Controller design for time-delay TS fuzzy systems with nonlinear consequents

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***Abstract*—This paper proposes a controller design method for time-delay Takagi-Sugeno fuzzy models with nonlinear consequents. We assume that both the input and the membership functions are affected by the delay. The nonlinearities in the consequents are handled by a slope-bounded condition. The controller design conditions are formulated as linear matrix inequalities. A numerical example illustrates the results.**

I. Introduction

There are numerous control systems that involve physical time lag. For instance, time-delay, also called dead-time, happens when sensors and actuators are not co-located. This phenomenon can appear in transportation [7], biological systems [1], networked control systems [9], etc. Time-delays are frequently non-negligible factors, a source of instability, thus, it is important to take them into account in the analysis and the design of the control systems.

The research concerning stability and stabilizing control of time-delay systems has been intensified in the past years. Lyapunov's direct method is an efficient way to analyse the stability of systems with time-delay. A complex overview on stability analysis for time-delay systems is presented in [5]: for slowly-varying and fast-varying delay, delay-dependent and delay-independent conditions, for both linear and nonlinear cases. The nonlinearities in [5] are handled directly, which makes the design conditions restrictive. Another way to handle model nonlinearities is the Takagi-Sugeno (TS) fuzzy modelling.

TS models are able to represent a large class of nonlinear time-delay systems as convex combination of local linear models. Stabilization and design conditions are generally formulated as Linear Matrix Inequalities (LMIs). In the last years the concept of time-delayed TS fuzzy models has become an important subject of interest. For example [16] developed stability conditions based on a quadratic convex combination using an augmented Lyapunov-Krasovskii functional for TS fuzzy systems with time-varying delay. A robust stabilization method for nonlinear time-delay systems has been proposed in [17], where a PDC controller has been used and the conditions have been developed based on an impulsive-time-dependent Lyapunov function. Improved stability and stabilization conditions for TS fuzzy system with time-varying delay have been presented in [6], where an augmented solution has been used to achieve delay-and-its-derivative-dependent stability conditions.

The mentioned methods do not include delay in the input, although they consider delay in the states. A delay in the input leads to delay in the membership functions of the controller, which motivates the research presented hereafter.

The TS models we consider are with nonlinear consequents. The purpose of this technique is to reduce the number of fuzzy rules and to handle nonlinearities that depend on unmeasured states. In the literature this idea of separating the nonlinearities has been exploited, e.g. in [3], [4], [10], [11], [12], but not for time-delay systems. In this paper we use a slope-bounded condition for handling the nonlinear part of the consequent.

Moreover, we consider TS models in the presence of time-varying input delay and assume that the membership functions may depend on both current and delayed states. In our previous research [13] we have considered slowly-varying delay. In this paper we tackle the problem of fast-varying delay, and propose conditions that depend on the maximum delay and the maximum of its derivative.

The structure of this paper is as follows: Section 2 reviews the necessary concepts for TS fuzzy systems and their basic properties. The design conditions are presented in Section 3. Section 4 illustrates the delayed condition on a numerical example. Section 5 concludes the paper.

Notations. Let $F = F^T \in R^{n\times n}$ be a real symmetric matrix; $F > 0$ and $F < 0$ mean that F is positive definite and negative definite, respectively. I denotes the identity matrix and 0 the zero matrix of appropriate dimensions. The symbol $*$ in a matrix indicates a transposed quantity in the symmetric position, for instance $\begin{pmatrix} P & * \\ A & P \end{pmatrix} = \begin{pmatrix} P & A^T \\ A & P \end{pmatrix}$, and $A + * = A + A^T$. The notation $\mathrm{diag}(f_1, ..., f_n)$, where $f_1, ..., f_n \in \mathbb{R}$, stands for the diagonal matrix, whose diagonal components are $f_1, ..., f_n$. $\|x\|$, where $x \in \mathbb{R}^{n_x}$, is the Euclidean norm of x. Throughout this paper, the following shorthand notations are used to represent convex sums of matrix expressions:

$$F_z = \sum_{i=1}^{s} q_i(z(t))F_i, \tag{1}$$

$$F_{z_\tau} = \sum_{i=1}^{s} q_i(z(t-\tau(t)))F_i, \tag{2}$$

$$F_{zz_\tau} = \sum_{i=1}^{s} q_i(z(t)) \sum_{j=1}^{s} q_j(z(t-\tau(t)))F_{ij}, \tag{3}$$

