H∞ control for discrete-time Takagi-Sugeno descriptor models: a delayed approach

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Abstract:

This paper provides conditions for H∞ control of discrete-time nonlinear descriptor models. The nonlinear model is represented by a Takagi-Sugeno one. Conditions are given in terms of linear matrix inequalities. Relaxed conditions are obtained via delayed Lyapunov functions and delayed control laws. Such an approach allows adding extra decision variables without increasing the number of LMI conditions. The benefits of the proposed approach are verified via a numerical example.

Keywords:

Descriptor models, Takagi-Sugeno models, linear matrix inequalities, H∞ control.

1 Introduction

The interest in Takagi-Sugeno (TS) models [1] has increased since the introduction of the sector nonlinearity approach [2]. This approach allows obtaining an exact TS representation of a nonlinear system; therefore the designed controller/observer applies directly to the nonlinear plant. The TS model is a collection of linear models blended together by nonlinear membership functions (MFs).

Due to their structure, TS models are studied via the direct Lyapunov method; conditions are usually given in terms of linear matrix inequalities (LMIs). These LMIs can be efficiently solved via convex optimization techniques [3], [4].

In the continuous-time case, the non-quadratic (NQ) approach is difficult to deal with. On the other hand, in the discrete-time case, important improvements using the NQ approach have been achieved [5]–[8]; in recent years the inclusion of delays in the controller/observer design has been used to relax LMI conditions without increasing the computational cost [9], [10].

TS descriptor models have been introduced in [11] to deal with nonlinear descriptor models which naturally appear in mechanical systems [12]. A TS descriptor model separates the nonlinear terms in both sides of the equation. This fact helps to reduce the computational burden since the number of local models is $r = 2^p$, where p is the number of nonlinear terms [13].

Works related to TS descriptor models in the continuous-time case include [13]–[15], while for the discrete-time case can be found in [16], [17].

This work aims to design controllers for disturbed TS descriptor models in discrete-time. It is based on the latest results in the literature and the well-known Finsler's Lemma [18]. Via Finsler's Lemma it is possible to separate the Lyapunov matrix from the controller gains, as well as handling the descriptor structure.

The paper is organized as follows: Section 2 introduces the problem to be studied, some useful shorthand notations and a motivating example; Section 3 provides the main results for disturbance attenuation; Section 4 illustrates the advantages of the approaches via a numerical example.

2 Notations and problem statement

Consider the following nonlinear discrete-time model in the descriptor form [12]

$$\begin{aligned} E(x)x(\kappa+1) &= A(x)x(\kappa)+B(x)u(\kappa) \\ y &= C(x)x(\kappa), \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the control input vector, $y \in \mathbb{R}^o$ is the output vector, and κ is the current sample. Matrices $A(x)$, $B(x)$, $\mathrm{C}(x)$, and $E(x)$ are assumed to be smooth in a compact set of the state space Ω. Moreover matrix $E(x)$ is assumed to be nonsingular for all $x(\kappa)$ in the compact set Ω.

Using the sector nonlinearity approach [19], in case of nonlinear descriptors, the p nonlinear terms in right hand-side of (1) are grouped in $h_i(z(\kappa))$, $i \in \{1,\ldots,2^p\}$, thus generating $r = 2^p$ linear models. In a similar way, the p_e nonlinearities in the left hand-side of (1), i.e., the nonlinearities in $E(x)$, are captured in $v_k(z(\kappa))$, $k \in \{1,\ldots,2^{p_e}\}$; they generate $r_e = 2^{p_e}$ linear models.

The MFs hold the convex-sum property, i.e.,

$$\begin{aligned} &0 \le h_i(z(\kappa)) \le 1, \quad \textstyle\sum_{i=1}^{r} h_i(z(\kappa)) = 1, \\ &0 \le v_k(z(\kappa)) \le 1, \quad \textstyle\sum_{k=1}^{r_e} v_k(z(\kappa)) = 1. \end{aligned}$$

Moreover, the MFs depend on the premise vector $z(\kappa)$, which is assumed to be known.

