Stabilization of TS fuzzy systems with time-delay and nonlinear consequents

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Abstract: This paper proposes a controller design method for Takagi-Sugeno fuzzy systems with nonlinear consequents when the input is affected by time-delay. We consider that the membership functions may depend on both current and delayed states. To handle the nonlinear consequents a slope bounded condition is used. The design conditions are formulated in terms of linear matrix inequalities. A numerical example illustrates the obtained results.

Keywords: TS fuzzy systems, time-delay, stabilization, Lyapunov synthesis, LMIs

1. INTRODUCTION

Time-delay systems are common in real-life, in particular in transportation (Laurain et al., 2017), biological systems (Chang and Chen, 2010), networked control systems (Ma et al., 2013), etc. When there is a significant distance between the actuator and the system, the effect of the actuator is not instantaneous, but happens with a delay. In networked controlled systems, where the controller and the actuator are not at the same location, the time-delay varies also based on the load of the communication network (Ma et al., 2013). Time-delays are usually non-negligible factors, and need to be considered in the controller design.

Therefore, stability and stabilizing control of time-delay systems are major research topics, and they have been widely explored in the past years. A general overview of delay-dependent and delay-independent stability analysis of linear time-delay systems and the Lyapunov functionals used to derive stability conditions have been presented in (Fridman, 2014), together with some results on stability and stabilization of nonlinear time-delay systems. The problem of stabilizing controller design for single input single output linear systems where the delay is a transport delay and input-dependent, has been explored in (Bresch-Pietri et al., 2014), where a prediction based controller has been designed. Yin et al. (2011) presents a robust H_∞ controller for Lur'e systems with bounded nonlinearities. Although there is an extension in the direction of nonlinear systems, the case of input delays are not discussed in (Yin et al., 2011), and the nonlinearities need to fulfill slope and sector bounded conditions.

Nonlinear time-delay systems are often represented in a polytopic form, among which widely used are the Takagi-Sugeno (TS) fuzzy models. The TS model is classically a convex combination of local linear models. Conditions in this framework are usually formulated as Linear Matrix Inequalities (LMIs), as these are easy to solve with existing convex optimization methods (Lendek et al., 2011).

Recently, stability analysis and controller design of time-delay TS models has attracted significant interest. (Yang et al., 2014) propose a stability criteria based on a quadratically convex combination approach for continuous-time TS systems with time-varying delay using an augmented Lyapunov-Krasovskii functional. (Zhang et al., 2020) develop conditions for the stabilization of TS models using a PDC controller based on an impulse-time-dependent Lyapunov function. (Gao et al., 2019) present improved delay-dependent conditions for stability and stabilization of TS models. However, in all the results, the delay in the input is rarely considered. This is an important issue, as, when using a fuzzy controller, a delay in the input leads to delay in the membership functions of the controller.

TS models are often developed using the sector nonlinearity approach (Ohtake et al., 2001), as this leads to a fuzzy model that is equivalent to the original nonlinear model in a convex set of the state-space. However, when using this approach the number of local models may increase exponentially with the number of nonlinearities, and the analysis or design problem may become computationally intractable. A possibility to avoid the exponential increase of the local models is to keep some of the nonlinearities in their original form and consider them as local nonlinearities. This idea has been used for both observer and controller design in several results, both in continuous and discrete-time case, see e.g., (Dong et al., 2009, 2010; Moodi and Farrokhi, 2013, 2015; Nagy and Lendek, 2019).

Therefore, in this paper we consider time-delay TS systems with nonlinear consequents. A similar approach has been presented by Moodi and Kazemy (2019), where the nonlinearities fulfill a sector bounded condition. In our approach we use a slope bounded condition, which can handle different types of nonlinearities. Furthermore, we consider TS models with time-delay in the input and where the membership functions may depend on both current and delayed states.

The rest of the paper is organized as follows. In Section 2 we introduce the necessary concepts for time-delay

TS fuzzy models with nonlinear consequents. Section 3 presents the considered controller structure and the conditions developed for controller design. The controller is illustrated on an example in Section 4 and a comparison with another result from the literature is also presented here. Section 5 concludes the paper.