where q_i, $i = 1, ..., s$ are nonlinear functions called the membership functions with the property:

$$q_i \in [0,\, 1],\, i = 1, ..., s, \quad \sum_{i=1}^{s} q_i(z) = 1. \tag{4}$$

II. Preliminaries and problem statement

The time-delay TS fuzzy model that we consider with nonlinear consequents:

$$\begin{aligned}\dot{x}(t) =& A_{zz_\tau} x(t) + D_{zz_\tau} x(t-\tau(t)) \\ &+ B_{zz_\tau} u(t-\tau(t)) + B_{zz_\tau} G\psi(Hx(t)),\end{aligned} \tag{5}$$

where $A_{zz_\tau} \in \mathbb{R}^{n_x \times n_x}$, $D_{zz_\tau} \in \mathbb{R}^{n_x \times n_x}$, $B_{zz_\tau} \in \mathbb{R}^{n_x \times n_u}$ and $G \in \mathbb{R}^{n_u \times r}$ represent the model matrices, i.e., $\sum_{i=1}^{s}\sum_{j=1}^{s} q_i(z(t))q_j(z(t-\tau(t)))A_{ij}x(t)$, etc.; $x(t) \in \mathbb{R}^{n_x}$ is the state vector, $u(t) \in \mathbb{R}^{n_u}$ is the control input, s is the number of rules, $z(t) \in \mathbb{R}^{n_z}$ is the premise vector; $\tau(t)$ is the varying time-delay, where τ is differentiable, $\dot{\tau} \leq d$, $d \in [0,\, 1)$ is a given constant, $\tau \leq h$, $h > 0$ is the maximum time-delay.

The quantity $\psi(Hx(t)) \in \mathbb{R}^r$ is an r-dimensional vector where $H \in \mathbb{R}^{r \times n_x}$ and each entry is a function of a linear combination of the states, i.e.

$$\psi_i = \psi_i(\sum_{j=1}^{n} H_{ij}x_j), \quad i = 1, ..., r.$$

To develop our results, the elements in the vector $\psi(Hx(t))$ must fulfill the following assumption.

Assumption 1: For any $i \in \{1, ..., r\}$ there exist constants $0 < b_i \leq \infty$, so that

$$0 \leq \frac{\psi_i(v) - \psi_i(w)}{v - w} \leq b_i, \quad \forall v, w \in \mathbb{R}, v \neq w. \tag{6}$$

As in [2], in view of (6), there exist $\delta_i(t) \in [0,\, b_i]$, so that for any v, $w \in \mathbb{R}$

$$\psi_i(v) - \psi_i(w) = \delta_i(t)(v - w). \tag{7}$$

We denote $\delta(t) = \mathrm{diag}(\delta_1(t), ..., \delta_r(t))$.

In this paper we consider state-feedback controller design and we assume that all the states are available. For simplicity in what follows we omit in the notation the explicit time dependence of the delay, i.e. we use τ instead of $\tau(t)$.

To develop our results the following lemmas and property are used.

Lemma 1: Let A and B be matrices of appropriate dimensions and ranks, with $B = B^T > 0$. Then

$$-A^T B^{-1} A \leq -A - A^T + B$$

Lemma 2 (Congruence): Given matrix $P = P^T$ and a full column rank matrix Q, it holds that

$$P > 0 \quad \Rightarrow \quad QPQ^T > 0.$$

Property 1: (Schur complement) Let $\mathcal{M} = \mathcal{M}^T = \begin{bmatrix} M_{11} & M_{12} \\ M_{12}^T & M_{22} \end{bmatrix}$, with M_{11} and M_{22} square matrices of appropriate dimensions. Then:

$$\begin{aligned}\mathcal{M} < 0 \Leftrightarrow & \begin{cases} M_{11} < 0 \\ M_{22} - M_{12}^T M_{11}^{-1} M_{12} < 0 \end{cases} \\ \Leftrightarrow & \begin{cases} M_{22} < 0 \\ M_{11} - M_{12} M_{22}^{-1} M_{12}^T < 0 \end{cases}\end{aligned} \tag{8}$$

The design conditions will be defined as a triple sum negativity problem having the form

$$\sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{k=1}^{s} q_i(z(t))\, q_j(z(t-\tau))\, q_k(z(t-\tau))F_{ijk} < 0, \tag{9}$$

with symmetric matrices F_{ijk}, and nonlinear functions q_i, $i = 1, ..., s$, satisfying the convex sum property in (4).

