Estimation for control – Practical Assignment 3
Discrete-time models, state feedback control

Logistics

- This practical assignment should be carried out by a group of maximum two students.
- The assignment solution consists of Matlab code and Simulink model. This code will be checked and run by the teacher during the lab class, and your attendance to the lab will only be registered if you have a working, original solution. Validated attendances for all the labs are necessary for eligibility to the exam. Moreover, at most two labs can be recovered at the end of the semester, which means accumulating three or more missing labs at any point during the semester automatically leads to final ineligibility.
- Discussing ideas among students is encouraged; however, directly sharing and borrowing pieces of code is forbidden, and any violation of this rule will lead to disqualification of the solution.

Assignment description

Prerequisite:

- A nonlinear model, with state vector $x = [x_1,\, x_2, ...x_n]^T$, and input vector $u = [u_1,\, u_2, ...u_m]^T$ where $n \geq 3$, is the number of state variables, $m \geq 1$, is the number of inputs, and the dynamic equation has the form:

  $$\begin{aligned} \dot{x} =& f(x,u) \\ y =& Cx, \end{aligned} \tag{1}$$

  where $f(x) = [f_1(x,u), f_2(x,u), ...f_n(x,u)]^T$ is a vector functions with at least one nonlinear term. It is assumed that all the states are measurable, and directly available so $C = I$ (identity matrix of appropriate dimension).
- nonlinear model – implementation in Matlab-Simulink
- linear state-space model

  $$\begin{aligned} \dot{x} =& Ax + Bu \\ y =& Cx + Du, \end{aligned} \tag{2}$$

  where $C = I$ as in (1), and $D = 0$ (zero matrix of appropriate dimension).
- state-feedback control
- comparison between linear and nonlinear

Requirements:

- discrete-time nonlinear model
- discrete-time linear model

$$
\begin{aligned}
x(k+1) &= Ax(k) + Bu(k) \\
y(k) &= Cx(k)
\end{aligned}
$$

- testing continuous-time state-feedback controller on discrete-time model
- discrete-time state feedback controller
- comparison between discrete-time and continuous-time models

Hint: For the discrete-time model the following approxiamtion can be used:

$$
\dot{x} \simeq \frac{x(k+1) - x(k)}{T_s}, \tag{3}
$$

where T_s is the sampling time.

Based on (3) equation (1) can be rewritten as:

$$
\frac{x(k+1) - x(k)}{T_s} = f(x(k), u(k)) \quad \Rightarrow \quad x(k+1) \quad = x(k) + T_s \left(f(x(k), u(k)) \right).
$$

Keep in mind that, in order to obtain stability for the discrete-time model, the eigen-values of the closed loop system have to be inside of the unit circle (not negative, as it was in the continuous-time case).