Notations. Let $F = F^T \in R^{n\times n}$ be a real symmetric matrix; $F > 0$ and $F < 0$ mean that F is positive definite and negative definite, respectively. I denotes the identity matrix and 0 the zero matrix of appropriate dimensions. The symbol $*$ in a matrix indicates a transposed quantity in the symmetric position, for instance $\begin{pmatrix} P & * \\ A & P \end{pmatrix} = \begin{pmatrix} P & A^T \\ A & P \end{pmatrix}$, and $A + * = A + A^T$. The notation $\mathrm{diag}(f_1, ..., f_n)$, where $f_1, ..., f_n \in \mathbb{R}$, stands for the diagonal matrix, whose diagonal components are $f_1, ..., f_n$; $\|x\|$, where $x \in \mathbb{R}^{n_x}$, is the Euclidean norm of x.

2. PRELIMINARIES AND PROBLEM STATEMENT

The time-delay TS fuzzy model is a convex combination of linear models, having the form:

$$\begin{aligned} \dot{x}(t) =& \sum_{i=1}^{s}\sum_{j=1}^{s} h_i(z(t))h_j(z(t-\tau(t))) \\ & (A_{ij}x(t) + D_{ij}x(t-\tau(t)) + B_{ij}u(t-\tau(t))) \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^{n_x}$ is the state vector, $u(t) \in \mathbb{R}^{n_u}$ is the control input, s is the number of rules, $z(t) \in \mathbb{R}^{n_z}$ is the premise vector. $\tau(t)$ is the varying time-delay and h_i, $i = 1, ..., s$ are nonlinear functions with the property

$$h_i \in [0,\, 1],\, i = 1, ..., s, \quad \sum_{i=1}^{s} h_i(z) = 1. \tag{2}$$

These nonlinear functions are called the membership functions. Matrices A_{ij}, B_{ij}, and D_{ij} represent the local models. Note that in this paper we consider state-feedback controller design, and we assume that all the states are available and accurately measured. For simplicity in what follows we omit in the notation the explicit time dependency of the delay function, i.e. we use τ instead of $\tau(t)$. Throughout this paper, the following shorthand notations are used to represent convex sums of matrix expressions:

$$F_z = \sum_{i=1}^{s} h_i(z(t))F_i, \tag{3}$$

$$F_{z_\tau} = \sum_{i=1}^{s} h_i(z(t-\tau))F_i, \tag{4}$$

$$F_{zz_\tau} = \sum_{i=1}^{s} h_i(z(t)) \sum_{j=1}^{s} h_j(z(t-\tau))F_{ij}. \tag{5}$$

Based on this notation, (1) can be rewritten as

$$\dot{x}(t) = A_{zz_\tau}x(t) + D_{zz_\tau}x(t-\tau) + B_{zz_\tau}u(t-\tau) \tag{6}$$

To develop our results the following lemmas and property are considered.

Lemma 1. (Congruence). Given matrix $P = P^T$ and a full column rank matrix Q, it holds that

$$P > 0 \quad \Rightarrow \quad QPQ^T > 0.$$

Estimation and control problems with varying time-delay will be defined as a triple sum negativity problem having the form

$$\sum_{i=1}^{s}\sum_{j=1}^{s}\sum_{k=1}^{s} h_i(z(t))\, h_j(z(t-\tau))\, h_k(z(t-\tau))F_{ijk} < 0, \tag{7}$$

with symmetric matrices F_{ijk}, and nonlinear functions h_i, $i = 1, ..., s$, satisfying the convex sum property in (2).

Lemma 2. (Tuan et al. (2001)). Equation (7) is satisfied if the following conditions hold

$$\begin{aligned} F_{ijj} &\leq 0 \\ \frac{2}{s-1}F_{ijj} + F_{ijk} + F_{ikj} &\leq 0 \quad \forall i, j, k = 1, ..., s,\, j \neq k. \end{aligned} \tag{8}$$

Property 1. (Schur complement) Let $\mathcal{M} = \mathcal{M}^T = \begin{bmatrix} M_{11} & M_{12} \\ M_{12}^T & M_{22} \end{bmatrix}$, with M_{11} and M_{22} square matrices of appropriate dimensions. Then:

$$\begin{aligned} \mathcal{M} < 0 &\Leftrightarrow \begin{cases} M_{11} < 0 \\ M_{22} - M_{12}^T M_{11}^{-1} M_{12} < 0 \end{cases} \\ &\Leftrightarrow \begin{cases} M_{22} < 0 \\ M_{11} - M_{12} M_{22}^{-1} M_{12}^T < 0 \end{cases} \end{aligned} \tag{9}$$