Sufficient LMI conditions are obtained using the following Lemma:

Lemma 3 ([15]): Equation (9) is satisfied if the following conditions hold

$$\begin{aligned} F_{ijj} &< 0 \\ \frac{2}{s-1}F_{ijj} + F_{ijk} + F_{ikj} &< 0 \quad \forall i, j, k = 1, ..., s,\, j \neq k. \end{aligned} \tag{10}$$

III. Main results

In this section we develop sufficient conditions for controller design. To this end, the following control law is considered:

$$u(t) = -K_z x(t) - G\psi(Hx(t)), \tag{11}$$

where $K_z = \sum_{k=1}^{s} q_k(z(t))K_k$ are the controller gains. Based on (5) and (11), the closed loop system is:

$$\begin{aligned}\dot{x}(t) =& A_{zz_\tau}x(t) + D_{zz_\tau}x(t-\tau) + B_{zz_\tau}G\psi(Hx(t)) \\ &+ B_{zz_\tau}(-K_{z_\tau}x(t-\tau) - G\psi(Hx(t-\tau))) \\ =& A_{zz_\tau}x(t) + (D_{zz_\tau} - B_{zz_\tau}K_{z_\tau})x(t-\tau) \\ &+ B_{zz_\tau}G(\psi(Hx(t)) - \psi(Hx(t-\tau))).\end{aligned} \tag{12}$$

Furthermore, using Assumption 1 we obtain:

$$\begin{aligned}\psi(Hx(t)) - \psi(Hx(t-\tau)) =& \delta(t)\left(Hx(t) - Hx(t-\tau)\right) \\ =& \delta(t)H\left(x(t) - x(t-\tau)\right),\end{aligned} \tag{13}$$

and for simplification we denote $\eta := H\left(x(t) - x(t-\tau)\right)$. This leads to the following form for (12):

$$\begin{aligned}\dot{x}(t) =& A_{zz_\tau}x(t) + (D_{zz_\tau} - B_{zz_\tau}K_{z_\tau})x(t-\tau) \\ &+ B_{zz_\tau}G\delta(t)\eta \\ \eta =& H\left(x(t) - x(t-\tau)\right).\end{aligned} \tag{14}$$

To develop the design conditions, we consider the Lyapunov functional [5]:

$$
\begin{aligned}
V(t,x,\dot{x}) =& x^T(t)Px(t) + \int_{t-h}^{t} x^T(s)Sx(s)ds \\
&+ h\int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)R\dot{x}(s)dsd\theta \\
&+ \int_{t-\tau}^{t} x^T(s)Qx(s)ds
\end{aligned} \tag{15}
$$

The following result can be formulated:

Theorem 1: Consider the closed loop system (14), and assume that τ is differentiable, $\dot{\tau} \le d$, $d \in [0, 1)$ is a given constant, $\tau \le h$, $h > 0$ is the maximum time-delay. If there exist matrices $\tilde{P} = \tilde{P}^T > 0$, $\tilde{R} = \tilde{R}^T > 0$, $\tilde{S} = \tilde{S}^T > 0$, $\tilde{S}_{12}$, $\tilde{Q} = \tilde{Q}^T > 0$, $\tilde{M} = \mathrm{diag}(m_1, ..., m_r) > 0$, N_i, $i = 1, ..., s$ and constant $\tilde{\epsilon} > 0$, so that $\begin{bmatrix}\tilde{R} & \tilde{S}_{12}\\ * & \tilde{R}\end{bmatrix} \ge 0$ and Lemma 3 holds with (16), where

$$
\begin{aligned}
\tilde{\Sigma}_{33} =& -(1-d)\tilde{Q} - 2\tilde{R} + \tilde{S}_{12} + \tilde{S}_{12}^T \\
\nu(\tilde{M}) =& -2\tilde{M}\mathrm{diag}(\frac{1}{b_1}, ..., \frac{1}{b_r}),
\end{aligned} \tag{17}
$$

then the closed loop system (14) is asymptotically stable. The controller gains can be recovered from $K_i = N_i\tilde{P}^{-1}$, $i = 1, ..s$.

