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# Finding a stabilizing switching law for switching nonlinear models

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This paper considers the stabilization of switching nonlinear models by switching between the subsystems. We assume that arbitrary switching between two subsystems is possible once a subsystem has been active for a predefined number of samples. We use a Takagi-Sugeno representation of the models and a switching Lyapunov function is employed to develop sufficient stability conditions. If the conditions are satisfied, we construct a switching law that stabilizes the system. The application of the conditions is illustrated on several examples.

**Keywords:** switching systems, TS systems, stabilizing switching law, non-quadratic Lyapunov function

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## 1. Introduction

Switching systems are a class of hybrid systems that switch between a family of modes or subsystems. In the last decades, stability analysis and stabilization of switching systems have attracted much attention, mostly in the continuous-time case, with linear subsystems.

For instance, linear switching systems where the switching laws can be arbitrarily chosen have been considered in (Altafini 2002) to study the reachable set of such systems. Stabilization and tracking conditions for continuous-time linear switching systems have been developed in (Baglietto et al. 2013; Battistelli 2013), delay-dependent stabilization in (Kim et al. 2008), and observability with unknown input has been investigated in (Boukhobza and Hamelin 2011). The results for switching systems have been applied in (Guan et al. 2013) for the decentralized stabilization of multiagent systems. State-feedback controller design for nonlinear switching systems has been presented in (Blanchini et al. 2007) and optimal control in (Bengea and DeCarlo 2005). A notable result, although for continuous-time linear switching systems is the one in (Ji et al. 2007), which concerns the design of switching sequences for stabilization and proves that it is sufficient for stabilization to employ a periodic switching law.

This paper deals with a particular class of nonlinear switching models. Such models can be found in numerous and various domains (Zwart et al. 2010; Venkataramanan et al. 2002; Pasamontes et al. 2011; Widyotriatmo and Hong 2012; Moustris and Tzafestas 2011; Zhao and Spong 2001; Li et al. 2012, 2014), such as automotive, network controlled application, DC converters, mobile robots, etc. In the case of automotive applications, switching approaches have been used for different parts of the vehicle: engine control, HCCI combustion (Liao et al. 2013), air path with turbocharger (Nguyen et al. 2012a,b, 2013), clutch actuator control (Langjord et al. 2008). Multi-cellular converters are components which require to control several switches with high frequency to lead to a desired level of conversion. A way to consider the different possible modes is to use a switching structure (Hauroigné et al. 2012).

In the previously listed applications, the problems addressed are the analysis of stability and/or the development of controller or observer. Originally, mainly continuous time switching systems were considered but recently discrete time approach have been developed (Chen et al. 2012; Duan and Wu 2012; Hetel et al. 2011).

We represent the switching nonlinear models by switching Takagi-Sugeno (TS) fuzzy systems (Takagi and Sugeno 1985). These are nonlinear, convex combinations of local linear models that have the advantage that they are able to exactly represent nonlinear systems on a compact set of the state-space, while allowing the use of classical approaches for linear systems. Stability and stabilization conditions for TS models have been recently derived using nonquadratic Lyapunov functions (Guerra and Vermeiren 2004; Kruszewski et al. 2008; Mozelli et al. 2009). The stability or design conditions are generally derived in the form of linear matrix inequalities (LMIs), which can be solved using convex optimization methods.

It is well-known that by switching between two – independently stable – subsystems the switching system can be destabilized and conversely, by switching between unstable subsystems, the states can be made to converge to zero. Our goal is to design the switching law that stabilizes a given switching system. Switched TS systems have been investigated mainly in the continuous case where the stability is based on the use of a quadratic Lyapunov function (Tanaka et al. 2001; Lam et al. 2002, 2004; Ohtake et al. 2006) or a piecewise one (Feng 2003, 2004). For discrete-time switching TS models, few results exist (Doo et al. 2003; Dong and Yang 2009). In this paper we consider arbitrary switching discrete-time systems and investigate under which conditions they can be stabilized by a suitable chosen switching law. Instead of attempting to stabilize each subsystem so that the overall system to be stable, we stabilize the system by switching between possible unstable subsystems. We use a graph representation of the switching system and employing a switching nonquadratic Lyapunov function, we construct a switching law that stabilizes a given system. Thus, the novelty and the contribution of the paper consist in 1) developing sufficient conditions for being able to stabilize a discrete-time switching nonlinear system simply by switching between the subsystems;

2) constructing a switching law that stabilizes the system; and 3) obtaining a stabilizing switching law that minimizes the lowest upper bound on the convergence rate.

The structure of the paper is as follows. Section 2 presents the switching TS models and the notations used in this paper. Conditions for stabilization and the construction of the switching law are developed in Section 3. Section 4 discusses the assumptions used and illustrates the constructions of the switching law on two numerical examples. Section 5 concludes the paper.

## 2. Notation and preliminaries

In this paper we analyze whether and under which conditions a switching law that stabilizes a given switching system can be constructed. For this, we consider subsystems of the form

$$\begin{aligned} \boldsymbol{x}(k+1) &= \sum_{i=1}^{r_j} h_{ji}(\boldsymbol{z}_j(k)) A_{ji} \boldsymbol{x}(k) \\ &= A_{jz} \boldsymbol{x}(k) \end{aligned} \tag{1}$$

where $j$ is the index of the subsystem, $j = 1, 2, \ldots, n_\mathrm{s}$, $n_\mathrm{s}$ being the number of the subsystems, $\boldsymbol{x}$ denotes the state vector, $r_j$ is the number of rules in the $j$th subsystem, $\boldsymbol{z}_j$ is the scheduling vector, $h_{ji}$, $i = 1, 2, \ldots, r_j$ are normalized membership functions, and $A_{ji}$, $i = 1, 2, \ldots, r_j$, $j = 1, 2, \ldots, n_\mathrm{s}$, are the local models. To motivate the research presented hereafter, consider the following example.

