# Maximizing guaranteed inter-event time in control of TS fuzzy systems

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**Abstract:** This paper analyses the problem of maximizing of the inter-event time when controlling nonlinear systems represented by Takagi-Sugeno fuzzy models. First a linear and a fuzzy controller and corresponding triggering conditions are designed. To maximize the guaranteed minimum inter-event time, an optimization problem is formulated. The results are illustrated on a numerical example.

*Keywords:* fuzzy systems, event triggered control, minimum guaranteed inter-event time

## 1. INTRODUCTION

Nowadays controllers for various processes are implemented on embedded microprocessors with access to a limited power supply, or processes are controlled through networks with limited bandwidth. Considering the constraints, the interest in event-triggered control (ETC) increased, as this method can conserve communication resources and prevent network congestion for network controlled systems (Tolić and Hirche, 2017).

Starting with the publications of Årzén (1999), research concerning ETC led to the development of numerous methods. For example, Tabuada (2007) investigated a method based on input-to-state stability and designed triggering conditions. This approach avoided the Zeno behavior and presented a way to compute the inter-event time.

To provide longer inter-event times, Girard (2015) proposed dynamic ETC, while Maalej et al. (2012) developed a dynamic triggering law based on the system states. This idea is similar to self triggered control, as the event is using only the previous state information. Extensive research on self triggering control was performed by Heemels et al. (2012), Zhang and Fridman (2021), etc. Recent results related to general event trigger controllers have been developed by Espitia et al. (2021).

A primary problem related to ETC is the Zeno behavior and how to avoid it (Lygeros et al., 2003). Zeno behavior means that the controller attempts to infinitely trigger in a finite time, which is not possible. To avoid this problem, when designing the ETC, a positive minimum inter-event time (MIET) should be imposed (Zhang and Fridman, 2021). Asymptotic convergence can be affected by imposing the MIET, leading to practical convergence (Berneburg and Nowzari, 2021). Another method to assure a positive MIET is to use a self triggered control (Anta and Tabuada, 2010) or periodic event-triggered control (Heemels et al., 2013). An analysis of the relationship between MIET and convergence rate was done by Zhang et al. (2024). Ghodrat and Marquez (2020) designed an event-triggered control for linear systems that can guarantee MIET in the case when the system is affected by disturbance. In a similar direction, Borgers and Heemels (2013) studied how disturbances affect the MIET. Borgers and Heemels (2014) extended this research and studied the conditions for a positive lower bound on MIET.

The majority of ETC algorithms use time-invariant feedback. To enhance the control performance for linear time-invariant systems, a linear time-varying controller based on Riccati-type differential equations was developed by Inoue et al. (2011). Ahmed-Ali et al. (2016) proved that, in contrast to constant gain, time-varying gain can increase the maximum sampling interval. In general, while the design tends to be more challenging, ETC algorithms with time-varying gain increase the control performance and conserve communication resources.

Results related to ETC for general nonlinear systems that consider a guaranteed computed MIET are limited, as the interest is mainly to prove its existence, but not the actual computation of a positive lower bound.

A way to represent nonlinear systems is using Takagi-Sugeno (TS) fuzzy models, usually in a bounded region of the state-space. The main advantage of TS models is that the local linear models are combined in a convex structure. Using this representation, event-triggered fuzzy control designs have been explored based on techniques like backstepping (Rathnayake and Diagne, 2024), sliding mode (Xu et al., 2025b), fault tolerant control (Fan and Yang, 2018), adaptive control (Xu et al., 2025a), memory-based event-triggered control (Hou and Dong, 2024), etc.

In all these results the MIET is not computed directly and it can only be observed from the simulations. However, the MIET has an important role when looking at the the practical feasibility of an ETC to be implemented on a hardware device with limited resources. In fact, for

⋆ This work was supported by the project "Romanian Hub for Artificial Intelligence-HRIA", Smart Growth, Digitization and Financial Instruments Program, MySMIS no. 351416.

choosing the actual hardware, a lower bound on the inter-event intervals is necessary. Our current work is motivated by this issue and focuses on the design of a controller that guarantees and maximizes the MIET. The contributions of this research are:

- a MIET is guaranteed for the controllers developed for TS fuzzy models
- conditions to design the controller together with the optimization problem to maximize the MIET are formulated

The structure of this paper is the following: Section 2 contains the problem statement, and the necessary background. Sufficient conditions for controller design are presented in Section 3. The optimization problem is formulated in Section 4. The developed conditions are illustrated on a numerical example in Section 5. Conclusions are drawn in Section 6.