Proof: Consider the candidate Lyapunov-Krasovskii functional (15). The derivative of V is

$$
\begin{aligned}
\dot{V}(t,x,\dot{x}) =& x^T(t)P\dot{x}(t) + \dot{x}^T(t)Px(t) \\
&+ h^2\dot{x}^T(t)R\dot{x}(t) - h\int_{t-h}^{t} \dot{x}^T(s)R\dot{x}(s)ds \\
&+ x^T(t)[S+Q]x(t) - x^T(t-h)Sx(t-h) \\
&- (1-\dot{\tau}(t))x^T(t-\tau)Qx(t-\tau),
\end{aligned} \tag{18}
$$

where $P = P^T > 0$, $R = R^T > 0$, $S = S^T > 0$, $Q = Q^T > 0$.

Based on [5]

$$
-\int_{t-h}^{t} \dot{x}^T(s)R\dot{x}(s)ds \le -\begin{bmatrix}e_1\\ e_2\end{bmatrix}^T \begin{bmatrix}R & S_{12}\\ * & R\end{bmatrix} \begin{bmatrix}e_1\\ e_2\end{bmatrix} \tag{19}
$$

where $e_1 = x(t) - x(t-\tau(t)), e_2 = x(t-\tau(t)) - x(t-h)$ and $\begin{bmatrix}R & S_{12}\\ * & R\end{bmatrix} \ge 0$, for some $S_{12} \in \mathbb{R}^{n_x \times n_x}$.

Using $\dot{\tau}(t) \le d$ and denoting

$$
\chi := \begin{bmatrix} x(t) \\ x(t-h) \\ x(t-\tau) \\ \delta(t)\eta \end{bmatrix} \tag{20}
$$

we obtain

$$
\dot{V}(t,x,\dot{x}) \le \chi^T\Delta\chi \tag{21}
$$

where Δ is defined in (22) and $\mathcal{B} = D_{zz_\tau} - B_{zz_\tau}K_{z_\tau}$, $\mathcal{C} = B_{zz_\tau}G$, $\Sigma_{33} = -(1-d)Q - 2R + S_{12} + S_{12}^T$.

Next we use Assumption 1 to determine relaxed conditions for $\dot{V} < 0$. Consider the inequality:

$$
\chi^T\Delta\chi + \chi^T\theta\chi \le 0, \tag{23}
$$

where

$$
\theta = \begin{bmatrix} \epsilon I & 0 & 0 & H^TM \\ * & 0 & 0 & 0 \\ * & * & 0 & -H^TM \\ * & * & * & \nu(M) \end{bmatrix} \tag{24}
$$

and $M = \mathrm{diag}(m_1, ..., m_r) > 0$.

Let us now examine $\chi^T\theta\chi$:

$$
\begin{aligned}
\chi^T\theta\chi =& \epsilon x(t)^Tx(t) + 2(x(t)-x(t-\tau))^TH^TM\delta(t)\eta \\
&+ (\delta(t)\eta)^T\nu(M)\delta(t)\eta.
\end{aligned} \tag{25}
$$

Since $\eta = H(x(t) - x(t-\tau))$, we have

$$
\begin{aligned}
-\chi^T\theta\chi =& -\epsilon x(t)^Tx(t) - 2\eta^TM\delta(t)\eta - (\delta(t)\eta)^T\nu(M)\delta(t)\eta \\
=& -\epsilon\|x(t)\|^2 - 2\eta^T\left(M\delta(t) - \delta(t)^TM\mathrm{diag}(\tfrac{1}{b_1}, ..., \tfrac{1}{b_r})\delta(t)\right)\eta.
\end{aligned} \tag{26}
$$

Both M and $\delta(t)$ are diagonal matrices, so we can examine the terms:

$$
m_i\delta_i(t) - m_i\frac{1}{b_i}\delta_i(t)^2 = m_i\delta_i(t)\left(1 - \delta_i(t)\tfrac{1}{b_i}\right). \tag{27}
$$