As mentioned before, a shortcoming of TS models obtained by the sector nonlinearity approach is that the number of local models may be exponential in the number of nonlinearities. In order to reduce the number of local models, we consider a form with nonlinear consequents, i.e.,

$$\begin{aligned} \dot{x}(t) =& A_{zz_\tau}x(t) + D_{zz_\tau}x(t-\tau) \\ &+ B_{zz_\tau}u(t-\tau) + B_{zz_\tau}G\psi(Hx(t)). \end{aligned} \tag{10}$$

The quantity $\psi(Hx(t)) \in \mathbb{R}^r$ is an r-dimensional vector where $H \in \mathbb{R}^{r\times n_x}$ and each entry is a function of a linear combination of the states, i.e.

$$\psi_i = \psi_i(\sum_{j=1}^{n} H_{ij}x_j), \quad i = 1, ..., r.$$

To develop our results, the elements in vector $\psi(Hx(t))$ must fulfill the following assumption.

Assumption 1. For any $i \in \{1, ..., r\}$ there exist constants $0 < b_i \leq \infty$, so that

$$0 \leq \frac{\psi_i(v) - \psi_i(w)}{v - w} \leq b_i, \quad \forall v, w \in \mathbb{R}, v \neq w. \tag{11}$$

As a remark, let us consider the case when the nonlinearities do not satisfy (11), but the following is true:

Assumption 2. For any $i \in \{1, ..., r\}$ there exist constants $0 \leq a_i < b_i \leq \infty$, so that

$$a_i \leq \frac{\psi_i(v) - \psi_i(w)}{v - w} \leq b_i, \quad \forall v, w \in \mathbb{R}, v \neq w. \tag{12}$$

If $a_i \neq 0$, a new function can be defined $\tilde{\psi}_i(v) := \psi_i(v) - a_i v$, which satisfies (11), with $\tilde{a}_i = 0$, and $\tilde{b}_i = b_i - a_i$, and the new terms are added to A_{zz_τ}. Assumption 2 intuitively bounds the rate of change of the nonlinearity, and corresponds to a global Lipschitz property of ψ_i, when ψ_i is continuously differentiable. This assumption is made by Arcak and Kokotović (1999, 2001); Chong et al. (2012); Draa et al. (2018).

As in (Chong et al., 2012), in view of (11), there exist $\delta_i(t) \in [0, b_i]$, so that for any $v, w \in \mathbb{R}$

$$\psi_i(v) - \psi_i(w) = \delta_i(t)(v - w). \tag{13}$$

Let $\delta(t) = \mathrm{diag}(\delta_1(t), ..., \delta_r(t))$.

A somewhat restrictive assumption we make is on the form of the nonlinear part, i.e. $B_{zz_\tau} G\psi(Hx(t))$. Note however that such a form often appears, e.g. for mechanical systems in classical state-space form obtained from Euler-Lagrange equations. To see this, let us consider the model of a robot arm

$$M(\theta)\ddot{\theta} = -F(\theta, \dot{\theta}) + u, \tag{14}$$

where u represents the torque; θ, $\dot{\theta}$ and $\ddot{\theta}$ are the angles, angular velocities and angular accelerations. $M(\theta)$ is the mass matrix, $F(\theta, \dot{\theta})$ contains the Centrifugal, Coriolis and gravity matrices. For the classical state-space representation equation (14) must be multiplied with the inverse of the mass matrix. In this context B_{zz_τ} is $M(\theta)^{-1}$.

3. MAIN RESULTS

In this section we develop sufficient conditions for controller design.

To this end, the following control law is considered:

$$u(t) = -K_z x(t) - G\psi(Hx(t)), \tag{15}$$

where $K_z = \sum_{k=1}^{s} h_k(z(t))K_k$, contains the TS fuzzy controller gains. Based on (10) and (15), the closed loop system is:

$$\begin{aligned} \dot{x}(t) =& A_{zz_\tau}x(t) + D_{zz_\tau}x(t-\tau) + B_{zz_\tau}G\psi(Hx(t)) \\ &+ B_{zz_\tau}(-K_{z_\tau}x(t-\tau) - G\psi(Hx(t-\tau))) \\ =& A_{zz_\tau}x(t) + (D_{zz_\tau} - B_{zz_\tau}K_{z_\tau})x(t-\tau) \\ &+ B_{zz_\tau}G(\psi(Hx(t)) - \psi(Hx(t-\tau))). \end{aligned} \tag{16}$$

