Estimation for control – Practical Assignment 9
Unknown input decoupling observer and fault detection

Logistics

- This practical assignment should be carried out by a group of maximum two students.
- The assignment solution consists of Matlab code and Simulink model. This code will be checked and run by the teacher during the lab class, and your attendance to the lab will only be registered if you have a working, original solution. Validated attendances for all the labs are necessary for eligibility to the exam. Moreover, at most two labs can be recovered at the end of the semester, which means accumulating three or more missing labs at any point during the semester automatically leads to final ineligibility.
- Discussing ideas among students is encouraged; however, directly sharing and borrowing pieces of code is forbidden, and any violation of this rule will lead to disqualification of the solution.

Unknown input decoupling observer design

In this laboratory we are going to study what happens when the system dynamics is disturbed with an unknown input signal. The difference compared to the previous lab is that now we do not need the restriction that the disturbance is constant. The decoupled observer can be used to treat systems with any disturbances. We consider the following model for the system:

$$\begin{aligned} \dot{x} &= A\,x + B\,u + E\,d \\ y &= C\,x, \end{aligned} \tag{1}$$

where $d \in \mathbb{R}^{n_d}$ is the disturbance vector and E is the disturbance matrix. The observer has the form:

$$\begin{aligned} \dot{z} &= F\,z + TB\,u + K\,y \\ \hat{x} &= z + H\,y, \end{aligned} \tag{2}$$

where z is an auxiliary state, $\hat{x}$ is the estimate of x and F, T, K and H will be defined later. The objective here is to estimate the system states, which means that $e = x - \hat{x} \to 0$, when $t \to \infty$. The error dynamics can be written in the following way:

$$\begin{aligned} \dot{e} =& \dot{x} - \dot{\hat{x}} = \; Ax + Bu + Ed - \big[\dot{z} + H\dot{y}\big] = \\ & Ax + Bu + Ed - \big[Fz + TBu + Ky + HC\dot{x}\big] = \\ & Ax + Bu + Ed - \big[Fz + TBu + Ky + HC(Ax + Bu + Ed)\big] \end{aligned} \tag{3}$$

We separate the term $K = K_1 + K_2$, so that, we have $Ky = K_1Cx + K_2\,y$. We collect the matrices for the signals to obtain:

$$\dot{e} = (A - K_1C - HCA)x + (B - TB - HCB)u + (E - HCE)d - Fz - K_2\,y. \tag{4}$$

In order to introduce e in the equation we subtract and add the $\hat{x}$ multiplied with the matrices of x:

$$\begin{aligned}\dot{e} =&(A-K_1C-HCA)x-(A-K_1C-HCA)\hat{x}+(A-K_1C-HCA)\hat{x}\\&+(B-TB-HCB)u+(E-HCE)d-Fz-K_2\,y=\\&(A-K_1C-HCA)e+(A-K_1C-HCA)\underbrace{(z+Hy)}_{\hat{x}}\\&+(B-TB-HCB)u+(E-HCE)d-Fz-K_2\,y=\\&(A-K_1C-HCA)e+(A-K_1C-HCA-F)z+\big((A-K_1C-HCA)H-K_2\big)\,y\\&+(B-TB-HCB)u+(E-HCE)d.\end{aligned} \tag{5}$$

At this point we obtained a complicated form for the system dynamics with many free terms. Next, we want to define the free terms so that the final equation is:

$$\dot{e} = (A-K_1C-HCA)e, \tag{6}$$

which means, the rest of the matrices are 0 (note that, the notation 0 defines a zero matrix of appropriate dimension):

$$\begin{aligned}A-K_1C-HCA-F =&0\\(A-K_1C-HCA)H-K_2 =&0\\B-TB-HCB =&0\\E-HCE =&0,\end{aligned} \tag{7}$$

From where we obtain:

$$\begin{aligned}F =&TA-K_1C\\K_2 =&FH\\T =&I-HC\\H =&E(CE)^*,\end{aligned} \tag{8}$$

where $(CE)^*$ is the pseudo-inverse of CE, which means $CE(CE)^* = I$ (use Matlab `pinv(CE)`). The main advantage of this method is that, the estimation error e is converging to 0 as $t \to \infty$ regardless of the presence of the unknown input.

Two conditions are necessary to be fulfilled in order to make this observer work.

- in order to compute H, we need the condition that $\mathrm{rank}(E) = \mathrm{rank}(CE)$
- the pair (TA, C) should be fully state observable to compute K_1, such that F is Hurwitz (all the eigenvalues are negative).