The term $\delta_i(t) \in [0, b_i]$, so $\left(1 - \delta_i(t)\frac{1}{b_i}\right) \ge 0$, and since $m_i > 0$, the following holds:

$$
2\eta^T\left(M\delta(t) - \delta(t)^TM\mathrm{diag}(\tfrac{1}{b_1}, ..., \tfrac{1}{b_r})\delta(t)\right)\eta \ge 0. \tag{28}
$$

Finally, we obtain

$$
-\chi^T\theta\chi \le -\epsilon\|x(t)\|^2. \tag{29}
$$

Therefore, if $\chi^T\Delta\chi + \chi^T\theta\chi < 0$, then $\dot{V} < 0$.

To obtain LMI conditions, consider the matrix inequality $\Delta + \theta < 0$. Then, applying the Schur complement on (30) we get (31).

Using Lemma 2 with

$$
\begin{bmatrix} P^{-1} & 0 & 0 & 0 & 0 \\ 0 & P^{-1} & 0 & 0 & 0 \\ 0 & 0 & P^{-1} & 0 & 0 \\ 0 & 0 & 0 & M^{-1} & 0 \\ 0 & 0 & 0 & 0 & I^{-1} \end{bmatrix},
$$

where $\mathcal{A} = P^{-1}A_{zz_\tau}^T + * + \epsilon P^{-2} + P^{-1}(S+Q-R)P^{-1}$, (31) becomes (32). Next, using the Schur complement on ϵP^{-2}, denoting $\tilde{P} = P^{-1}$, $\tilde{S} = P^{-1}SP^{-1}$, $\tilde{Q} = P^{-1}QP^{-1}$, $\tilde{M} = M^{-1}$, $N_{z_\tau} = K_{z_\tau}P^{-1}$, $\tilde{\epsilon} = \frac{1}{\epsilon}$, $P\tilde{S}_{12}P = S_{12}$ and $P\tilde{R}P = R$ and using the inequality $-P^{-1}\tilde{R}P^{-1} \le \tilde{R} - 2\tilde{P}$, we obtain (33).

The conditions in Theorem 1 are obtained by applying Lemma 1 on (33) and congruence with $\begin{bmatrix}P^{-1} & 0\\ 0 & P^{-1}\end{bmatrix}$ on $\begin{bmatrix}R & S_{12}\\ * & R\end{bmatrix}$. ■

$$F_{ijk} =$$
$$\begin{bmatrix} \tilde{P}A_{ij}^T + * + \tilde{S} + \tilde{Q} - \tilde{R} & \tilde{S}_{12} & D_{ij}\tilde{P} - B_{ij}N_k + \tilde{R} - \tilde{S}_{12} & B_{ij}G\tilde{M} + \tilde{P}H^T & h\tilde{P}A_{ij} & \tilde{P} \\ * & -\tilde{R} - \tilde{S} & \tilde{R} - \tilde{S}_{12}^T & 0 & 0 & 0 \\ * & * & \tilde{\Sigma}_{33} & -\tilde{P}H^T & h\tilde{P}(D_{ij}^T - K_k^T B_{ij}^T) & 0 \\ * & * & * & \nu(\tilde{M}) & h\tilde{M}(B_{ij}G)^T & 0 \\ * & * & * & * & -\tilde{R} & 0 \\ * & * & * & * & * & -\tilde{\epsilon}I \end{bmatrix} \tag{16}$$

$$\Delta = \begin{bmatrix} A_{zz_\tau}^T P + * + h^2 A_{zz_\tau}^T R A_{zz_\tau} + S + Q - R & S_{12} & P\mathcal{B} + h^2 A_{zz_\tau}^T R\mathcal{B} + R - S_{12} & (P + h^2 A_{zz_\tau}^T R)\mathcal{C} \\ * & -R - S & R - S_{12}^T & 0 \\ * & * & \Sigma_{33} + h^2\mathcal{B}^T R\mathcal{B} & h^2\mathcal{B}^T R\mathcal{C} \\ * & * & * & h^2\mathcal{C}^T R\mathcal{C} \end{bmatrix} \tag{22}$$