Furthermore, by using Assumption 1 we obtain:

$$\begin{aligned} \psi(Hx(t)) - \psi(Hx(t-\tau)) =& \delta(t)(Hx(t) - Hx(t-\tau)) \\ =& \delta(t)H(x(t) - x(t-\tau)), \end{aligned} \tag{17}$$

and for simplification we denote $\eta := H(x(t) - x(t-\tau))$. This leads to the following form for (16):

$$\begin{aligned} \dot{x}(t) =& A_{zz_\tau}x(t) + (D_{zz_\tau} - B_{zz_\tau}K_{z_\tau})x(t-\tau) \\ &+ B_{zz_\tau}G\delta(t)\eta \\ \eta =& H(x(t) - x(t-\tau)). \end{aligned} \tag{18}$$

In (Fridman, 2014) several candidate Lyapunov functionals are presented to prove stability for delayed-time systems. We chose the following simple one for our approach:

$$V(t, x) = x^T(t)Px(t) + \int_{t-\tau}^{t} x^T(s)Qx(s)ds \tag{19}$$

This form of the Lyapunov function allows us to find delay-independent conditions, i.e. the conditions depend only on the derivative of the delay function but not the magnitude. In the following theorem we summarize the main result of this work.

Theorem 1. Consider the closed loop system (18), and assume that τ is differentiable, $\dot{\tau} \leq d$ and $d \in [0, 1)$ is a given constant. If there exist matrices $P = P^T > 0$, $Q = Q^T > 0$, $M = \mathrm{diag}(m_1, ..., m_r) > 0$, N_i, $i = 1, ..., s$, and constant $\epsilon > 0$ so that

$$\begin{aligned} F_{ijj} \leq& 0 \\ \frac{2}{s-1}F_{ijj} + F_{ijk} + F_{ikj} \leq& 0 \quad \forall i, j, k = 1, ..., s,\ j \neq k. \end{aligned} \tag{20}$$

where

$$F_{ijk} = \begin{bmatrix} PA_{ij}^T + * + Q & D_{ij}P - B_{ij}N_k & B_{ij}GM + PH^T & P \\ * & -(1-d)Q & -PH^T & 0 \\ * & * & \nu(M) & 0 \\ * & * & * & -\epsilon I \end{bmatrix}, \tag{21}$$

and $\nu(M) = -2M\mathrm{diag}(\frac{1}{b_1}, ..., \frac{1}{b_r})$, then the closed loop system (18) is asymptotically stable

Proof. Consider the following candidate Lyapunov-Krasovskii functional:

$$V(t, x) = x^T(t)\tilde{P}x(t) + \int_{t-\tau}^{t} x^T(s)\tilde{Q}x(s)ds, \tag{22}$$

where $\tilde{P} = \tilde{P}^T > 0$ and $\tilde{Q} = \tilde{Q}^T > 0$. The derivative of V along the trajectories of x is

$$\begin{aligned} \dot{V}(t, x) =& 2\dot{x}(t)^T\tilde{P}x(t) + x^T(t)\tilde{Q}x(t) \\ &- (1 - \dot{\tau}(t))x^T(t-\tau)\tilde{Q}x(t-\tau) \end{aligned} \tag{23}$$

Using $\dot{\tau}(t) \leq d$, and denoting

$$\chi := \begin{bmatrix} x(t) \\ x(t-\tau) \\ \delta(t)\eta \end{bmatrix} \tag{24}$$

we obtain

$$\dot{V}(t, x) \leq \chi^T \begin{bmatrix} \mathcal{A} & \tilde{P}(D_{zz_\tau} - B_{zz_\tau}K_{z_\tau}) & \tilde{P}B_{zz_\tau}G \\ * & -(1-d)\tilde{Q} & 0 \\ * & * & 0 \end{bmatrix} \chi \tag{25}$$

where $\mathcal{A} = A_{zz_\tau}^T\tilde{P} + \tilde{P}A_{zz_\tau} + \tilde{Q}$.