Notation: The following shorthand notation is employed to represent convex sums of matrix expressions:

$$\Upsilon_h = \sum_{i=1}^{r} h_i(z(\kappa))\Upsilon_i,\ \Upsilon_{h^+} = \sum_{l=1}^{r} h_l(z(\kappa+1))\Upsilon_l,$$

$$\Upsilon_h^{-1} = \left(\sum_{i=1}^{r} h_i(z(\kappa))\Upsilon_i\right)^{-1},\ \Upsilon_v = \sum_{k=1}^{r_e} v_k(z(\kappa))\Upsilon_k.$$

An asterisk $(*)$ is used in matrix expressions to denote the transpose of the symmetric element; for in-line expressions it denotes the transpose of the terms on its left side. Arguments will be omitted when their meaning is obvious. In what follows, $x_{\kappa+}$ and x_κ stand for $x(\kappa+1)$ and $x(\kappa)$ respectively.

Based on the definitions above, the TS descriptor model

$$\begin{aligned} E_v x_{\kappa+} &= A_h x_\kappa + B_h u_\kappa \\ y &= C_h x_\kappa, \end{aligned} \tag{2}$$

exactly represents the nonlinear one (1); where matrices A_i , B_i, C_i $i \in \{1,\ldots,r\}$ represent the *i*-th linear right-hand side model (2) and E_k, $k \in \{1,\ldots,r_e\}$ represent the *k*-th linear left-hand side model of the TS descriptor model. Recall that (2) is an exact rewriting of (1).

When dealing with TS models, the MFs must be dropped off in order to obtain LMI constraints. The following relaxation scheme will be employed due to its good compromise between effectiveness and computational complexity.

Relaxation Lemma [20]: Let Υ_{ij}^k be matrices of proper dimensions. Then $\Upsilon_{hh}^v < 0$ holds if

$$\begin{aligned} &\Upsilon_{ii}^k < 0, \quad \forall i,k \\ &\frac{2}{r-1}\Upsilon_{ii}^k + \Upsilon_{ij}^k + \Upsilon_{ji}^k < 0, \quad i \ne j, \quad \forall k, \end{aligned} \tag{3}$$

for $i,j \in \{1,\ldots,r\}$, $k \in \{1,\ldots,r_e\}$.

Finsler's Lemma [18]: Let $x \in \mathbb{R}^n$, $Q = Q^T \in \mathbb{R}^{n\times n}$, and $R \in \mathbb{R}^{m\times n}$ such that $rank(R) < n$; the following expressions are equivalent:

a) $x^T Q x < 0$, $\forall x \in \{x \in \mathbb{R}^n : x \ne 0, Rx = 0\}$.

b) $\exists M \in \mathbb{R}^{n\times m} : Q + MR + R^T M^T < 0$.

Property 1. Let $X = X^T > 0$ and Y matrices of appropriate sizes. The following expression holds:

$$\left(Y-X\right)^{T}X^{-1}\left(Y-X\right)\geq 0$$
$$\Leftrightarrow Y^{T}X^{-1}Y\geq Y+Y^{T}-X.$$

The following Example 1 points out the importance of keeping the descriptor form instead of computing the classical state space: $x_{\kappa+}=A\left(x\right)x_{\kappa}+B\left(x\right)u_{\kappa}$.

Example 1. Consider the following system in nonlinear descriptor form (1) with

$$E\left(x\right)=\begin{bmatrix}1.4 & -0.7\cos\left(x_{1}\right)\\ 0.7\cos\left(x_{1}\right) & 2.3\end{bmatrix},$$
$$A\left(x\right)=\begin{bmatrix}-0.9 & 0.6\\ -1 & \cos\left(x_{2}\right)+2.5\end{bmatrix},$$
$$B\left(x\right)=\begin{bmatrix}0.5\\ 1\end{bmatrix},\quad C\left(x\right)=\begin{bmatrix}0.4\\ 0.5+x_{2}\end{bmatrix}^{T}.$$