$$\begin{bmatrix} A_{zz_\tau}^T P + * + S + Q - R + \epsilon I & S_{12} & P\mathcal{B} + R - S_{12} & PB_{zz_\tau}G + H^T M \\ * & -R - S & R - S_{12}^T & 0 \\ * & * & \Sigma_{33} & -H^T M \\ * & * & * & \nu(M) \end{bmatrix} + \begin{bmatrix} h^2 A_{zz_\tau}^T R A_{zz_\tau} & 0 & h^2 A_{zz_\tau}^T R\mathcal{B} & h^2 A_{zz_\tau}^T R B_{zz_\tau} G \\ * & 0 & 0 & 0 \\ * & * & h^2\mathcal{B}^T R\mathcal{B} & h^2\mathcal{B}^T R B_{zz_\tau} G \\ * & * & * & h^2 (B_{zz_\tau}G)^T R B_{zz_\tau} G \end{bmatrix} \leq 0 \tag{30}$$

$$\begin{bmatrix} A_{zz_\tau}^T P + * + S + Q - R + \epsilon I & S_{12} & P\mathcal{B} + R - S_{12} & PB_{zz_\tau}G + H^T M & hA_{zz_\tau}^T \\ * & -R - S & R - S_{12}^T & 0 & 0 \\ * & * & \Sigma_{33} & -H^T M & h\mathcal{B}^T \\ * & * & * & \nu(M) & h(B_{zz_\tau}G)^T \\ * & * & * & * & -R^{-1} \end{bmatrix} \leq 0 \tag{31}$$

$$\begin{bmatrix} \mathcal{A} & P^{-1}S_{12}P^{-1} & \mathcal{B}P^{-1} + P^{-1}(R - S_{12})P^{-1} & B_{zz_\tau}GM^{-1} + P^{-1}H^T & hP^{-1}A_{zz_\tau}^T \\ * & -P^{-1}(R + S)P^{-1} & P^{-1}(R - S_{12}^T)P^{-1} & 0 & 0 \\ * & * & P^{-1}\Sigma_{33}P^{-1} & -P^{-1}H^T & hP^{-1}\mathcal{B}^T \\ * & * & * & M^{-1}\nu(M)M^{-1} & hM^{-1}(B_{zz_\tau}G)^T \\ * & * & * & * & -R^{-1} \end{bmatrix} \leq 0 \tag{32}$$

$$\begin{bmatrix} \tilde{P}A_{zz_\tau}^T + * + \tilde{S} + \tilde{Q} - \tilde{R} & \tilde{S}_{12} & D_{zz_\tau}\tilde{P} - B_{zz_\tau}N_{z_\tau} + \tilde{R} - \tilde{S}_{12} & B_{zz_\tau}G\tilde{M} + \tilde{P}H^T & h\tilde{P}A_{zz_\tau}^T & \tilde{P} \\ * & -\tilde{R} - \tilde{S} & \tilde{R} - \tilde{S}_{12}^T & 0 & 0 & 0 \\ * & * & \tilde{\Sigma}_{33} & -\tilde{P}H^T & h\tilde{P}\mathcal{B}^T & 0 \\ * & * & * & \nu(\tilde{M}) & h\tilde{M}(B_{zz_\tau}G)^T & 0 \\ * & * & * & * & \tilde{R} - 2\tilde{P} & 0 \\ * & * & * & * & * & -\tilde{\epsilon}I \end{bmatrix} \leq 0 \tag{33}$$

IV. EXAMPLE

In this section we illustrate the use of the conditions of Theorem 1, then we compare our results with the results obtained using the conditions presented by [13]. In the literature the case when the input is delayed is rarely considered. A result for such a case is presented in [8], however, it considers TS models with classic consequents, not nonlinear ones.

