implemented in the engine test bench. Figures 2 presents the results of the simulation of the air-fuel ratio reference tracking. As one can see, the proposed controller converges more quickly to the reference and keeps closer.

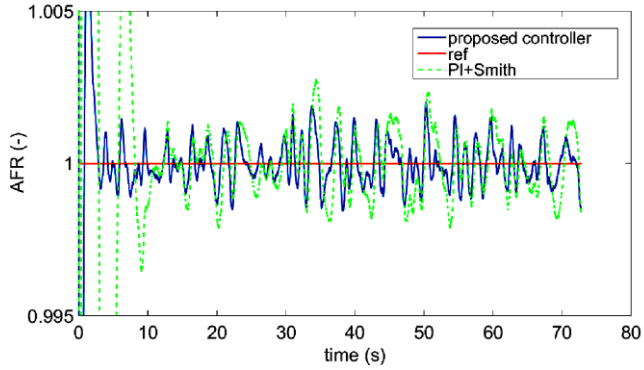

Fig. 2. Simulation results of the air-fuel ratio reference tracking

A second scenario is designed to evaluate the ability of the controllers to reject external disturbances. Figure 3 presents the engine speed profile that is realized on the engine test bench. This signal is injected in the simulation signals as disturbance.

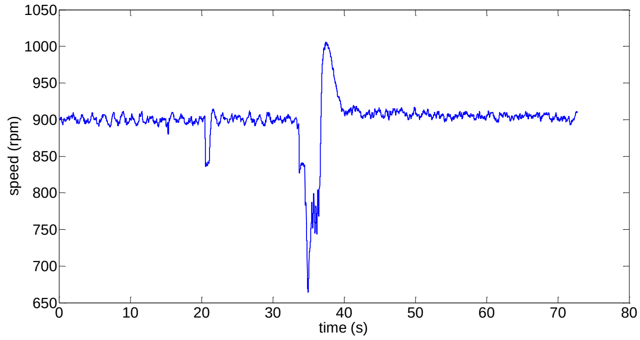

Fig. 3. Measured engine speed used for the idle speed scenario

Figure 4 and 5 present the air-fuel ratio output signal from the two models controlled by the PI + Smith controller and the proposed one, and the associated command signals. As one can see, the proposed controller has less overshoot than the PI + Smith controller and it recovers faster, meaning that it keeps the air-fuel ratio closer to the stoichiometric proportions, improving the efficiency of the catalytic converter.

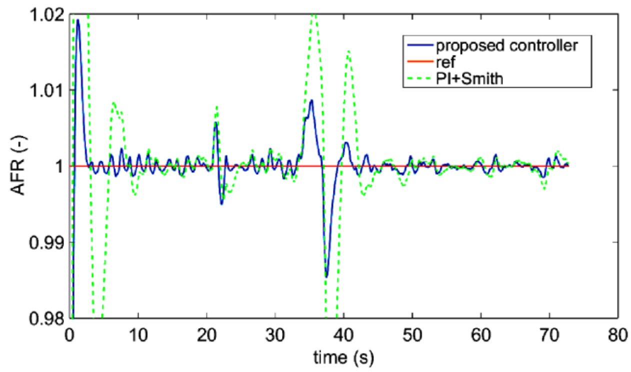

Fig. 4. Disturbance rejection results

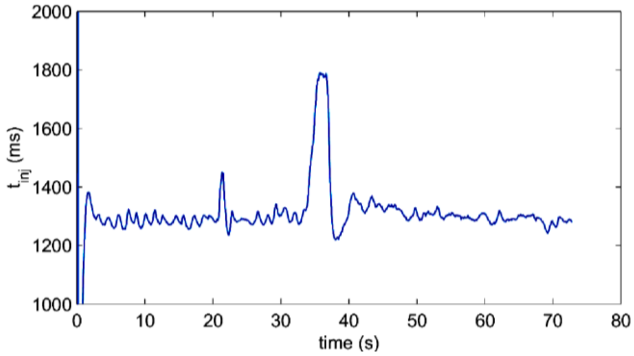

Fig. 5. Command signal of the injection timing

The third scenario assesses the ability of the controllers to handle variations of the transport delay. Figure 6 presents the engine speed profile realized on the engine test bench and used in simulation. Large variations of speed have been done with the driver pedal to simulate realistic driving conditions. Since the variable transport delay expression only depends on the engine speed, see equation (4), the delay is therefore varying.