**Example 1.** *Consider a system with three subsystems as follows:*

$$\Sigma_1 : \boldsymbol{x}(k+1) = h_{1,1}(\boldsymbol{x}(k)) A_{1,1} \boldsymbol{x}(k) + h_{1,2}(\boldsymbol{x}(k)) A_{1,2} \boldsymbol{x}(k)$$

$$A_{1,1} = \begin{pmatrix} -0.44 & -0.26 \\ -0.65 & 0.62 \end{pmatrix} \qquad A_{1,2} = \begin{pmatrix} 1.1 & -0.2 \\ 0.53 & -0.27 \end{pmatrix}$$

$$h_{1,1} = \frac{1 - \sin(x_1(k))}{2} \qquad h_{1,2} = 1 - h_{1,1}$$

$$\Sigma_2 : \boldsymbol{x}(k+1) = h_{2,1}(\boldsymbol{x}(k)) A_{2,1} \boldsymbol{x}(k) + h_{2,2}(\boldsymbol{x}(k)) A_{2,2} \boldsymbol{x}(k)$$

$$A_{2,1} = \begin{pmatrix} 0.02 & 0.6 \\ -0.22 & -0.44 \end{pmatrix} \qquad A_{2,2} = \begin{pmatrix} 0.32 & -0.15 \\ -1 & 0.8 \end{pmatrix}$$

$$h_{2,1} = \frac{1 - \cos(x_1(k))}{2} \qquad h_{2,2} = 1 - h_{2,1}$$

$$\Sigma_3 : \boldsymbol{x}(k+1) = h_{3,1}(\boldsymbol{x}(k)) A_{3,1} \boldsymbol{x}(k) + h_{3,2}(\boldsymbol{x}(k)) A_{3,2} \boldsymbol{x}(k)$$

$$A_{3,1} = \begin{pmatrix} 2 & 0.5 \\ 0.5 & 2 \end{pmatrix} \qquad A_{3,2} = \begin{pmatrix} 1 & 0.1 \\ 0.5 & 3 \end{pmatrix}$$

$$h_{3,1} = \frac{1 - \exp(-x_1^2(k))}{2} \qquad h_{3,2} = 1 - h_{3,1}$$

*where $x_1(k)$ denotes the first element of the state vector $\boldsymbol{x}$ at time $k$. One can switch from each subsystem to any other one and any subsystem can be active for any number of samples. However, $A_{1,2}$ and $A_{2,2}$ are not Schur and both $A_{3,1}$ and $A_{3,2}$ have eigenvalues larger than 1. Our goal is to find a switching law that stabilizes the switching system above.*

*Since none of the subsystems is stable, no existing result in the literature can prove the stability of this system. Moreover, just switching to one subsystem and keeping it continuously active is not a solution. However, by switching continuously between the first and second subsystem, the* *states converge to zero. This can be proven by using a periodic Lyapunov function, such as the one proposed in (Lendek et al. 2013). Consequently, in order to stabilize the system, when starting from the third subsystem, one can switch to the first or second one and then switch between these two.*

For the easier notation, we use a directed graph representation of the switching system (1). The graph associated to (1) is $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, with $\mathcal{V}$ being the set of vertices representing the subsystems and $\mathcal{E}$ the set of admissible transitions or switches. Thus, $(v_i, v_j) \in \mathcal{E}$ if a switch from subsystem $i$ to subsystem $j$ is possible. Note that we assume that self-transitions are also possible: these correspond to the subsystem being active for more than one sample. With a slight abuse of notation, we also use the notation $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{W}\}$, with $\mathcal{W}$ being a matrix of weights associated to the vertices and edges. By convention, if a transition between two different subsystems is not possible, the corresponding weight is $\infty$.

A path $\mathcal{P}(v_i, v_j)$ between two vertices $v_i$ and $v_j$ in the graph $\mathcal{G}$ is a sequence of vertices $\mathcal{P}(v_i, v_j) = [v_{p_1}, v_{p_2}, \ldots, v_{p_{n_p}}]$ so that $v_i = v_{p_1}$, $v_j = v_{p_{n_p}}$, and $(v_{p_k}, v_{p_{k+1}}) \in \mathcal{E}$, $k = 1, 2, \ldots, n_p - 1$. A path between two vertices is in general not unique. A cycle $\mathcal{C} = [c_1, c_2, \ldots, c_{n_c}, c_1]$ is a path having the same initial and final vertex. Two cycles are equivalent if the vertices in one are a cyclic permutation of the vertices in the other. In this paper, when referring to paths and cycles[1], we mean elementary paths and cycles, i.e., paths in which each vertex may appear only once. A graph is strongly connected if there is a path between any two vertices in $\mathcal{V}$.

In a weighted graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{W}\}$ we define the weight of a path $W(\mathcal{P}(v_i, v_j))$ as the product of all vertices and edges that appear in the path, i.e.,

$$W([v_{p_1}, v_{p_2}, \ldots, v_{p_{n_p}}]) = \prod_{k=1}^{n_p} w_{p_k, p_k} \cdot \prod_{k=1}^{n_p - 1} w_{p_k, p_{k+1}}$$

The weight of a cycle is similarly defined. A cycle is subunitary, if its weight is less than 1.

A path in a graph associated to a switching system corresponds to a switching law. A cycle in a graph associated to a switching system corresponds to a periodic switching law.

Let us illustrate the notions above on an example.

**Example 2.** *Consider a switching system composed of four subsystems:*

$$\boldsymbol{x}(k+1) = A_{i,z}\boldsymbol{x}(k)$$

*for* $i = 1, 2, 3, 4$*, and with admissible switches* $(1, 2)$*,* $(2, 1)$*,* $(2, 3)$*,* $(3, 1)$*,* $(4, 3)$*,* $(1, 4)$*. Next to this, each subsystem can be active for more than one sample. The corresponding graph representation is illustrated in Figure 1.*

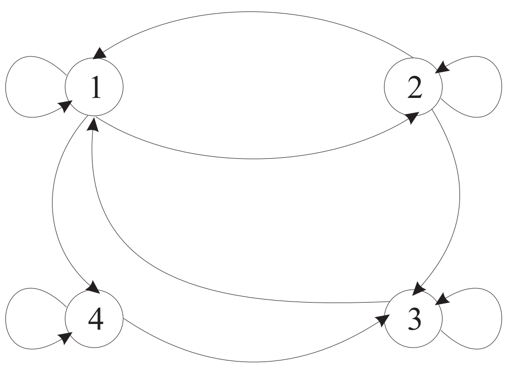


Figure 1. Graph representation of the switching system in Example 2.

[1]In case of cycles, due to the notation, the first and last vertex are the same.

