Estimation for control – Practical Assignment 1
Nonlinear models

Logistics

- This practical assignment should be carried out by a group of maximum two students.
- The assignment solution consists of Matlab code and Simulink model. This code will be checked and run by the teacher during the lab class, and your attendance to the lab will only be registered if you have a working, original solution. Validated attendances for all the labs are necessary for eligibility to the exam. Moreover, at most two labs can be recovered at the end of the semester, which means accumulating three or more missing labs at any point during the semester automatically leads to final ineligibility.
- Discussing ideas among students is encouraged; however, directly sharing and borrowing pieces of code is forbidden, and any violation of this rule will lead to disqualification of the solution.

Assignment description

In this assignment you need to find a nonlinear model of a real system with state vector $x = [x_1,\ x_2, ...x_n]^T$, and input vector $u = [u_1,\ u_2, ...u_m]^T$ where $n \geq 3$, is the number of state variables, $m \geq 1$, is the number of inputs. The dynamic equation has the form:

$$\dot{x} = f(x, u)$$

where $f(x) = [f_1(x, u), f_2(x, u), ...f_n(x, u)]^T$ is a vector function with at least one nonlinear term. A numerical example which fulfills these requirements can be the following. Consider the state vector $x = [x_1\, x_2\, x_3]^T$ and the model:

$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} &= f(x_1, x_2, x_3, u) = \begin{bmatrix} f_1(x_1, x_2) \\ f_2(x_1, x_2, x_3) \\ f_3(x_2, u) \end{bmatrix} = \begin{bmatrix} x_1 + 2x_2 \\ x_1 x_3 + x_2^2 \\ x_2 + u \end{bmatrix} \\ y &= x_1 \end{aligned} \tag{1}$$

As it can be seen the model is described by three equations, and the second equation contains two nonlinear terms: $x_1 x_3$ and x_2^2.

Possible issues what you may encounter during the search for models:

- The model is not directly described using the notation x, but with some other variable names, like θ, α etc. In this case you need to rename them, to obtain the classical form.
- The model is described with a second (or higher) order differential equation, for instance:

  $$\ddot{\alpha} = sin(\alpha) + \dot{\alpha} + \tau$$

  In this case you need to convert it into a a system of differential equations containing only first order terms. For instance by denoting $x_1 = \alpha$, $x_2 = \dot{\alpha}$ and $u = \tau$, we obtain:

  $$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \sin(x_1) + x_2 + \tau \end{aligned}$$

Requirements:

- Find a physical system with at least 3 states and having a nonlinear model. Determine the meaning of the states, inputs and outputs (measurements).
- Implement the model in Simulink, using a Matlab function block
- Test your model with no input and with several initial conditions, find the physical meaning of the evolution of the states
- Test your model with zero initial condition, and several input signals, different from zero. Find the physical meaning.