Via the sector nonlinearity approach, a TS descriptor models results with $r_{e}=2$ and $r=4$ due to the number of nonlinearities on the left-hand side and on the right-hand side. To compute the classical TS model, it is necessary to calculate $E^{-1}\left(x\right)$, resulting in $x_{\kappa+}=E^{-1}\left(x\right)\left(A\left(x\right)x_{\kappa}+B\left(x\right)u_{\kappa}\right)$, with

$$E^{-1}\left(x\right)=\begin{bmatrix}\dfrac{32.86}{46+7\cos^{2}\left(x_{1}\right)} & \dfrac{10\cos\left(x_{1}\right)}{46+7\cos^{2}\left(x_{1}\right)}\\ \dfrac{-10\cos\left(x_{1}\right)}{46+7\cos^{2}\left(x_{1}\right)} & \dfrac{20}{46+7\cos^{2}\left(x_{1}\right)}\end{bmatrix}$$

This means that four different nonlinearities have to be considered, which results in $r=16$ local models. One can see that the number of LMI conditions for classical TS models can grow quickly and may be leading to a numerical intractability. ◊

Throughout this paper, we consider a TS descriptor model under external disturbances, i.e.,

$$\begin{aligned}E_{v}x_{\kappa+}&=A_{h}x_{\kappa}+B_{h}u_{\kappa}+D_{h}w_{\kappa}\\ y&=C_{h}x_{\kappa}+J_{h}w_{\kappa},\end{aligned}\tag{4}$$

where $w\in\mathbb{R}^{q}$ is the vector of external disturbances. Matrices D_{h} and J_{h} share the same definition as A_{h}. The controller design with disturbance attenuation is presented in the following section.

3 Main results

For controller design, the following non-PDC control law is used

$$u_{\kappa}=F_{(\cdot)}H_{(\cdot)}^{-1}x_{\kappa},\tag{5}$$

where $F_{(\cdot)}$ and $H_{(\cdot)}$ will be defined later on. Note that the state is available for control purposes.

The TS descriptor model (4) together with the control law (5) writes

$$\begin{aligned}E_{v}x_{\kappa+}&=\left(A_{h}+B_{h}F_{(\cdot)}H_{(\cdot)}^{-1}\right)x_{\kappa}+D_{h}w_{\kappa}\\ y&=C_{h}x_{\kappa}+J_{h}w_{\kappa},\end{aligned}\tag{6}$$

Note that the closed-loop model (6) can be rewritten as

$$\begin{bmatrix}A_{h}+B_{h}F_{(\cdot)}H_{(\cdot)}^{-1} & -E_{v} & D_{h}\end{bmatrix}\begin{bmatrix}x_{\kappa}\\ x_{\kappa+}\\ w_{\kappa}\end{bmatrix}=0.\tag{7}$$

A system performs disturbance attenuation with a factor $\gamma>0$ if the following well-known condition holds [21]

$$\Delta V\left(x_{\kappa}\right)+y_{\kappa}^{T}y_{\kappa}-\gamma^{2}w_{\kappa}^{T}w_{\kappa}<0\tag{8}$$

3.1 Non-quadratic approach

In what follows, conditions for stabilization are given via the non-quadratic Lyapunov function

$$V\left(x_\kappa\right)=x_\kappa^T\left(\sum_{i=1}^{r}h\left(z\left(\kappa\right)\right)P_i\right)^{-1}x_\kappa \\ =x_\kappa^T P_h^{-1}x_\kappa, \tag{9}$$

with $P_h=P_h^T>0$, $P_h^{-1}=X_h$.

Theorem 1 (NQ): The TS descriptor model (6) is asymptotically stable with disturbance attenuation at least γ if there exist matrices $P_h=P_h^T>0$, H_{hv}, and F_{hv} such that the following inequality holds

$$\begin{bmatrix} -H_{hv}-H_{hv}^T+P_h & (*) & 0 & (*) \\ A_hH_{hv}+B_hF_{hv} & \Gamma^{(2,2)} & (*) & 0 \\ 0 & D_h^T & -\gamma^2 I & (*) \\ C_hH_{hv} & 0 & J_h & -I \end{bmatrix}<0 \tag{10}$$

with $\Gamma^{(2,2)}=-E_vP_{h^+}-P_{h^+}E_v^T+P_{h^+}$.