*The graph is $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, with $\mathcal{V} = \{1, 2, 3, 4\}$ and*

$$\mathcal{E} = \{(1,1), (1,2), (1,4), (2,1), (2,2), (2,3), (3,1), (3,3), (4,3), (4,4)\}$$

*Possible paths between vertices 1 and 3 are $\mathcal{P}(1,3) = [1, 2, 3]$ and $\mathcal{P}(1,3) = [1, 4, 3]$. The sequence $[1, 2, 1]$ is a cycle and is equivalent to $[2, 1, 2]$.*

*Let the associated weight matrix be given by*

$$\mathcal{W} = \begin{pmatrix} 1 & 0.5 & \infty & 1 \\ 0.5 & 2 & 2 & \infty \\ 3 & \infty & 1 & \infty \\ \infty & \infty & 1 & 2 \end{pmatrix}$$

*where $\infty$ corresponds to an inadmissible switch. The graph with the weights given in $\mathcal{W}$ is illustrated in Figure 2.*

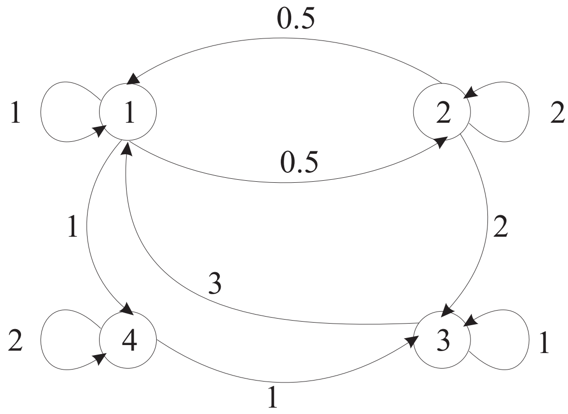


Figure 2. Graph representation of the switching system with weights in Example 2.

*The weight of the path $\mathcal{P}(1,3) = [1, 2, 3]$ is $W(\mathcal{P}(1,3)) = w_{11}w_{12}w_{22}w_{23}w_{33} = 2$. The weight of the cycle $\mathcal{C} = [1, 2, 1]$ is $W(\mathcal{C}) = w_{11}w_{12}w_{22}w_{21} = 0.5 < 1$ so this cycle is subunitary. Since in the graph above there exists a path between any two vertices, the graph is strongly connected.*

Once activated, a subsystem may be active continuously for at least $p_i^m \in \mathbb{N}^+$ and at most $p_i^M \in \mathbb{N}^+$ samples, that are assumed known. Our goal is to find an admissible activation sequence such that the switching system is stabilized.

0 and $I$ denote the zero and identity matrices of appropriate dimensions, and a $(*)$ denotes the term induced by symmetry. The subscript $z + m$ (as in $A_{1z+m}$) stands for the scheduling vector being evaluated at the current sample plus $m$th instant, i.e., $\boldsymbol{z}(k + m)$.

In what follows, we will make use of the following results:

**Lemma 1.** *(Skelton et al. 1998) Consider a vector $\boldsymbol{x} \in \mathbb{R}^{n_x}$ and two matrices $Q = Q^T \in \mathbb{R}^{n_x \times n_x}$ and $R \in \mathbb{R}^{m \times n_x}$ such that rank$(R) < n_x$. The two following expressions are equivalent:*

*(1)* $\boldsymbol{x}^T Q \boldsymbol{x} < 0$, $\boldsymbol{x} \in \{\boldsymbol{x} \in \mathbb{R}^{n_x}, \boldsymbol{x} \neq 0, R\boldsymbol{x} = 0\}$
*(2)* $\exists M \in \mathbb{R}^{m \times n_x}$ *such that* $Q + MR + R^T M^T < 0$

Analysis and design for TS models often lead to double-sum negativity problems of the form

$$\boldsymbol{x}^T \sum_{j=1}^{r} \sum_{k=1}^{r} h_j(\boldsymbol{z}(k)) h_k(\boldsymbol{z}(k)) \Gamma_{j,k} \boldsymbol{x} < 0 \tag{2}$$

where $\Gamma_{j,k}$, $j, k = 1, 2, \ldots, r$ are matrices of appropriate dimensions.

**Lemma 2.** *(Tuan et al. 2001) The double-sum* (2) *is negative, if*

$$\begin{aligned}&\Gamma_{ii} < 0\\ &\frac{2}{r-1}\Gamma_{ii} + \Gamma_{ij} + \Gamma_{ji} < 0, \quad i, j = 1, 2, \ldots, r, \, i \neq j\end{aligned}$$

## 3. Stabilizing switching law

We now consider the problem of determining a switching law which stabilizes a given switching system. The following assumptions are made:

(1) a stable subsystem cannot be active for an infinite number of samples
(2) the graph associated to the system is strongly connected: from any initial subsystem there exists a path to any other subsystem

Let us discuss the assumptions above. Recall that our goal is to obtain convergence of all the states to zero. Since there is no control input, we have to do this by either keeping continuously active a subsystem that is stable or by switching between the subsystems.

Assumption 1 is related to the existence of stable subsystems. If there exists a stable subsystem that can be active for an infinite number of samples, the states converge to zero if this subsystem is kept active. If the initial condition implies a different subsystem, stabilization is obtained by switching to the stable subsystem and keeping it active. Thus, on the level of the whole switching system, the problem can be reformulated as finding a path from each subsystem to this stable one. If such a path exists, the problem is solved. However, if no stable subsystem can be continuously active, a law that continuously switches between subsystems has to be constructed to obtain convergence of the states to zero.

Assumption 2 is related to the fact that the switching law may not contain all the subsystems. However, if the associated graph is strongly connected, meaning that there exists a path from any subsystem to any other subsystem, it is possible to switch from any initial subsystem to one that is contained in the switching law. The case when Assumption 2 is not satisfied will be discussed later on.

Recall that we consider the associated directed graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where the vertices $\mathcal{V} = \{v_1, v_2, \ldots, v_{n_s}\}$ correspond to the subsystems, and each edge $e_{i,j} = (v_i, v_j) \in \mathcal{E}$ corresponds to an admissible transition.

In what follows, we build a weight-adjacency matrix, that assigns to each admissible transition, including self-transitions, a weight. By convention, if $(v_i, v_j) \notin \mathcal{E}$, for $i \neq j$, $i, j = 1, 2, \ldots, n_s$ the corresponding weight $w_{i,j} = \infty$, and if $(v_i, v_i) \notin \mathcal{E}$, $i = 1, 2, \ldots, n_s$, then the corresponding weight $w_{i,i} = 1$. For all other edges, the weight will be determined by the properties of the corresponding switch.

For this, consider the Lyapunov function

$$V(\boldsymbol{x}(k)) = \boldsymbol{x}(k)^T P_{i,z} \boldsymbol{x}(k) \tag{3}$$

with $P_{i,z} = P_{i,z}^T > 0$, for the $i$th subsystem, $i = 1, 2, \ldots, n_s$.