## 2. PRELIMINARIES AND PROBLEM STATEMENT

**Notations**. Consider a real symmetric matrix $F = F^T \in R^{n\times n}$. We denote with $F > 0$ ($F < 0$) if $F$ is positive (negative) definite definite. We denote the identity matrix with $I$ and the zero matrix with 0. The transposed quantity in a symmetric position is indicated by the symbol $(*)$, i.e. $\begin{pmatrix} P & (*) \\ A & P \end{pmatrix} = \begin{pmatrix} P & A^T \\ A & P \end{pmatrix}$, and $A + (*) = A + A^T$. For $x \in \mathbb{R}^{n_x}$, $\|x\|$ represents the Euclidean norm. To simplify the notations, $F_{z(t)} = \sum_{i=1}^{r} q_i(z(t))F_i$ is used to represent convex sums. Nonlinear membership functions $q_i$, $i = 1, ..., r$, have the property $q_i \in [0, 1]$, $i = 1, \ldots, r$, $\sum_{i=1}^{r} q_i(z(t)) = 1$, $\forall z(t)$.

In a set $\mathcal{D} \subset \mathbb{R}^{n_x}$ of the state-space, with $0 \in \mathcal{D}$ we consider the model

$$\dot{x}(t) = A_{z(t)}x(t) + B_{z(t)}u(t) \tag{1}$$

with the state vector $x(t) \in \mathbb{R}^{n_x}$ and the control input $u(t) \in \mathbb{R}^{n_u}$. The vector of scheduling variables that depends on the states are denoted by $z$. We use $r$ for the number of linear models, and $A_{z(t)} = \sum_{i=1}^{r} q_i(z(t))A_i$, $B_{z(t)} = \sum_{i=1}^{r} q_i(z(t))B_i$.

Our goal is to design an ETC of the form

$$u(t) = -K_{z(t_k)}x(t_k) \quad t \in [t_k, t_{k+1}) \tag{2}$$

where $t_k$, $k = 0, 1, \ldots$ are the triggering instants such that the controller stabilizes the system and the inter-event time, $\tau_k = t_{k+1} - t_k$, $k = 0, 1, \ldots$ is maximized.

To simplify the notation, the explicit continuous time-dependence of the variables is not indicated, i.e., $x(t)$ will be noted as $x$. The dependence on the triggering instant is specified, i.e. $x(t_k)$ represents the state value at time $t_k$.

To design the controller we will use the following result. Let a dynamic system given as:

$$\dot{x} = f(x, e) \tag{3}$$

where $e$ denotes the errors affecting the system. Then,

*Definition 1.* (Tabuada, 2007) A smooth function $V : \mathbb{R}^n \to \mathbb{R}^+$ is said to be an ISS Lyapunov function for system (3) if there exist $\kappa_\infty$ functions $\underline{w}$, $\overline{w}$, $w$, and $\gamma$ satisfying

$$\underline{w}(\|x\|) \le V(x) \le \overline{w}(\|x\|)$$
$$\frac{\partial V}{\partial x} f(x, e) \le -w(\|x\|) + \gamma(\|e\|)$$

## 3. CONTROLLER DESIGN

Our goal is to design controller (2) and to optimize its gains such that the closed-loop system is asymptotically stable and the inter-event time $\tau_k$ is maximized. Two control structures are proposed: a linear and a fuzzy controller. The linear one has the advantage of simpler design and reduced implementation complexity. Moreover, it requires minimal assumptions and provides a simpler triggering condition. We also consider a fuzzy controller to provide more flexibility and reduce conservatism, but at the cost of increased computational complexity. To design the controllers, Lyapunov synthesis will be used, employing a common quadratic Lyapunov function of the form $V = x^T P^{-1} x$ with $P = P^T > 0$.