Proof: The variation of the Lyapunov function (9) is

$$\Delta V\left(x_\kappa\right)=x_{\kappa+}^T X_{h^+}x_{\kappa+}-x_\kappa^T X_h x_\kappa<0 \tag{11}$$

The expression $\Delta V\left(x_\kappa\right)$ together with condition (8) can be written as

$$\begin{bmatrix} x_\kappa \\ x_{\kappa+} \\ w_\kappa \end{bmatrix}^T \begin{bmatrix} C_h^TC_h-X_h & 0 & C_h^TJ_h \\ 0 & X_{h^+} & 0 \\ J_h^TC_h & 0 & \begin{pmatrix} J_h^TJ_h \\ -\gamma^2 I \end{pmatrix} \end{bmatrix} \begin{bmatrix} x_\kappa \\ x_{\kappa+} \\ w_\kappa \end{bmatrix}<0 \tag{12}$$

Via Finsler's Lemma, inequality (12) under constraint (7) results in

$$\begin{bmatrix} C_h^TC_h-X_h & 0 & C_h^TJ_h \\ 0 & X_{h^+} & 0 \\ J_h^TC_h & 0 & J_h^TJ_h-\gamma^2 I \end{bmatrix} \\ +M\begin{bmatrix} A_h+B_hF_{(\cdot)}H_{(\cdot)}^{-1} & -E_v & D_h \end{bmatrix}+(*)<0 \tag{13}$$

where the matrix $M\in\mathbb{R}^{(2n+q)\times n}$ can be arbitrarily selected. By selecting $H_{(\cdot)}=H_{hv}$ and $M=\begin{bmatrix} 0 \\ H_{hv}^{-T} \\ 0 \end{bmatrix}$ it gives

$$\begin{bmatrix} C_h^TC_h-X_h & 0 & C_h^TJ_h \\ 0 & X_{h^+} & 0 \\ J_h^TC_h & 0 & J_h^TJ_h-\gamma^2 I \end{bmatrix} \\ +\begin{bmatrix} 0 \\ H_{hv}^{-T} \\ 0 \end{bmatrix}\begin{bmatrix} A_h+B_hF_{hv}H_{hv}^{-1} & -E_v & D_h \end{bmatrix}+(*)<0 \tag{14}$$

By using the congruence property with the full-rank matrix $diag\left(H_{hv}^T,P_{h^+},I\right)$, (14) yields

$$\begin{bmatrix} -H_{hv}^TX_hH_{hv} & (*) & 0 \\ A_hH_{hv}+B_hF_{hv} & -E_vP_{h^+}-P_{h^+}E_v^T+P_{h^+} & (*) \\ 0 & D_h^T & -\gamma^2 I \end{bmatrix} \\ +\begin{bmatrix} H_{hv}^TC_h^T \\ 0 \\ J_h^T \end{bmatrix}\begin{bmatrix} C_hH_{hv} & 0 & J_h \end{bmatrix}<0 \tag{15}$$

Applying Property 1 over the position (1,1) and the Schur complement it yields (10). □

Remark 1: Note that (10) can be written in terms of LMI conditions via the Relaxation Lemma since γ^2 is not multiplied by any decision variable. Thus the optimal value can searched by minimizing γ^2.

3.2 Delayed non-quadratic approach

A way to obtain more relaxed results by augmenting the number of decision variables while the number of LMI constraints remains the same has been shown in [9]. The main idea is to introduce a delay in the Lyapunov function. This new Lyapunov function allows changing the structure of the controller gains. Therefore, two options can be taken:

OP1. $V(x_\kappa) = x_\kappa^T \left(\sum_{i=1}^{r} h(z(\kappa+1)) P_i\right)^{-1} x_\kappa$.