Fault detection

This observer can also be used to detect if a fault appears. We can treat actuator and sensor faults. Lets consider the following two models:

$$\begin{aligned}\dot{x} =&A\,x+B\,u+E\,d\\y =&C\,x+f_s,\end{aligned} \tag{9}$$

where f_s is the sensor fault. We denote the residual with

$$r := y - C\hat{x} = Cx + f_s - C\hat{x} = Ce + f_s. \tag{10}$$

We have direct access to the residual, because y is the measured output, $C\hat{x}$ is computed. The e is 0 in steady state with no sensor fault, which means also r is zero in this case. When a faulty measurement appears, we can see the effect on the residual.

Next lets consider the case when we have an actuator fault.

$$\begin{aligned} \dot{x} =& A\,x + B\,u + E\,d + B f_a \\ y =& C\,x, \end{aligned} \tag{11}$$

where f_a is the actuator fault. The error dynamics with (11) is similar to (6) plus the effect of the fault:

$$\dot{e} = Fe + TBf_a. \tag{12}$$

Based on (12) we can see that even if F is Hurwitz, the error dynamics will not converge to 0 due to the effect of the actuator fault f_a. If the error is not zero in steady-state, then also the residual cannot be 0.

Example

Consider the following linear model:

$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} &= \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ -2 & -3 & 0 \\ 0 & 0 & -2 \end{bmatrix}}_{\text{A}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 \\ 4 & 0 \\ 0 & 1 \end{bmatrix}}_{\text{B}} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 2 \\ 0 \\ 0.5 \end{bmatrix}}_{\text{E}} d \\ y &= \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{C}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}. \end{aligned} \tag{13}$$

Based on the model we can see that the disturbance has an effect on both x_1 and x_3. The rank condition is fulfilled $\operatorname{rank}(E) = \operatorname{rank}(CE) = 1$, so based on (8) we obtain:

$$H = \begin{bmatrix} 0.94 & 0.23 \\ 0 & 0 \\ 0.23 & 0.05 \end{bmatrix}, \quad T = \begin{bmatrix} 0.05 & 0 & -0.23 \\ 0 & 1 & 0 \\ -0.23 & 0 & 0.94 \end{bmatrix}. \tag{14}$$

The (TA, C) pair is observable, which leads to:

$$K_1 = \begin{bmatrix} 1.97 & 0.32 \\ -2.04 & -0.02 \\ -0.16 & -0.86 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -1.82 & -0.45 \\ 0.04 & 0.01 \\ -0.08 & -0.02 \end{bmatrix}, \quad F = \begin{bmatrix} -1.97 & 0.05 & 0.14 \\ 0.04 & -3 & 0.02 \\ 0.16 & -0.23 & -1.02 \end{bmatrix}. \tag{15}$$

So the obtained unknown input decoupling observer has the form:

$$\dot{z} = \underbrace{\begin{bmatrix} -1.97 & 0.05 & 0.14 \\ 0.04 & -3 & 0.02 \\ 0.16 & -0.23 & -1.02 \end{bmatrix}}_{F} z + \underbrace{\begin{bmatrix} 0.05 & 0 & -0.23 \\ 0 & 1 & 0 \\ -0.23 & 0 & 0.94 \end{bmatrix}}_{T} \underbrace{\begin{bmatrix} 0 & 0 \\ 4 & 0 \\ 0 & 1 \end{bmatrix}}_{B} u$$

$$+ \left(\underbrace{\begin{bmatrix} 1.97 & 0.32 \\ -2.04 & -0.02 \\ -0.16 & -0.86 \end{bmatrix}}_{K_1} + \underbrace{\begin{bmatrix} -1.82 & -0.45 \\ 0.04 & 0.01 \\ -0.08 & -0.02 \end{bmatrix}}_{K_2} \right) y \tag{16}$$

$$\hat{x} = z + \underbrace{\begin{bmatrix} 0.94 & 0.23 \\ 0 & 0 \\ 0.23 & 0.05 \end{bmatrix}}_{H} y,$$

Requirements:

- Design a linear continuous-time unknown input decoupling observer
- Test the obtained observer with a sinusoidal disturbance input d
- Plot the error (e) and the residual (r)
- Apply a constant disturbance after 10 seconds of normal working on one of the measured outputs (just on the output!)
- Can you see the fault on the sensor by looking at the residual r?
- Apply a constant disturbance after 10 seconds on one of the inputs
- Can you see the fault on the actuator by looking at the residual r?
- Test the obtained observer with a piece-wise constant disturbance d

Hint: Useful Matlab functions `obsv`, `place`.