### *3.1 Linear controller*

We start with a linear controller i.e., $K_i = K$, $i = 1, 2, \ldots, r$, resulting in

$$u(t) = -Kx(t_k) \quad t \in [t_k, t_{k+1}) \tag{4}$$

The closed-loop system can be written as:

$$\dot{x} = A_z x + B_z u(t_k) = A_z x - B_z K x(t_k) \tag{5}$$

where $K$ is the controller gain to be determined, together with the triggering instants $t_k$, $k = 0, 1, \ldots$. The dynamics (5) can be expressed as

$$\dot{x} = (A_z - B_z K)x + B_z K e \tag{6}$$

where $e = x - x(t_k)$ denotes the error. The derivative of the Lyapunov function $V = x^T P^{-1} x$ is:

$$\dot{V} = x^T P^{-1}(A_z - B_z K)x + x^T P^{-1} B_z K e + (*) \tag{7}$$

where $(*)$ represents the symmetric part of all the terms before it. $\dot{V}$ is bounded as:

$$\dot{V} \le -a x^T x + b\|x\|\|e\| \tag{8}$$

if there exists $a > 0$ so that $P^{-1}(A_z - B_z K) + (*) \le -aI$, $\forall z(t)$, and $b \ge \max_z \|P^{-1}(B_z K)\|$, $b > 0$. Then, $\dot{V} < 0$ as long as $-a\|x\|^2 + b\|x\|\|e\| < 0$, i.e. $\|e\| < \frac{a}{b}\|x\|$. Thus, the triggering condition $\|e\| \ge \sigma\|x\|$ from (Tabuada, 2007) is obtained for $\sigma \in (0, \frac{a}{b})$.

Using the same approach as Tabuada (2007) for the dynamics (6), we bound $\dot{x}$ as:

$$\|\dot{x}\| \le \alpha\|x\| + \alpha\|e\| \tag{9}$$

for some $\alpha > 0$ such that

$$\alpha \ge max(\|A_z - B_z K\|, \|B_z K\|)$$

holds $\forall z$. As at the triggering instant $t_k$, the error is reset $e(t_k) = 0$, the time required for $\|e\|$ to reach $\sigma\|x\|$ or, $\frac{\|e\|}{\|x\|}$ to reach $\sigma$ is (Tabuada, 2007) the inter-event time $\tau_k = t_{k+1} - t_k$. Considering the dynamics of $\frac{\|e\|}{\|x\|}$, recalling that $e = x - x(t_k)$, and using (9), we have:

$$\begin{aligned}
\frac{d}{dt}\frac{\|e\|}{\|x\|} &= \frac{d}{dt}\frac{(e^T e)^{\frac{1}{2}}}{(x^T x)^{\frac{1}{2}}} \\
&= \frac{1}{x^T x}\left(\frac{e^T \dot{x}\|x\|}{\|e\|} - \frac{x^T \dot{x}\|e\|}{\|x\|}\right) \\
&\le \alpha\left(1 + \frac{\|e\|}{\|x\|})\right)\left(1 + \frac{\|e\|}{\|x\|}\right).
\end{aligned}$$

With the notation $y = \frac{\|e\|}{\|x\|}$, the above inequality is transformed into $\dot{y} \leq \alpha(1+y)^2$. Let $\phi(t, \phi_0)$ be the solution of $\dot{\phi} = \alpha(1+\phi)^2$, having $\phi(0, \phi_0) = \phi_0 = 0$. Using the comparison lemma (Khalil, 2002) we get $y(t) \leq \phi(t, \phi_0)$. Since $\phi(t, 0) = \frac{\alpha t}{1-\alpha t}$ we obtain

$$\tau = \frac{\sigma}{\alpha(1+\sigma)} \tag{10}$$

The following theorem summarizes this result:

*Theorem 2.* The event-triggered controller (4) asymptotically stabilizes (1) using the triggering law $\|e\| \geq \sigma\|x\|$, with $\sigma \in (0, \frac{a}{b})$, if there exist matrices $P = P^T > 0$ and $K$, and $a$, $b$, and $\alpha$ positive constants, so that the following conditions hold $\forall z$:

$$\begin{aligned} &P^{-1}(A_z - B_z K) + (*) \leq -aI \\ &2\|P^{-1}B_z K\| \leq b \\ &\alpha \geq \max(\|A_z - B_z K\|, \|B_z K\|) \end{aligned} \tag{11}$$

Furthermore, the minimum guaranteed interevent time is at least $\tau = \frac{\sigma}{\alpha(1+\sigma)}$.