OP2. $V(x_\kappa) = x_\kappa^T \left(\sum_{i=1}^{r} h(z(\kappa-1)) P_i\right)^{-1} x_\kappa$.

With OP1 the new structure of the controller gains can include an extra MF $\sum_{i=1}^{r} h(z(\kappa+1))$. However the knowledge of the future state is needed, thus OP1 is not suitable. OP2 allows introducing $\sum_{i=1}^{r} h(z(\kappa-1))$, which is perfectly implementable. The following Theorem establishes conditions for disturbance attenuation with these ideas.

Theorem 2 (DNQ): The TS descriptor model (6) is asymptotically stable and the disturbance attenuation is at least γ if there exist matrices $P_h = P_h^T > 0$, H_{hh^-v}, and F_{hh^-v} such that the following inequality holds

$$\begin{bmatrix} -H_{hh^-v} - H_{hh^-v}^T + P_{h^-} & (*) & 0 & (*) \\ A_h H_{hh^-v} + B_h F_{hh^-v} & \Gamma^{(2,2)} & (*) & 0 \\ 0 & D_h^T & -\gamma^2 I & (*) \\ C_h H_{hh^-v} & 0 & J_h & -I \end{bmatrix} < 0, \tag{16}$$

with $\Gamma^{(2,2)} = -E_v P_h - P_h E_v^T + P_h$.

Proof: Consider the Lyapunov function $V(x_\kappa) = x_\kappa^T \left(\sum_{i=1}^{r} h(z(\kappa-1)) P_i\right)^{-1} x_\kappa$ and its variations as follows

$$\Delta V(x_\kappa) = x_{\kappa+}^T X_h x_{\kappa+} - x_\kappa^T X_{h^-} x_\kappa < 0. \tag{17}$$

The expression (17) together with condition (8) can be written as

$$\begin{bmatrix} x_\kappa \\ x_{\kappa+} \\ w_\kappa \end{bmatrix}^T \begin{bmatrix} C_h^T C_h - X_{h^-} & 0 & C_h^T J_h \\ 0 & X_h & 0 \\ J_h^T C_h & 0 & \begin{pmatrix} J_h^T J_h \\ -\gamma^2 I \end{pmatrix} \end{bmatrix} \begin{bmatrix} x_\kappa \\ x_{\kappa+} \\ w_\kappa \end{bmatrix} < 0 \tag{18}$$

Via Finsler's Lemma, inequality (18) under constraint (7) results in

$$\begin{bmatrix} C_h^T C_h - X_{h^-} & 0 & C_h^T J_h \\ 0 & X_h & 0 \\ J_h^T C_h & 0 & J_h^T J_h - \gamma^2 I \end{bmatrix} + M \begin{bmatrix} A_h + B_h F_{(\cdot)} H_{(\cdot)}^{-1} & -E_v & D_h \end{bmatrix} + (*) < 0 \tag{19}$$

Choosing $M = \begin{bmatrix} 0 \\ H_{hh^-v}^{-T} \\ 0 \end{bmatrix}$, $H_{(\cdot)} = H_{hh^-v}$, $F_{(\cdot)} = F_{hh^-v}$, and using the congruence property with $diag\left(H_{hh^-v}^T, P_h, I\right)$, (19) yields

$$\begin{bmatrix} -H_{hh^-v}^T X_{h^-} H_{hh^-v} & (*) & 0 \\ A_h H_{hh^-v} + B_h F_{hh^-v} & -E_v P_h - P_h E_v^T + P_h & (*) \\ 0 & D_h^T & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} H_{hh^-v}^T C_h^T \\ 0 \\ J_h^T \end{bmatrix} \begin{bmatrix} C_h H_{hh^-v} & 0 & J_h \end{bmatrix} < 0 \tag{20}$$

Applying the Schur complement and Property 1 over the position (1,1), it yields (16).□

A more relaxed result can be obtained if the congruence property with $diag\left(H_{hh^-v}^T, G_{hhh^-}^T, I\right)$ is applied to (19). This result is summarized in Corollary 1.