Using these Lyapunov functions our goal is to determine upper bounds on their increase for each switch, i.e., finding constants $\delta_{i,j} > 0$ so that $V(\boldsymbol{x}(k+1)) \leq \delta_{i,j} V(\boldsymbol{x}(k))$ for the transition $(v_i, v_j)$.

For any $(v_i,\, v_j) \in \mathcal{E}$ we have

$$
\begin{aligned}
&V(\boldsymbol{x}(k+1)) - \delta_{i,j} V(\boldsymbol{x}(k)) < 0 \\
&\boldsymbol{x}(k+1)^T P_{j,z+1} \boldsymbol{x}(k+1) - \delta_{i,j} \boldsymbol{x}(k)^T P_{i,z} \boldsymbol{x}(k) < 0 \\
&\begin{pmatrix} \boldsymbol{x}(k) \\ \boldsymbol{x}(k+1) \end{pmatrix}^T \begin{pmatrix} -\delta_{i,j} P_{i,z} & 0 \\ 0 & P_{j,z+1} \end{pmatrix} \begin{pmatrix} \boldsymbol{x}(k) \\ \boldsymbol{x}(k+1) \end{pmatrix} < 0
\end{aligned}
$$

At the same time, the system's dynamics can be written as

$$
\begin{pmatrix} A_{i,z} & -I \end{pmatrix} \begin{pmatrix} \boldsymbol{x}(k) \\ \boldsymbol{x}(k+1) \end{pmatrix} = 0
$$

Using Lemma 1, we have $V(\boldsymbol{x}(k+1)) - \delta_{i,j} V(\boldsymbol{x}(k)) < 0$ if there exists $\overline{M}_{i,j,z}$ so that

$$
\begin{pmatrix} -\delta_{i,j} P_{i,z} & 0 \\ 0 & P_{j,z+1} \end{pmatrix} + \overline{M}_{i,j,z} \begin{pmatrix} A_{i,z} & -I \end{pmatrix} + (*) < 0
$$

Choosing $\overline{M}_{i,j,z} = \begin{pmatrix} 0 \\ M_{i,j,z} \end{pmatrix}$ we obtain the sufficient conditions

$$
\begin{pmatrix} -\delta_{i,j} P_{i,z} & (*) \\ M_{i,j,z} A_{i,z} & -M_{i,j,z} - M_{i,j,z}^T + P_{j,z+1} \end{pmatrix} < 0 \tag{4}
$$

To find all $\delta_{i,j}$, one has to solve (4) for all $(v_i,\, v_j) \in \mathcal{E}$.

Once all $\delta_{i,j}$ are available, define the weight matrix as $\mathcal{W} = [w_{i,j}]$ with

$$
w_{i,j} = \begin{cases} \delta_{i,i}^{p_i^m - 1} & \text{if } \delta_{i,i} > 1 \\ \delta_{i,i}^{p_i^M - 1} & \text{if } \delta_{i,i} < 1 \\ \delta_{i,j} & \text{if } i \neq j \end{cases} \tag{5}
$$

Then, the following result can be stated:

**Theorem 1.** *The switching system (1) is asymptotically stabilizable by a switching law, if its associated graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{W}\}$ contains a subunitary cycle $\mathcal{C}_n$. Furthermore, for the ith initial subsystem, $i = 1,\, 2,\, \ldots,\, n_s$, the stabilizing switching law is given by $\mathcal{P}(v_i, v_j)\mathcal{C}_n$, where $v_i$ denotes the vertex corresponding to the initial subsystems, $\mathcal{P}(v_i, v_j)$ is a path to vertex $v_j$, with $v_j \in \mathcal{C}_n$.*

**Proof.** *Consider the switching system (1) and recall that the associated graph description is $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{W}\}$, where the vertices $\mathcal{V} = \{v_1,\, v_2,\, \ldots,\, v_{n_s}\}$ correspond to the subsystems, each edge $e_{i,j} = (v_i,\, v_j) \in \mathcal{E}$ corresponds to an admissible transition between two subsystems, and the weight matrix $\mathcal{W}$ is the one constructed above.*

*Assume that there exists a subsunitary cycle, i.e., there exists $\mathcal{C}_n = \{v_{c1},\, v_{c2},\, \ldots,\, v_{cp},\, v_{c1}\}$ such that the product of the edges and nodes in this cycle is subunitary, and let this product be denoted by $\delta_n$. For any subsystem $i$ such that $v_i \in \mathcal{C}_n$, we have $V(\boldsymbol{x}(k+n_c)) < \delta_n V(\boldsymbol{x}(k))$, i.e., after a full cycle, the corresponding Lyapunov function increases at most $\delta_n$ times, with $\delta_n < 1$. Consequently, the periodic switching law $\mathcal{C} = [c_1,\, c_2,\, \ldots,\, c_p,\, c_1]$ stabilizes the periodic system given by this cycle.*

*Let us now see the subsystems (if any) that are not in $\mathcal{C}_n$. Since we assumed that the $\mathcal{G}$ is strongly connected (Assumption 2), for all $v_i \notin \mathcal{C}$, there exists a path $\mathcal{P}(v_i, v_j)$ from the ith subsystem to a subsystem $j$ on the cycle. Consider the switching law $\mathcal{P}(v_i, v_j)\mathcal{C}_n$, i.e., first a switching law that leads to the cycle and then the periodic switching law corresponding to the cycle. Since none of the* *subsystems have a finite escape time, even though during the switches corresponding to $\mathcal{P}(v_i, v_j)$ the Lyapunov function might increase, during the periodic switching it will eventually decrease and consequently stabilize the system.*

It should be noted that, if it exists, the periodic stabilizing law may not be unique. Let us construct the weight matrix as $\mathcal{W} = [w_{i,j}]$ with

$$w_{i,j} = \begin{cases} \delta_{i,i}^{q_i} & \text{if } i = j \\ \delta_{i,j} & \text{if } i \neq j \end{cases}$$

with $q_i \in \{p_i^m - 1, p_i^m, \ldots, p_i^M - 1\}$. If the associated graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{W}\}$ contains a subunitary cycle $\mathcal{C}_n$, then $\mathcal{C}_n$ will give a stabilizing periodic law. The proof is similar to that of Theorem 1.