### 3.2 Fuzzy controller

Consider now the fuzzy controller:

$$u = \sum_{i=1}^{r} q_i(z(t_k))K_i x(t_k) = K_{z(t_k)}x(t_k),\ t \in [t_k, t_{k+1}). \tag{12}$$

In this case there is a mismatch between the controller gains calculated continuously, $K_z$, and the gains applied to the system when the triggering is enabled, $K_{z(t_k)}$. This mismatch is denoted by $\Delta K = K_z - K_{z(t_k)}$, $t \in [t_k, t_{k+1})$, and we include it into the triggering function. The closed-loop system is expressed as:

$$\begin{aligned} \dot{x} &= (A_z - B_z K_z)x + B_z K_z e + B_z \Delta K x(t_k) \\ &= (A_z - B_z K_z)x + (B_z K_z\ \ B_z)\begin{pmatrix} x - x(t_k) \\ \Delta K x(t_k) \end{pmatrix} \end{aligned} \tag{13}$$

The derivative of the Lyapunov function $V = x^T P^{-1} x$ along the trajectories of (13) can be written as:

$$\begin{aligned} \dot{V} &= x^T P^{-1}(A_z - B_z K_z)x + \\ &+ x^T P^{-1}(B_z K_z\ \ B_z)\begin{pmatrix} x - x(t_k) \\ \Delta K_z x(t_k) \end{pmatrix} + (*) \end{aligned} \tag{14}$$

where $(*)$ denotes the symmetric part. $V$ is now an ISS Lyapunov function in the sense of Definition 1 for system (13) with respect to the error $\tilde{e} = \begin{pmatrix} x - x(t_k) \\ \Delta K x(t_k) \end{pmatrix}$.

Assuming that there exists $a > 0$ so that $P^{-1}(A_z - B_z K_z) + (*) \leq -aI$, $\forall z(t)$, (14) can be bounded as:

$$\dot{V} \leq -a x^T x + b\|x\|\|\tilde{e}\| \tag{15}$$

where $b \geq \max_z \|P^{-1}(B_z K_z\ \ B_z)\|$, $b > 0$. Thus, in this case, the triggering condition $\|\tilde{e}\| = \sigma\|x\|$, with $\sigma \in (0, \frac{a}{b})$ ensures the negativity of $\dot{V}$.

The dynamics $\dot{x}$ can again be bounded as (9) where $\alpha > 0$ is a positive constant such that

$$\alpha \geq \max(\|A_z - B_z K_z\|, \|(B_z K_z\ \ B_z)\|) \tag{16}$$

hold $\forall z$. The guaranteed MIET is expressed as (10), with $\alpha$ given in (16), and the following result is formulated:

*Theorem 3.* The event-triggered controller (2) asymptotically stabilizes the system (1) using the triggering law $\|\left((x - x(t_k))^T\ \ (\Delta K_z x(t_k))^T\right)\| \geq \sigma\|x\|$, with $\sigma \in (0, \frac{a}{b})$, if there exist matrices $P = P^T > 0$ and $K_i$, $i = 1, 2, \ldots, r$, and positive constants $a$, $b$, and $\alpha$ so that $\forall z$ the following conditions hold:

$$\begin{aligned} &P^{-1}(A_z - B_z K_z) + (*) \leq -aI \\ &2\|P^{-1}(B_z K_z\ \ B_z)\| \leq b \\ &\alpha \geq \max(\|A_z - B_z K_z\|, \|(B_z K_z\ \ B_z)\|) \end{aligned} \tag{17}$$

The guaranteed MIET is at least $\tau = \frac{\sigma}{\alpha(1+\sigma)}$.