Figure 7 and 8 present the air-fuel ratio and the command signal of the injection timing. As one can see, the proposed controller which is triggered according to the crank-angle sampling value $T_s = 180$ and which includes the fixed delay in its state, is less impacted by variations of the transport delay. It has less overshoot than the PI controller and its Smith predictor structure adapted for time-varying delays.

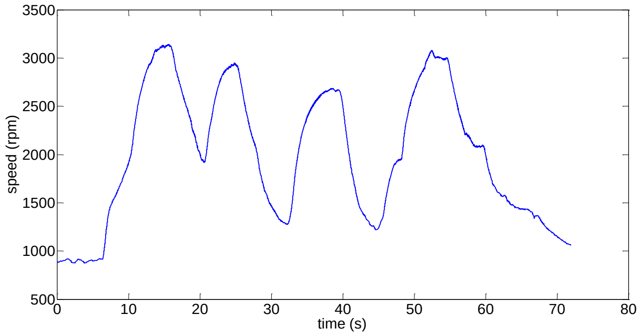

Fig. 6. Measured engine speed used for the varying speed scenario

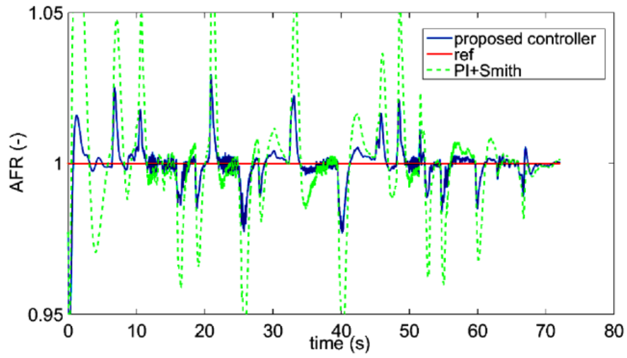

Fig. 7. Simulation results of the air-fuel ratio control in case of varying speed

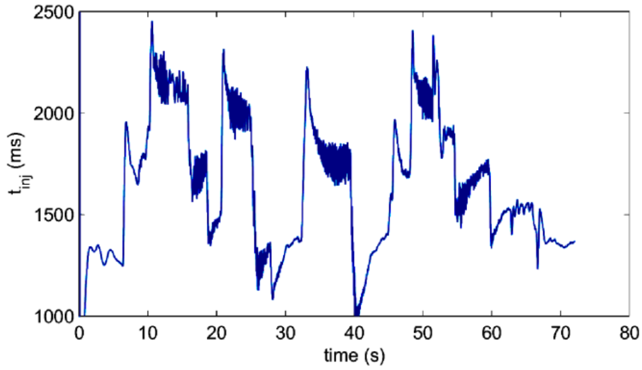

Fig. 8. Command signal of the injection timing

In conclusion, in all the three scenarios, the proposed controller performs better than the benchmark one.

VI. CONCLUSIONS

This paper has presented an original way to systematically design a controller for a system suffering from variable transport delay. This controller has been developed in the applicative context of the air-fuel ratio control of a gasoline engine. Once the model has been established and identified, a transformation to the crank-angle domain has been realized to fix the variable transport delay. The Takagi-Sugeno representation has been used to handle the nonlinear behavior. This paper proposed a methodology less conservative than the existing ones to design a controller by integrating both the delay and the error in the state. A linear state-feedback controller has been designed by applying the Lyapunov direct method and by solving LMI conditions. Simulations results have highlighted the efficiency and the interest of the proposed methodology compared with a PI controller in addition to a Smith predictor structure adapted for time-varying delays. Now that simulations have validated the ability of the controller to maintain the air-fuel ratio at the stoichiometric reference and to reject external disturbances, the controller can be implemented into the electronic control unit of the engine test bench to realize real-time experiments.

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