A quantifiable difference in the stabilizing switching laws consists in the guaranteed lowest upper bound on the convergence rate obtained. With the weight matrix constructed as above, once the periodic law becomes active, the Lyapunov function decreases in each cycle $\delta_n$ times, with

$$\begin{aligned} \delta_n &= \prod_{k=1}^{n_p} w_{p_k,p_k} \cdot \prod_{k=1}^{n_p-1} w_{p_k,p_{k+1}} \\ &= \prod_{k=1}^{n_p} \delta_{v_{p_k},v_{p_k}}^{q_{v_{p_k}}} \cdot \prod_{k=1}^{n_p-1} \delta_{v_{p_k},v_{p_{k+1}}} \end{aligned}$$

where $(v_{p_k}, v_{p_{k+1}}) \in \mathcal{C}_n$. Since each element in the product is positive, the product is minimized by choosing the minimal admissible $q_{v_{p_k}}$ if $\delta_{v_{p_k},v_{p_k}} > 1$ and the maximal admissible $q_{v_{p_k}}$ if $\delta_{v_{p_k},v_{p_k}} < 1$. Thus, by choosing the weight matrix (5), we obtain the periodic law with the lowest upper bound on the convergence rate.

It has to be noted that we use the one-sum Lyapunov function $V = \boldsymbol{x}_k^T P_{i,z} \boldsymbol{x}_k$ for the easier notation and derivation. The conditions (4) can be easily derived for the n-sum Lyapunov function $V = \boldsymbol{x}_k^T P_{i,\underbrace{z\ldots z}_{n}} \boldsymbol{x}_k$ and using the n-sum matrix $M_{i,j,\underbrace{z\ldots z}_{n}}$, leading to (Kruszewski et al. 2008; Megretski 1996)

$$\begin{pmatrix} -\delta_{i,j} P_{i,\underbrace{z\ldots z}_{n}} & (*) \\ M_{i,j,\underbrace{z\ldots z}_{n}} A_{i,z} & -M_{i,j,\underbrace{z\ldots z}_{n}} - M_{i,j,\underbrace{z\ldots z}_{n}}^T + P_{j,\underbrace{z+1\ldots z+1}_{n}} \end{pmatrix} < 0$$

On the other hand, conditions to determine that there is no switching law that can stabilize a given system can also be obtained. Consider the Lyapunov function (3) and let us now find lower bounds on the increase of the Lyapunov function in one sample, i.e., constants $\omega_{i,j} > 0$ so that $V(\boldsymbol{x}(k+1)) \geq \omega_{i,j} V(\boldsymbol{x}(k))$, if the transition is $(v_i, v_j)$.

For any $(v_i,\, v_j) \in \mathcal{E}$ we have

$$\begin{aligned} & V(\boldsymbol{x}(k+1)) - \omega_{i,j} V(\boldsymbol{x}(k)) > 0 \\ & \boldsymbol{x}(k+1)^T P_{j,z+1} \boldsymbol{x}(k+1) - \omega_{i,j} \boldsymbol{x}(k)^T P_{i,z} \boldsymbol{x}(k) > 0 \\ & \begin{pmatrix} \boldsymbol{x}(k) \\ \boldsymbol{x}(k+1) \end{pmatrix}^T \begin{pmatrix} -\omega_{i,j} P_{i,z} & 0 \\ 0 & P_{j,z+1} \end{pmatrix} \begin{pmatrix} \boldsymbol{x}(k) \\ \boldsymbol{x}(k+1) \end{pmatrix} > 0 \end{aligned}$$

Combining it with the system's dynamics and choosing $\overline{M}_{i,j,z} = \begin{pmatrix} M_{i,j,z} \\ 0 \end{pmatrix}$ in Finsler's lemma, we obtain the conditions

$$\begin{pmatrix} -M_{i,j,z}A_{i,z} + (*) + \omega_{i,j}P_{i,z} & M_{i,j,z} \\ (*) & P_{j,z+1} \end{pmatrix} > 0 \tag{6}$$

Now, define the lower weight matrix as $\underline{\mathcal{W}} = [\underline{w}_{i,j}]$ with

$$\underline{w}_{i,j} = \begin{cases} \omega_{i,i}^{p_i^m} & \text{if } \omega_{i,i} > 1 \\ \omega_{i,i}^{p_i^M} & \text{if } \omega_{i,i} < 1 \\ \omega_{i,j} & \text{if } i \neq j \end{cases}$$

and the following result can be stated:

**Corollary 1.** *The switching system* (1) *is not stabilizable by a switching law, if all the cycles in the associated graph* $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \underline{\mathcal{W}}\}$ *are supraunitary.*

## 4. Discussion and examples

The first issue that has to be noted is that due to the term $\delta P$, the conditions (4) are BMIs. Although the solution will not be optimal, however, these BMIs can be solved in several ways using two-step LMI conditions. A first possibility is

(1) Solve (4) for all $i = j$, i.e., for each subsystem. In this case, the $P_{i,z}$s are independent, and each condition can be formulated as a *gevp* problem. For this step, we have the conditions

$$\begin{pmatrix} -\delta_{i,i}P_{i,z} & (*) \\ M_{i,i,z}A_{i,z} & -M_{i,i,z} - M_{i,i,z}^T + P_{i,z+1} \end{pmatrix} < 0$$

for which sufficient conditions can be formulated using e.g., Lemma 2, with

$$\Gamma_{j,k}^{i,l} = \begin{pmatrix} -\delta_{i,i}P_{i,j} & (*) \\ M_{i,i,j}A_{i,k} & -M_{i,i,j} - M_{i,i,j}^T + P_{i,l} \end{pmatrix} < 0$$

for $i = 1, 2, \ldots, n_{\mathrm{s}}$. For each subsystem, the *gevp* problem can be independently solved. Note that in the expression above $i$ denotes the index of the subsystem and $l$ corresponds to a different time instant.

(2) Once the $P_{i,z}$s are obtained, using them one can solve the conditions for different subsystems, i.e,

$$\begin{pmatrix} -\delta_{i,j}P_{i,z} & (*) \\ M_{i,j,z}A_{i,z} & -M_{i,j,z} - M_{i,j,z}^T + P_{j,z+1} \end{pmatrix} < 0$$

with $i, j = 1, 2, \ldots, n_{\mathrm{s}}$, $i \neq j$, for the decision variables $\delta_{i,j}$ and $M_{i,j,z}$. Sufficient LMI conditions can again be formulated using e.g., Lemma 2.