## 4. OPTIMIZATION PROBLEM

Our goal is to maximize the inter-event time, i.e.

$$maximize \quad \tau = \frac{\sigma}{\alpha(1+\sigma)} \tag{18}$$

with $\sigma \in (0, \frac{a}{b})$ so that the following constraints are satisfied $\forall z$ for the linear controller:

$$\begin{aligned} &P^{-1}(A_z - B_z K) + (*) \leq -aI \\ &2\|P^{-1}B_z K\| \leq b \\ &\alpha \geq \max(\|A_z - B_z K\|, \|B_z K\|) \end{aligned} \tag{19}$$

while for the fuzzy controller we have the constraints

$$\begin{aligned} &P^{-1}(A_z - B_z K) + (*) \leq -aI \\ &2\|P^{-1}B_z K\| \leq b \\ &\alpha \geq \max(\|A_z - B_z K\|, \|(B_z K_z\ \ B_z)\|) \end{aligned} \tag{20}$$

The decision variables are the elements of $P$ and $K$ matrices. We solve the optimization problem with two methods: a gradient based method (Waltz et al., 2006) and the global optimization method *PSO* (Kennedy and Eberhart, 1995). We use the functions *fmincon* and *particleswarm* readily implemented in Matlab. *Fmincon* uses a gradient-based algorithm and offers fast convergence, but it may converge to a local optimum. *PSO* is a global optimization method, but has the drawback of increased computational effort.

### 4.1 Gradient-based method

As neither *fmincon*, nor *PSO* can take matrices as decision variables, we combine the matrices $P$ and $K$ in a vector as follows. Consider the elements in $P = P^T > 0$ and $K$:

$$P = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ (*) & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ (*) & (*) & \cdots & p_{nn} \end{bmatrix} \quad K = \begin{bmatrix} k_{11} & k_{12} & \cdots & k_{1n} \\ k_{21} & k_{22} & \cdots & k_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ k_{m1} & k_{m2} & \cdots & k_{mn} \end{bmatrix}.$$

The decision vector is built as:

$$\hat{x} = \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1n} & p_{22} & \cdots & p_{2n} & \cdots & p_{nn} \\ k_{11} & k_{12} & \cdots & k_{1n} & \cdots & k_{m1} & k_{m2} & \cdots & k_{mn} \end{pmatrix}$$

To obtain a feasible initial point, we generate random values for the upper triangular part of the $P_0$ matrix, i.e. $[\ p_{11}\ p_{12}\ \cdots\ p_{1n}\ p_{22}\ \cdots\ p_{2n}\ \cdots p_{nn}]$. After that, we we build the matrix.

To ensure that the initial starting point is feasible and stabilizes the closed-loop system, we verify that $P_0 > 0$ and solve

$$(A_z - B_z K_0)P_0 + (*) \leq 0 \tag{21}$$

to obtain $K_0$. If (21) is unfeasible or $P_0 < 0$, a new random $P_0$ will be generated and the entire process is redone until a solution is found. Having now the initial matrices $P_0$ and $K_0$ we can build the initial point $\hat{x}_0$.

Recall the objective function (18) that we want to maximize. To compute the initial inter-event time, constraints (19) are transformed into equations:

$$\begin{aligned} a &= -\max(eig(P^{-1}(A_z - B_zK) + (*))) \\ b &= 2\|P^{-1}B_zK\| \\ \alpha &= \max(\|A_z - B_zK\|, \|B_zK\|) \end{aligned} \tag{22}$$

We take $\sigma = \frac{a}{b}$, thus the objective function is $\tau = \frac{a}{\alpha(a+b)}$. *fmincon* considers by default that the nonlinear constraints are $< 0$, thus we define them as:

$$\begin{aligned} c_1 &= \max(eig((P^{-1}(A_z - B_zK) + (*)))) \\ c_2 &= -2\|P^{-1}B_zK\| \\ c_3 &= -\max(\|A_z - B_zK\|, \|B_zK\|) \\ c_4 &= \max(-eig(P)). \end{aligned} \tag{23}$$

After the optimization is run, we obtain the optimal vector $\hat{x}_{opt}$. We build the matrices $P_{opt}$ and $K_{opt}$ and we perform a stabilization check on the closed loop system (6). If the closed loop system is stable, we compute the inter event time $\tau$ and save the parameters to be used in the simulation. If the closed loop system is unstable, the entire process is redone. As this method finds a local optimum, we run the optimization several times from different, randomly chosen initial points.