Corollary 1 (DNQ): The TS descriptor model (6) is asymptotically stable and the disturbance attenuation is at least γ if there exist matrices $P_h = P_h^T > 0$, H_{hh^-v}, G_{hhh^-}, and F_{hh^-v} such that the following inequality holds

$$\begin{bmatrix} \Gamma^{(1,1)} & (*) & (*) & (*) & (*) \\ \Gamma^{(2,1)} & \Gamma^{(2,2)} & (*) & (*) & (*) \\ 0 & G_{hhh^-} & -P_h & (*) & (*) \\ 0 & D_h^T & 0 & -\gamma^2 I & (*) \\ C_h H_{hh^-v} & 0 & 0 & J_h & -I \end{bmatrix} < 0 \tag{21}$$

with $\Gamma^{(1,1)} = -H_{hh^-v} - H_{hh^-v}^T + P_{h^-}$, $\Gamma^{(2,1)} = A_h H_{hh^-v} + B_h F_{hh^-v}$, and $\Gamma^{(2,2)} = -E_v G_{hhh^-} - G_{hhh^-}^T E_v^T$.

Proof: Recall (19) and by applying the congruence property with $diag\left(H_{hh^-v}^T, G_{hhh^-}^T, I\right)$, it gives

$$\begin{bmatrix} -H_{hh^-v}^T X_{h^-} H_{hh^-v} & (*) & 0 \\ A_h H_{hh^-v} + B_h F_{hh^-v} & \Gamma^{(2,2)} & (*) \\ 0 & D_h^T & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} H_{hh^-v}^T C_h^T \\ 0 \\ J_h^T \end{bmatrix} \begin{bmatrix} C_h H_{hh^-v} & 0 & J_h \end{bmatrix} < 0 \tag{22}$$

with $\Gamma^{(2,2)} = -E_v G_{hhh^-} - G_{hhh^-}^T E_v^T + G_{hhh^-}^T X_h G_{hhh^-}$. Using the Property 1 on block (1,1) and twice the Schur complement, (22) gives the desired result (21), thus concluding the proof. □

Remark 2: In Corollary 1, the choice for matrix G_{hhh^-} allows obtaining extra degrees of freedom without increasing the number of LMIs to be satisfied. The number of extra free matrices is r^3.

Remark 3: The classical TS model is a special case of the TS descriptor one when $E_v = I$.

Table 1 summarizes the proposed approaches in terms of number of decision variables, where n stands for the number of states, m the number of inputs, r the number of linear models in the right-hand side, and r_e the number of linear models in the left-hand side.

Table 1.

Approach	No. of Decision Variables
Theorem 1	$\frac{n(n+1)}{2} \times r + (r \times r_e) \times (n^2 + n \times m)$
Theorem 2	$\frac{n(n+1)}{2} \times r + (r^2 \times r_e) \times (n^2 + n \times m)$
Corollary 1	$\frac{n(n+1)}{2} \times r + (r^2 \times r_e) \times (n^2 + n \times m)$ $+ n^2 \times r^3$

4 Example

The results are illustrated via the following numerical example.

Example 2. Recall the nonlinear descriptor model in Example 1 when it is under external perturbations $w(\kappa)$. Considering the compact set $\Omega = \{x : x_1 \in \mathbb{R}, |x_2| \le 1\}$, the representation in the form (4) gives $r_e = 2$ and $r = 4$ with local matrices as follows

$$E_1 = \begin{bmatrix} 1.4 & -0.7 \\ 0.7 & 2.3 \end{bmatrix}, \quad A_1 = A_2 = \begin{bmatrix} -0.9 & 0.6 \\ -1 & 3.5 \end{bmatrix},$$

$$E_2 = \begin{bmatrix} 1.4 & 0.7 \\ -0.7 & 2.3 \end{bmatrix}, \quad A_3 = A_4 = \begin{bmatrix} -0.9 & 0.6 \\ -1 & 3.05 \end{bmatrix},$$

$$B_i = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}, \quad C_2 = C_4 = \begin{bmatrix} 0.4 \\ -0.5 \end{bmatrix}^T,$$

$$C_1 = C_3 = \begin{bmatrix} 0.4 \\ 1.5 \end{bmatrix}^T, \quad D_1 = D_3 = \begin{bmatrix} 0 \\ 0.3 + 0.5\alpha \end{bmatrix}^T,$$

$$D_2 = D_4 = \begin{bmatrix} 0 \\ 0.3 - 0.5\alpha \end{bmatrix}^T, \quad J_i = 0.2\alpha,$$