Another possibility is solving all the conditions (4) first for the decision variables $P_{i,z}$ and $M_{i,j,z}$, $i, j = 1, 2, \ldots, n_{\mathrm{s}}$ and afterward for $\delta_{i,j}$ and $M_{i,j,z}$ $i, j = 1, 2, \ldots, n_{\mathrm{s}}$, minimizing at the same time $\delta_{i,j}$. Alternatively, one can use a path-following algorithm or available BMI solvers, such as *penbmi* (Kočvara and Stingl 2008).

Since $\delta_{i,j}$, $i, j = 1, 2, \ldots, n_\mathrm{s}$, correspond to an upper bound on the increase or decrease of the Lyapunov function during a transition from the $i$th to the $j$th subsystem, the conditions (4) always have a solution with large enough $\delta_{i,j}$, $i, j = 1, 2, \ldots, n_\mathrm{s}$. A special case is when the transition is a self-transition, and the constant is subunitary, i.e., $\delta_{i,i} < 1$, in which case this constant corresponds to the decay-rate of the subsystem. We do not require that $\delta_{i,j}$, $i, j = 1, 2, \ldots, n_\mathrm{s}$, are all subunitary. Indeed, our goal is to find a subunitary cycle, for which it may be enough if there exists a single $\delta_{i,j} < 1$.

The problem of finding a subunitary cycle can be approached in two ways. On the one hand, one can first solve the conditions (4) for the whole graph and find the subunitary cycles. This can be done by using the logarithm of $\delta_{i,j}$ as weights in the graph and employing the methods proposed e.g., in (Cherkassky and Goldberg 1999; Yamada and Kinoshita 2002; Hanusa 2009). On the other hand, one can generate all elementary cycles in the graph, and solve independently the conditions (4) for each cycle until a subunitary cycle is found. This approach will be illustrated in what follows.

**Example 3.** *Consider the switching system composed of four subsystems, with the associated graph from Example 2, i.e.,*

$$\boldsymbol{x}(k+1) = A_{i,z}\boldsymbol{x}(k)$$

*for* $i = 1, 2, 3, 4$, *and with admissible switches* $(1,2)$, $(2,1)$, $(2,3)$, $(3,1)$, $(4,3)$, $(1,4)$. *The graph can be defined as* $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, *with* $\mathcal{V} = \{1, 2, 3, 4\}$ *and*

$$\mathcal{E} = \{(1,1), (1,2), (1,4), (2,1), (2,2), (2,3), (3,1), (3,3), (4,3), (4,4)\}$$

*The local matrices and membership functions are given as:*

$$\begin{aligned}
A_{1,1} &= \begin{pmatrix} 0.5 & 0.2 \\ 0.85 & 0.2 \end{pmatrix} & A_{1,2} &= \begin{pmatrix} 0.75 & 0.3 \\ 0.2 & 0.5 \end{pmatrix} \\
h_{1,1} &= \frac{1 - \sin(x_1(k))}{2} & h_{1,2} &= 1 - h_{1,1} \\
A_{2,1} &= \begin{pmatrix} 1.3 & 0.6 \\ -0.22 & 0.5 \end{pmatrix} & A_{2,2} &= \begin{pmatrix} 0.5 & -0.15 \\ -0.8 & 0.8 \end{pmatrix} \\
h_{2,1} &= \frac{1 - \cos(x_1(k))}{2} & h_{2,2} &= 1 - h_{2,1} \\
A_{3,1} &= \begin{pmatrix} 0.8 & 0.41 \\ 0.6 & 0.2 \end{pmatrix} & A_{3,2} &= \begin{pmatrix} 0.06 & 0.81 \\ 0.35 & 0.1 \end{pmatrix} \\
h_{3,1} &= 1 - \exp(-x_1^2(k)) & h_{3,2} &= 1 - h_{3,1} \\
A_{4,1} &= \begin{pmatrix} 0.45 & 0.4 \\ 0.8 & 0.4 \end{pmatrix} & A_{4,2} &= \begin{pmatrix} 0.8 & 0.2 \\ 0.5 & 0.6 \end{pmatrix} \\
h_{4,1} &= \frac{1 - \tanh(x_1(k))}{2} & h_{4,2} &= 1 - h_{4,1}
\end{aligned}$$

*where* $x_1(k)$ *denotes the first element of the state vector* $\boldsymbol{x}$ *at time* $k$. *Each subsystem can be active for at least* 2 *and at most* 5 *samples, i.e.,* $p_i^m = 2$ *and* $p_i^M = 5$, $i = 1, 2, 3, 4$. *Our goal is to find a switching law that can stabilize this system.*

*The first subsystem is stable. However, this subsystem can only be active for at most* 5 *samples. The remaining subsystems are not stable: both local models of the second subsystem, the first local model of the third and the second local model of the fourth subsystem have eigenvalues larger than* *one.*

*Let us now inspect the associated graph of admissible switches. Although the number of switches is quite large, there are only three elementary cycles:* $\mathcal{C}_1 = [1,\ 2,\ 1]$, $\mathcal{C}_2 = [1,\ 2,\ 3,\ 1]$, *and* $\mathcal{C}_3 = [1,\ 4,\ 3,\ 1]$. *All the remaining cycles are equivalent to one of these three. To ease the computations, we consider just the weights for the three enumerated cycles. We solve the conditions* (4) *using the two-step procedure explained above*[2], *i.e., first solving the* gevp *problems associated to each subsystem and then the interconnection LMIs. We obtain:*

*(1) Cycle* $\mathcal{C}_1 = [1,\ 2,\ 1]$*:*

$$\delta = \begin{pmatrix} 0.8333 & 1.6743 & \times & \times \\ 2.0096 & 1.3564 & \times & \times \\ \times & \times & \times & \times \\ \times & \times & \times & \times \end{pmatrix}$$

*The weight matrix*[3] *is*

$$\mathcal{W}_1 = \begin{pmatrix} 0.4822 & 1.6743 & \times & \times \\ 2.0096 & 1.3564 & \times & \times \\ \times & \times & \times & \times \\ \times & \times & \times & \times \end{pmatrix}$$

*The weight of the cycle* $W(\mathcal{C}_1) = 2.20 > 1$, *consequently no conclusion can be drawn.*

*(2) Cycle* $\mathcal{C}_2 = [1,\ 2,\ 3,\ 1]$*:*

$$\delta = \begin{pmatrix} 0.8333 & 3.1171 & \times & \times \\ \times & 1.3564 & 1.5379 & \times \\ 0.5840 & \times & 1.1657 & \times \\ \times & \times & \times & \times \end{pmatrix}$$

*The weight matrix is*

$$\mathcal{W}_2 = \begin{pmatrix} 0.4822 & 3.1171 & \times & \times \\ \times & 1.3564 & 1.5379 & \times \\ 0.5840 & \times & 1.1657 & \times \\ \times & \times & \times & \times \end{pmatrix}$$

*The weight of the cycle* $W(\mathcal{C}_2) = 2.13 > 1$, *consequently no conclusion can be drawn.*

*(3) Cycle* $\mathcal{C}_3 = [1,\ 4,\ 3,\ 1]$*:*

$$\delta = \begin{pmatrix} 0.8333 & \times & \times & 6.0713 \\ \times & 1.3564 & \times & \times \\ 0.3544 & \times & 1.1657 & \times \\ \times & \times & 0.6673 & 1.0643 \end{pmatrix}$$

[2] Similar results are obtained using *penbmi* (Kočvara and Stingl 2008).
[3] $\times$ denotes an element that is not of interest for the particular case.