Let us now consider the fuzzy controller (12). $P$ has the same structure as before and the $i^{th}$ controller gain $K_i$, $i = 1, 2, \dots r$ is:

$$K_i = \begin{bmatrix} k_{11}^{(i)} & k_{12}^{(i)} & \cdots & k_{1n}^{(i)} \\ k_{21}^{(i)} & k_{22}^{(i)} & \cdots & k_{2n}^{(i)} \\ \vdots & \vdots & \ddots & \vdots \\ k_{m1}^{(i)} & k_{m2}^{(i)} & \cdots & k_{mn}^{(i)} \end{bmatrix}$$

The decision vector is therefore modified as:

$$\hat{x} = \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1n} & p_{22} & \cdots & p_{2n} & \cdots & p_{nn} \\ k_{11}^{(1)} & k_{12}^{(1)} & \cdots & k_{1n}^{(1)} & \cdots & k_{m1}^{(1)} & k_{m2}^{(1)} & \cdots & k_{mn}^{(1)} \\ \cdots & k_{11}^{(r)} & k_{12}^{(r)} & \cdots & k_{1n}^{(r)} & \cdots & k_{m1}^{(r)} & k_{m2}^{(r)} & \cdots & k_{mn}^{(r)} \end{pmatrix} \tag{24}$$

The steps for the generation of the initial point are the same. We have to modify (22) as:

$$\begin{aligned} a &= -\max(eig(P^{-1}(A_z - B_zK_z) + (*))) \\ b &= 2\|P^{-1}\left(B_zK_z \;\; B_z\right)\| \\ \alpha &= \max(\|A_z - B_zK\|, \|\left(B_zK_z \;\; B_z\right)\|) \end{aligned} \tag{25}$$

and we change the constraints (23) into:

$$\begin{aligned} c_1 &= \max(eig(P^{-1}(A_z - B_zK) + (*))) \\ c_2 &= -2\|P^{-1}\left(B_zK_z \;\; B_z\right)\| \\ c_3 &= -\max(\|A_z - B_zK\|, \|\left(B_zK_z \;\; B_z\right)\|) \\ c_4 &= \max(-eig(P)) \end{aligned} \tag{26}$$

As everything is defined now for the fuzzy case, we can run *fmincon* for the objective function (18) together with the constraints (26) to get the optimal vector $\hat{x}_{opt}$. The decision matrices $P_{opt}$ and $K_{z_{opt}}$ are built from the optimized vector and a stabilization check of the closed loop system is done. As before, if the closed-loop system is stable, the matrices are saved and the inter-event time $\tau$ is calculated, otherwise a new initial point is generated and the entire process is redone.

### *4.2 PSO based approach*

Particle swarm optimization (PSO) is a population-based optimization technique. A possible solution is represented by a particle that moves in the search space. The movement is influenced by the particle's own best position and the swarm best position.

For the actual optimization we use the *particleswarm* function from Matlab which generates an initial swarm automatically. Lower and upper bound are set to limit the explored space. The vector of decision variables is built as in Section 4.1, and the goal is to maximize the objective function (18) together with the constraints (22).

An important difference from the *fmincon* is that *particleswarm* can not consider directly the nonlinear constraints. Thus, a penalization function is needed that considers both the constraints and the objective function. We define this function as:

$$f(\hat{x}) = \begin{cases} \tau, & \text{if (23) is satisfied} \\ -10^5, & \text{otherwise} \end{cases} \tag{27}$$

Similarly to Section 4.1, once we obtain the the optimal vector $\hat{x}_{opt}$, we build the matrices $P_{opt}$ and $K_{opt}$ from the vector and we perform a stabilization check on the closed loop system (6). If the closed loop system is stable, we compute the inter event time $\tau$ and save the parameters to be used in the simulation.

For the fuzzy controller (12) the algorithm is the same, but we consider a fuzzy $K_z$, thus changing the decision vector into (24), and correspondingly modifying the penalty function (27).