$i \in \{1, \ldots, r\}$, where α is a real-valued parameter.

The MFs are defined as follows: $v_1 = \frac{\cos(x_1)+1}{2}$, $v_2 = 1 - v_1$, $h_1 = \omega_0^1 \omega_0^2$, $h_2 = \omega_0^1 \omega_1^2$, $h_3 = \omega_1^1 \omega_0^2$, and $h_4 = \omega_1^1 \omega_1^2$; their corresponding weighting functions are $\omega_0^1 = \frac{\cos(x_2)+1}{2}$, $\omega_1^1 = 1 - \omega_0^1$, $\omega_0^2 = \frac{x_2+1}{2}$, and $\omega_1^2 = 1 - \omega_0^2$. The MFs hold the convex-sum property on the compact set Ω.

Employing conditions given in this work the minimal value for γ^2 is calculed for $\alpha \in [-2, 0]$.

Figure 1 shows the results for Theorem 1 ($\bigcirc$), Theorem 2 ($\times$), and Corollary 1 ($+$).

Note that for this example, the number of decision variables in Theorem 1 is 60, in Theorem 2 is 204, while for Corollary 1 is 460. The number of linear models for the classical TS representation is $r = 16$, recall Example 1, thus similar conditions as in [9] lead to $r^3 + r = 4112$ LMI conditions; while for any of the proposed approaches the number of LMI is 132. This fact shows why is important to keep the descriptor form and to construct the TS descriptor model.

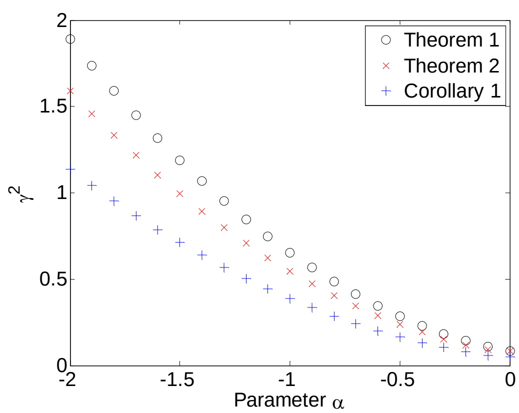

Figure 1. γ^2 values in Example 2.

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References

[1] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *IEEE Transactions on Systems, Man and Cybernetics*, vol. 15, no. 1, pp. 116–132, 1985.

[2] H. Ohtake, K. Tanaka, and H. Wang, "Fuzzy modeling via sector nonlinearity concept," in *Joint 9th IFSA World Congress and 20th NAFIPS International Conference*, Vancouver, Canada, 2001, pp. 127–132.

[3] S. Boyd, L. El Ghaoul, E. Feron, and V. Balakrishnan, *Linear matrix inequalities in system and control theory*. Philadelphia, PA: Society for Industrial and Applied Mathematics, 1994.

[4] C. Scherer and S. Weiland, *Linear Matrix Inequalities in Control*, Lecture Notes, Dutch Institute for Systems and Control. Delft University, The Netherlands, 2005.

[5] T. M. Guerra and L. Vermeiren, "LMI-based relaxed nonquadratic stabilization conditions for nonlinear systems in the Takagi–Sugeno's form," *Automatica*, vol. 40, no. 5, pp. 823–829, 2004.

[6] A. Kruszewski, R. Wang, and T. M. Guerra, "Nonquadratic stabilization conditions for a class of uncertain nonlinear discrete time TS fuzzy models: a new approach," *IEEE Transactions on Automatic Control*, vol. 53, no. 2, pp. 606–611, 2008.