*The weight matrix is*

$$\mathcal{W}_3 = \begin{pmatrix} 0.4822 & \times & \times & 6.0713 \\ \times & 1.3564 & \times & \times \\ 0.3544 & \times & 1.1657 & \times \\ \times & \times & 0.6673 & 1.0643 \end{pmatrix}$$

*The weight of the cycle* $W(\mathcal{C}_3) = 0.8590 < 1$, *therefore we have the stabilizing periodic law* $[1, 1, 1, 1, 1, 4, 4, 3, 3, 1, \ldots]$.

*The only subsystem that is not part of the periodic switching law that has been found is subsystem 2. However, if this is the initial subsystem, it can be stabilized by switching to any other subsystem (e.g., the first one) and then applying the periodic law. Such a trajectory is illustrated in Figure 3. The initial states were* $\boldsymbol{x}_0 = [-10\,10]^T$, *the initially active subsystem the second, and the trajectory has been obtained by first switching to the first subsystem and then applying the periodic law. As can be seen, the states converge to zero. The switching law is illustrated in Figure 4, and the value of the Lyapunov function in Figure 5. Although the value of the Lyapunov function increases for some switches, it decreases during a cycle.*

*Note that the periodic laws* $[1, 1, 1, 1, 1, 4, 4, 4, 3, 3, 1, \ldots]$ *and* $[1, 1, 1, 1, 1, 4, 4, 4, 4, 3, 3, 1, \ldots]$ *also stabilize the system. However, in this case the weights of the corresponding cycles are* 0.9143 *and* 0.9731, *respectively, thus, when applying one of these laws, the states will converge slower to zero.*

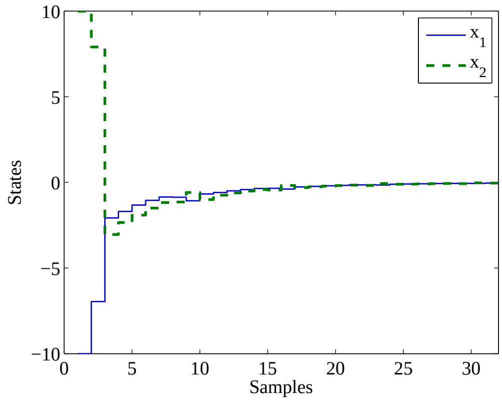


Figure 3. Trajectory of the states of the switching system in Example 3.

Let us now return to the assumption that the graph associated to the switching system should be strongly connected. Indeed, if this graph is strongly connected and if there exists a subunitary cycle, then starting from any subsystem, the states can be stabilized to zero, as shown in Section 3. Let us assume now that the graph is not strongly connected. In this case, the existence of a stabilizing switching law depends on the initial subsystem. We illustrate this on an example.

**Example 4.** *Consider a switching system composed of seven subsystems, with the associated graph illustrated in Figure 6.*

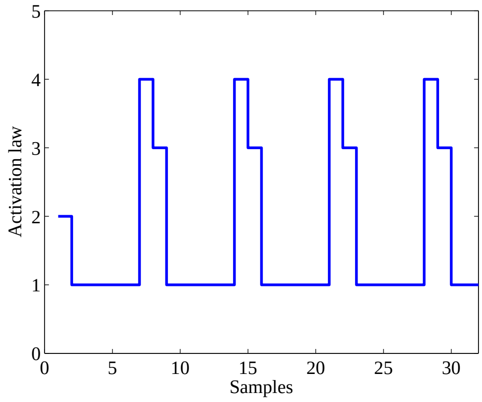


Figure 4. The switching law for Example 3.

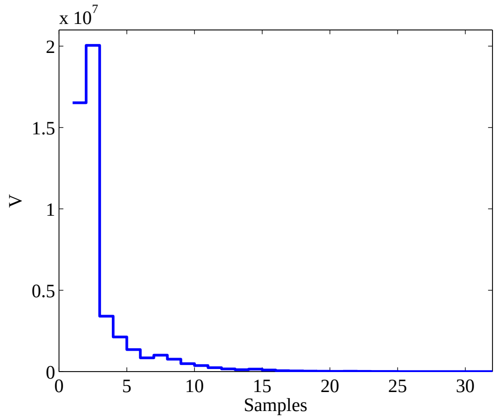


Figure 5. The value of the Lyapunov function in Example 3.

*Let us assume that after solving the BMIs in* (4)*, the associated weight matrix is given by*

$$\mathcal{W} = \begin{pmatrix} 1 & 2 & \infty & \infty & \infty & \infty & \infty \\ \infty & 1 & 0.2 & \infty & \infty & \infty & \infty \\ 0.5 & \infty & 1.5 & 2 & \infty & \infty & \infty \\ \infty & \infty & \infty & 1 & \infty & \infty & 1.5 \\ \infty & \infty & \infty & 1 & 1.2 & 0.5 & \infty \\ \infty & \infty & \infty & \infty & 0.75 & 1.5 & \infty \\ \infty & \infty & \infty & 0.5 & \infty & \infty & 1.5 \end{pmatrix}$$

*where* $\infty$ *corresponds to an inadmissible switch. It can easily be seen from Figure 6 that there are two stable periodic switching laws: one involving subsystems* 1, 2, *and* 3, *with weight* $W_{1,2,3} = 1 \cdot 2 \cdot 1 \cdot 0.2 \cdot 1.5 \cdot 0.5 = 0.3$ *and a second involving subsystems* 5 *and* 6 *with weight* $W_{5,6} = 1.2 \cdot 0.5 \cdot 1.5 \cdot 0.75 = 0.675$*. Consequently, starting from subsystems* 1, 2, 3, 5, *or* 6*, the switching system can be stabilized, by choosing the corresponding periodic switching law. Let us inspect now subsystems* 4 *and* 7*. From these subsystems there are no paths to any other subsystems, and the weight of the cycle* $W(\mathcal{C}_{4,7}) = 1.125 > 1$*, therefore no conclusion can be drawn.*