## 5. EXAMPLE

The results of inter-event time maximization are illustrated in this section on the numerical example from Su et al. (2021). We set the initial conditions to $x = [0.5\ 0.4]^T$ and consider the exogenous disturbance to be 0 resulting the fuzzy model:

$$\dot{x}(t) = A_{z(t)}x(t) + B_{z(t)}u(t)$$

with 2 rules,

$$A_1 = \begin{bmatrix} -1.2 & 0.3 \\ 1 & -0.8 \end{bmatrix} A_2 = \begin{bmatrix} 0.6 & -0.5 \\ -1.2 & -0.4 \end{bmatrix},$$

$$B_1 = [0\ 2]^T,\ B_2 = B_1,$$

$$q_1(x_1) = \frac{1 - \sin(x_1)}{2}, \quad q_2(x_1) = \frac{1 + \sin(x_1)}{2}$$

The open loop is not asymptotically stable, thus a controller is needed for stabilization. Since *fmincon* is a gradient-based method which depends on the starting point, we ran the algorithm for 60 initial points. For the linear controller case, the best result obtained with *fmincon* is $\tau_{opt} = 0.0079$ and the corresponding gain matrix is: $K = (-2.5757\ 0.7362)$. The inter-event times can be seen in Fig. 1. In the simulation, the actual inter-event time is 0.0254 i.e., the guaranteed MIET is satisfied. It can be observed in Fig. 2 that the closed-loop system is stabilized.

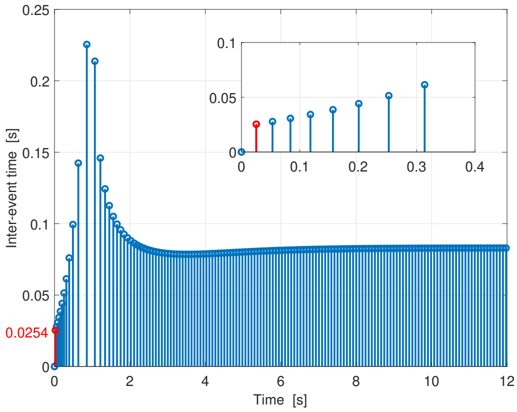


Fig. 1. Inter-event times for linear controller and *fmincon*

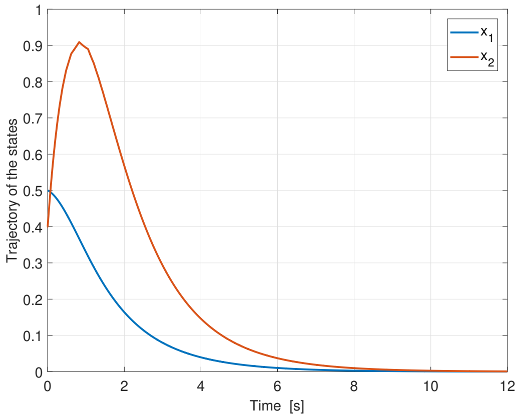


Fig. 2. Trajectory of the closed-loop system

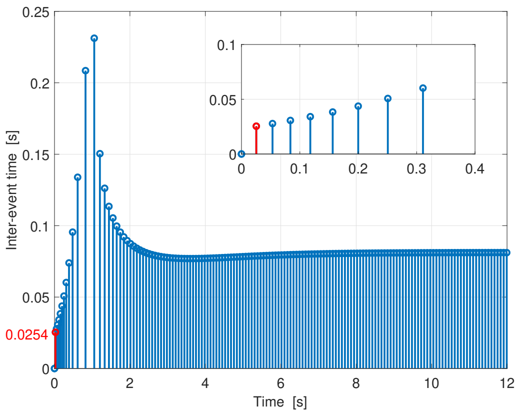


Fig. 3. Inter-event times for linear controller and *PSO*

Considering the optimization with *particleswarm* for the linear controller, the closed-loop system is stabilized with the controller gain: $K = (-2.5381\ 0.7180)$ and $\tau_{opt} = 0.0079$. In Fig. 3 we can see the inter-event times for a particular trajectory, with the minimum being $\tau = 0.0254$, thus almost the same result is obtained as with *fmincon*.

For the fuzzy controller with *fmincon* we obtain the controller gains: $K_1 = (-2.8128\ 0.8461)$ and $K_2 = (-2.7343\ 1.0362)$. Fig. 4 shows the actual inter-event times, with the smallest 0.0279 being greater than the guaranteed $\tau_{opt} = 0.0079$.