[7] B. Ding, "Homogeneous polynomially nonquadratic stabilization of discrete-time Takagi-Sugeno systems via nonparallel distributed compensation law," *IEEE Transactions on Fuzzy Systems*, vol. 18, no. 5, pp. 994–1000, 2010.

[8] D.-H. Lee, J.-B. Park, and Y.-H. Joo, "Improvement on nonquadratic stabilization of discrete-time Takagi-Sugeno fuzzy systems: multiple-parameterization approach," *IEEE Transactions on Fuzzy Systems*, vol. 18, no. 2, pp. 425–429, 2010.

[9] T. M. Guerra, H. Kerkeni, J. Lauber, and L. Vermeiren, "An efficient Lyapunov function for discrete T–S models: observer design," *IEEE Transactions on Fuzzy Systems*, vol. 20, no. 1, pp. 187–192, 2012.

[10] Zs. Lendek, T. M. Guerra, and J. Lauber, "Construction of extended Lyapunov functions and control laws for discrete-time TS systems," in *2012*

IEEE International Conference on Fuzzy Systems (FUZZ-IEEE), Brisbane, Australia, 2012, pp. 1–6.

[11] T. Taniguchi, K. Tanaka, K. Yamafuji, and H. O. Wang, "Fuzzy descriptor systems: stability analysis and design via LMIs," in *Proceedings of the American Control Conference*, California, USA, 1999, vol. 3, pp. 1827–1831.

[12] D. Luenberger, "Dynamic equations in descriptor form," *IEEE Transactions on Automatic Control*, vol. 22, no. 3, pp. 312–321, 1977.

[13] T. Taniguchi, K. Tanaka, and H. O. Wang, "Fuzzy descriptor systems and nonlinear model following control," *IEEE Transactions on Fuzzy Systems*, vol. 8, no. 4, pp. 442–452, 2000.

[14] K. Guelton, S. Delprat, and T. M. Guerra, "An alternative to inverse dynamics joint torques estimation in human stance based on a Takagi–Sugeno unknown-inputs observer in the descriptor form," *Control Engineering Practice*, vol. 16, no. 12, pp. 1414–1426, 2008.

[15] V. Estrada-Manzo, T. M. Guerra, Zs. Lendek, and M. Bernal, "Improvements on non-quadratic stabilization of continuous-time Takagi-Sugeno descriptor models," in *2013 IEEE International Conference on Fuzzy Systems*, Hyderabad, India, 2013, pp. 1–6.

[16] Z. Wang, Y. Shen, X. Zhang, and Q. Wang, "Observer design for discrete-time descriptor systems: an LMI approach," *Systems & Control Letters*, vol. 61, no. 6, pp. 683–687, 2012.

[17] M. Chadli and M. Darouach, "Novel bounded real lemma for discrete-time descriptor systems: Application to control design," *Automatica*, vol. 48, no. 2, pp. 449–453, Feb. 2012.

[18] M. de Oliveira and R. Skelton, "Stability tests for constrained linear systems," *Perspectives in Robust Control*, vol. 268, pp. 241–257, 2001.

[19] K. Tanaka and H. O. Wang, *Fuzzy Control Systems Design and Analysis: a Linear Matrix Inequality Approach*. New York: John Wiley & Sons, Inc., 2001.

[20] H. D. Tuan, P. Apkarian, T. Narikiyo, and Y. Yamamoto, "Parameterized linear matrix inequality techniques in fuzzy control system design," *IEEE Transactions on Fuzzy Systems*, vol. 9, no. 2, pp. 324–332, 2001.

[21] K. Tanaka, T. Taniguchi, and H. O. Wang, "Model-based fuzzy control of TORA system: fuzzy regulator and fuzzy observer design via LMIs that represent decay rate, disturbance rejection, robustness, optimality" in *1998 IEEE International Conference on Fuzzy Systems*, Anchorage, Alaska, 1998, pp. 313–318.