To summarize, our goal has been to stabilize a switching system, i.e., obtain convergence of all the states to zero. The only instrument to do this – since there is no control input – is by switching between the subsystems. We construct a periodic switching law that stabilizes the system. This switching law may not contain all the subsystems. If the subsystems are strongly connected, even

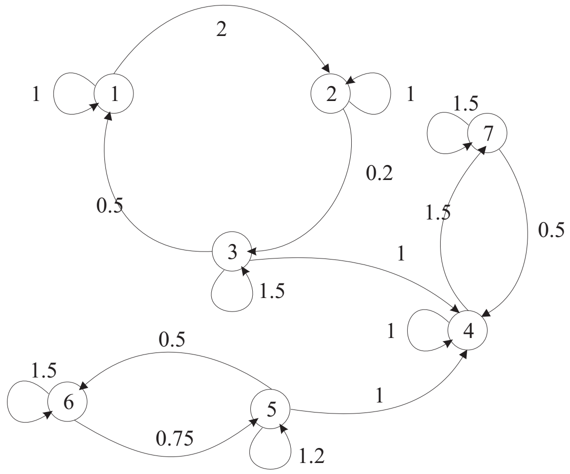


Figure 6. Graph representation of the switching system in Example 4.

if a subsystem is not part of the switching law – such as subsystem 2 in Example 3, which is only activated if it is the starting subsystem – there exists a path to switch to a subsystem that is part of the switching law. If the graph is not strongly connected, we might not be able to switch to a subsystem that is "on" the switching law, and therefore each strongly connected subgraph has to be treated separately. Such a case is discussed in Example 4: starting from subsystems 4 or 7, we cannot switch to a subsystem contained in a stabilizing switching law and thus, for such initial conditions, the system may not be stabilized.

To compare our results to recent ones from the literature, we adapt Example 1 from (Zhang et al. 2009).

**Example 5.** *Consider the switching system with two linear discrete-time subsystems with the system matrices (Zhang et al. 2009)*

$$A_1 = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \qquad A_2 = \begin{pmatrix} 1.5 & 1 \\ 0 & 1.5 \end{pmatrix}$$
$$B_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \qquad B_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

*Since in the present approach we do not consider controller gain design, the control gains have been designed separately and have the values*

$$K_1 = \begin{pmatrix} 0.1 & 1 \end{pmatrix} \qquad K_2 = \begin{pmatrix} 1.5 & 1 \end{pmatrix}$$

*Thus, the closed-loop system matrices are (with a slight abuse of notation)*

$$A_1 = \begin{pmatrix} 1.9 & -1 \\ -0.2 & 0 \end{pmatrix} \qquad A_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1.5 \end{pmatrix}$$

*It can easily be verified that none of the matrices is Schur. However, by simply switching between the subsystems, the states will converge to zero, thus a stabilizing switching law is* $[1, 2, 1, 2, \ldots]$. *The state trajectories of the closed-loop system, starting from* $\boldsymbol{x} = [5, 5]^T$ *and using this switching law, are illustrated in Figures 7 and 8.*

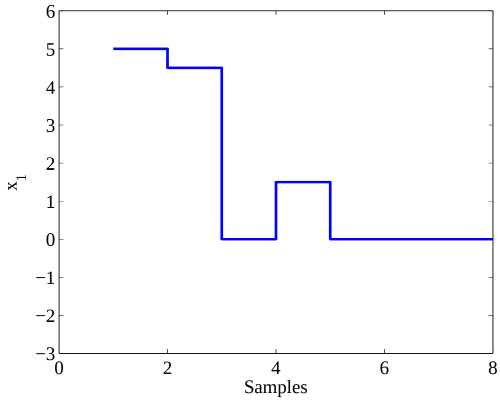


Figure 7. Trajectory of $x_1$ from Example 5.

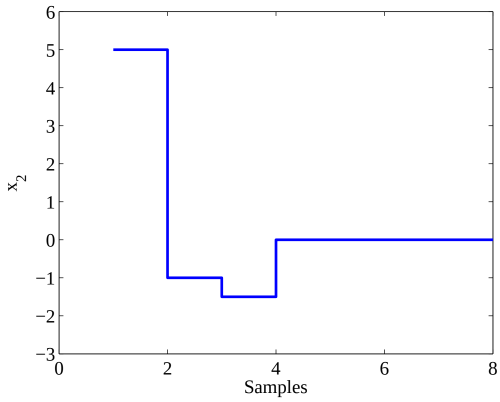


Figure 8. Trajectory of $x_2$ from Example 5.

*Let us now extend this system to a TS one, with local matrices*

$$A_{1,1} = \begin{pmatrix} 1.9 & -1 \\ -0.2 & 0 \end{pmatrix} \qquad A_{1,2} = \begin{pmatrix} 1.9 & -.8 \\ -0.2 & 0 \end{pmatrix}$$
$$A_{2,1} = \begin{pmatrix} 0 & 0 \\ 0 & 1.5 \end{pmatrix} \qquad A_{2,2} = \begin{pmatrix} 0 & 0.5 \\ 0 & 1.5 \end{pmatrix}$$

*Note that this system may represent a buck-boost power converter with uncertainty in the inductance, which appears in many practical problems.*

*For this system, not being linear, the results in (Zhang et al. 2009) no longer apply. However, following our approach, we have*

$$\delta = \begin{pmatrix} 4.02 & 0.27 \\ 0.27 & 2.25 \end{pmatrix}$$

*and the weight of the cycle* $[1, 2, 1]$ *is* $4.02 \cdot 0.27 \cdot 2.25 \cdot 0.27 = 0.65 < 1$, *thus a stabilizing switching law is* $[1, 2, 1, 2, \ldots]$.

## 5. Conclusions

In this paper we have investigated stabilization of switching nonlinear systems by a suitably chosen switching law. For this, we first developed conditions that, when satisfied, guarantee that the system is stabilizable by switching law. We employed a switching Lyapunov function to derive the conditions. If the conditions are satisfied, we proceed to construct the switching law that stabilizes all the states to zero. The developed conditions and switching laws have been illustrated on examples.

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