Using *particleswarm* function for the fuzzy controller, the obtained gains are: $K_1 = (-2.7347\ 0.7590)$ and $K_2 =$

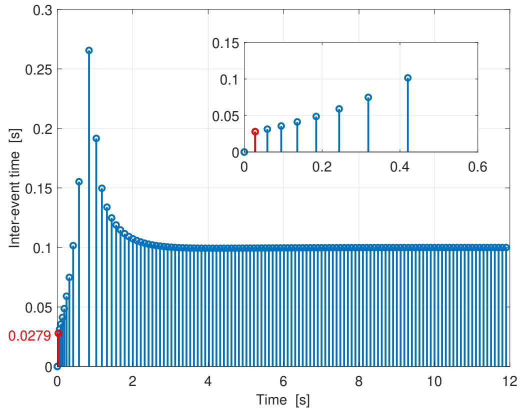


Fig. 4. Inter-event times for fuzzy controller and *fmincon*

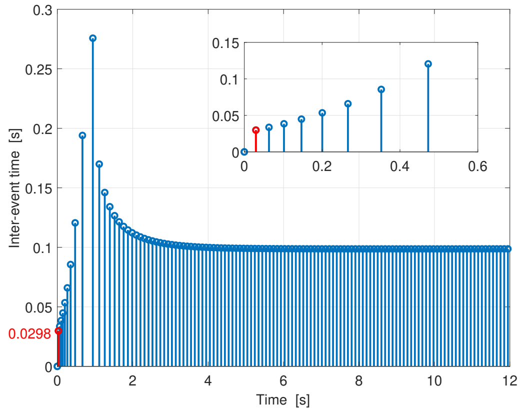


Fig. 5. Inter-event times for fuzzy controller and *PSO*

$(-2.6023\ 1.0794)$. The inter-event times are presented in Fig. 5. The guaranteed MIET is $\tau_{opt} = 0.008$ while the simulated one is $\tau = 0.0298$.

As can be seen, there is a significant gap between the guaranteed MIET and the inter-event times observed in simulation. This is due to the fact that the results are conservative (e.g., due to the common quadratic Lyapunov function, or using the same bound on $\|A_z - B_z K\|$ and$\|B_z K\|$) and the guaranteed MIET is actually a worst-case lower bound, rather than a prediction of performance. Considering the case of PSO optimization, the small difference between the linear and fuzzy controllers may be due to the fact that a linear controller suffices. However, in the simulations, a slightly better MIET is obtained by the fuzzy controller.

Note that the controller from (Su et al., 2021), although it depends on the triggering instants, it is computed and applied continuously based on the current error, see eq. (33) in (Su et al., 2021). Applying that controller in an event-triggered manner, i.e., with $u(t) = u(t_k)$, $t \in [t_k, t_{k+1})$, leads to a significantly reduced inter-event time, as shown in Fig. 6.

## 6. CONCLUSIONS

This paper developed event-triggered linear and fuzzy state feedback controllers for TS fuzzy systems. An optimization problem to maximize the inter-event time is formulated. A gradient-based and a global optimization

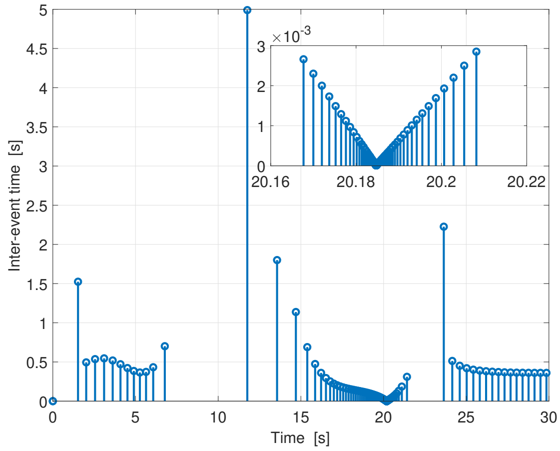


Fig. 6. Inter-event times for sliding mode control

method are used to solve it. The proposed design is illustrated on a numerical example. In the future, we will strive to reduce the conservativeness of the design.

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