To illustrate the conditions developed, consider the following nonlinear system:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-2 & -0.5\\ 0 & -6+\sin(x_1)\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}2 & 1\\ 0.9+0.1\sin(x_1) & 4+\sin(x_1(t-\tau))\end{bmatrix}\begin{bmatrix}x_1(t-\tau)\\ x_2(t-\tau)\end{bmatrix} + \begin{bmatrix}0\\ 0.75+0.25\sin(x_1)\end{bmatrix}(u(t-\tau)) + \begin{bmatrix}0\\ -0.375-0.125\sin(x_1)\end{bmatrix}(\alpha_1(x_1)+\alpha_2(x_2)), \tag{34}$$

where $\alpha_1(x_1)$ and $\alpha_2(x_2)$ are two nonlinear functions which satisfy Assumption 1. For the simulations we consider

$$\alpha_1(v) = \alpha_2(v) = \cos(v) + v, \tag{35}$$

and the constants that satisfy Assumption 1 are $b_1 = b_2 = 2$, but the obtained results are valid for any other nonlinear functions which satisfy Assumption 1 with b_1 and b_2. In the following we compare our approach to that of [13]. We analyze the maximum delay and variation of delay for which stabilization can be achieved.

For the rest of the nonlinearities we use the sector nonlinearity approach [14] and obtain the local matrices:

$$A_{11} = A_{12} = \begin{bmatrix}-2 & -0.5\\ 0 & -5\end{bmatrix},\ A_{21} = A_{22} = \begin{bmatrix}-2 & -0.5\\ 0 & -7\end{bmatrix},$$
$$D_{11} = \begin{bmatrix}2 & 1\\ 0.8 & 3\end{bmatrix},\ D_{12} = \begin{bmatrix}2 & 1\\ 0.8 & 5\end{bmatrix},\ G = \begin{bmatrix}-0.5 & 1\end{bmatrix},$$
$$D_{21} = \begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix},\ D_{22} = \begin{bmatrix}2 & 1\\ 1 & 5\end{bmatrix},\ H = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},$$
$$B_{11} = B_{12} = \begin{bmatrix}0\\ 0.5\end{bmatrix},\ B_{21} = B_{22} = \begin{bmatrix}0\\ 1\end{bmatrix},$$
$$q_1(z) = \frac{1-sin(z)}{2},\ q_2(z) = 1 - q_1(z),\ z = x_1.$$

The values of the maximum delay, h, and the maximum derivative of the delay, d, for which feasible solutions have been obtained are given in Fig. 4. As it can be seen, for small delays our approach gives feasible solutions for larger variations of the delay, while the approach in [13] gives feasible solutions for (very) slowly-varying large delays.

In what follows, we test the controller for a specific case. The time-delay function, τ, is varying with $\dot{\tau}(t) \le d = 0.5$, and has the form: $\tau(t) = 0.25 + 0.25\cos(2t)$, thus $h = 0.5$.

The initial conditions for the state vector is $x_0 = \begin{bmatrix}1 & 2\end{bmatrix}^T$. The open-loop system without the control is unstable. This can be seen in Fig 1.

Applying Theorem 1, the obtained control gains for $\tau(t) \le h = 0.5$ and $\dot{\tau}(t) \le d = 0.5$ are the following:

$$K_1 = \begin{bmatrix}6.84 & 4.82\end{bmatrix},\quad K_2 = \begin{bmatrix}6.88 & 7.35\end{bmatrix} \tag{36}$$

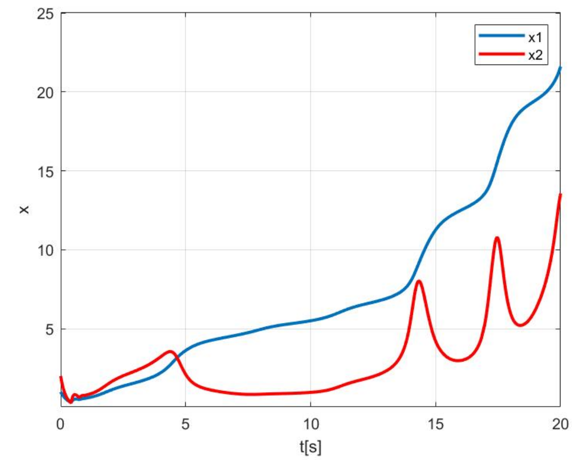

Fig. 1. Unstable open-loop system

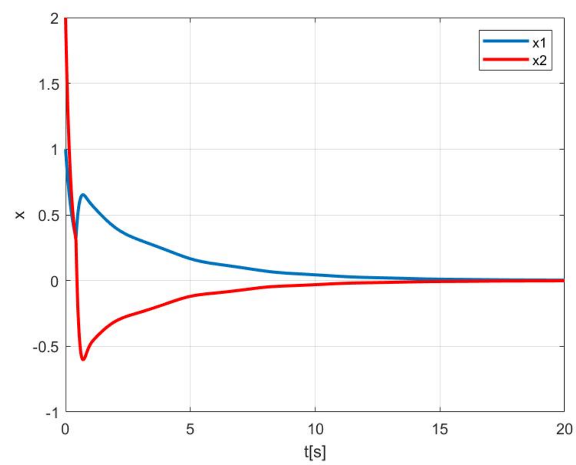

Fig. 2. Convergence of closed-loop states

The rest of the matrices are:

$$\tilde{P} = \begin{bmatrix}0.1722 & -0.1406\\ -0.1406 & 0.3833\end{bmatrix},\ \tilde{R} = \begin{bmatrix}0.2156 & -0.1291\\ -0.1291 & 0.1801\end{bmatrix},$$
$$\tilde{Q} = \begin{bmatrix}0.0127 & 0.0819\\ 0.0819 & 0.9347\end{bmatrix},\ \tilde{S} = \begin{bmatrix}0.1027 & -0.0605\\ -0.0605 & 0.2189\end{bmatrix},$$
$$\tilde{S}_{12} = \begin{bmatrix}-0.1085 & 0.0782\\ 0.0069 & -0.0913\end{bmatrix},\ \tilde{M} = \begin{bmatrix}0.7723 & 0\\ 0 & 0.6050\end{bmatrix},$$
$$\nu(\tilde{M}) = \begin{bmatrix}-0.7723 & 0\\ 0 & -0.6050\end{bmatrix}.$$

The obtained results can be seen in Fig. 2, i.e, this control stabilizes the system. The time evolution of control law can be seen in Fig. 3.

Note that even though the controller gains are close to each other, no feasible solutions were found when the design conditions were defined with a single control gain, thus a fuzzy controller is necessary.

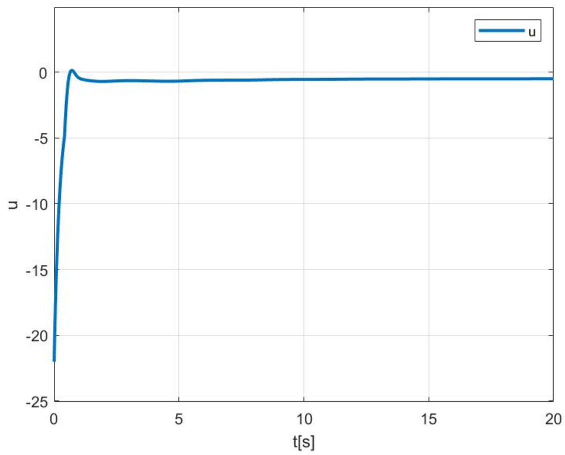

Fig. 3. Time evolution of control law

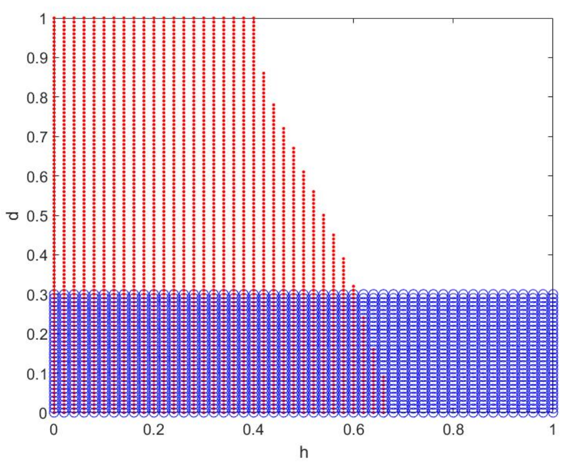

Fig. 4. Feasible solutions, '.' - Theorem 1, 'o' - Theorem 1 from [13]

V. Conclusions and Future Work

This paper considered stabilization for time-delay Takagi-Sugeno fuzzy systems with nonlinear consequents. The conditions were developed under the assumption that the membership functions may depend both on current and delayed states. The nonlinearity in the consequents was handled by a slope-bounded condition. The conditions were formulated as linear matrix inequalities and their use illustrated on a numerical example. In our future work we will consider more general Lyapunov functions and apply the conditions on a real application.

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