Next we consider the following inequality:

$$\begin{bmatrix} \mathcal{A} + \tilde{\epsilon}I & \tilde{P}(D_{zz_\tau} - B_{zz_\tau}K) & \tilde{P}B_{zz_\tau}G + H^T\tilde{M} \\ * & -(1-d)\tilde{Q} & -H^T\tilde{M} \\ * & * & \nu(\tilde{M}) \end{bmatrix} \leq 0, \tag{26}$$

where $\tilde{M} = \mathrm{diag}(\tilde{m}_1, ..., \tilde{m}_r) > 0$. Using (26) in combination with (25) to obtain

$$\dot{V}(t, x) + \chi^T \begin{bmatrix} \tilde{\epsilon}I & 0 & H^T\tilde{M} \\ * & 0 & -H^T\tilde{M} \\ * & * & \nu(\tilde{M}) \end{bmatrix} \chi \leq 0, \tag{27}$$

leads to

$$\begin{aligned} \dot{V}(t, x) \leq& -\tilde{\epsilon}x(t)^Tx(t) - 2(x(t) - x(t-\tau))^TH^T\tilde{M}\delta(t)\eta \\ &- (\delta(t)\eta)^T\nu(\tilde{M})\delta(t)\eta. \end{aligned} \tag{28}$$

Since $\eta^T = (x(t) - x(t-\tau))^TH^T$, we have

$$\begin{aligned} &\dot{V}(t, x) \leq \\ &-\tilde{\epsilon}x(t)^Tx(t) - 2\eta^T\tilde{M}\delta(t)\eta - (\delta(t)\eta)^T\nu(\tilde{M})\delta(t)\eta \\ &\quad = -\tilde{\epsilon}\|x(t)\|^2 \\ &-2\eta^T\left(\tilde{M}\delta(t) - \delta(t)^T\tilde{M}\mathrm{diag}(\frac{1}{b_1}, ..., \frac{1}{b_r})\delta(t)\right)\eta. \end{aligned} \tag{29}$$

Both $\tilde{M}$ and $\delta(t)$ are diagonal matrices, so we can examine the terms:

$$\tilde{m}_i\delta_i(t) - \tilde{m}_i\frac{1}{b_i}\delta_i(t)^2 = \tilde{m}_i\delta_i(t)\left(1 - \delta_i(t)\frac{1}{b_i}\right). \tag{30}$$

The term $\delta_i(t) \in [0,\, b_i]$, so $\left(1 - \delta_i(t)\frac{1}{b_i}\right) \geq 0$, and since $\tilde{m}_i > 0$, the following holds,

$$2\eta^T \left(\tilde{M}\delta(t) - \delta(t)^T \tilde{M} \mathrm{diag}(\frac{1}{b_1}, ..., \frac{1}{b_r})\delta(t)\right) \eta \geq 0. \quad (31)$$

Finally, we obtain

$$\dot{V}(t,x) \leq -\tilde{\epsilon}\|x(t)\|^2. \quad (32)$$

Now we consider (26). Lemma 2 can be used to define sufficient conditions, but due to the terms $\tilde{P}B_{ij}K_k$ the inequalities are bilinear. To remove this restriction, we pre and post multiply with (Congruence, Lemma 1):

$$\begin{bmatrix} \tilde{P}^{-1} & 0 & 0 \\ 0 & \tilde{P}^{-1} & 0 \\ 0 & 0 & \tilde{M}^{-1} \end{bmatrix},$$

and (26) becomes

$$\begin{bmatrix} \mathcal{B} & (D_{zz_\tau} - B_{zz_\tau}K)\tilde{P}^{-1} & B_{zz_\tau}G\tilde{M}^{-1} + \tilde{P}^{-1}H^T \\ * & -(1-d)\tilde{P}^{-1}\tilde{Q}\tilde{P}^{-1} & -\tilde{P}^{-1}H^T \\ * & * & \nu(\tilde{M}^{-1}) \end{bmatrix} \leq 0, \quad (33)$$

where $\mathcal{B} = \tilde{P}^{-1}A_{zz_\tau} + A^T_{zz_\tau}\tilde{P}^{-1} + \tilde{\epsilon}\tilde{P}^{-2} + \tilde{P}^{-1}\tilde{Q}\tilde{P}^{-1}$. Next, using the Schur complement on $\tilde{\epsilon}\tilde{P}^{-2}$, and denoting $P = \tilde{P}^{-1}$, $Q = \tilde{P}^{-1}\tilde{Q}\tilde{P}^{-1}$, $M = \tilde{M}^{-1}$, $N = K\tilde{P}^{-1}$ and $\epsilon = \frac{1}{\tilde{\epsilon}}$ we obtain the following inequality:

$$\begin{bmatrix} PA^T_{zz_\tau} + * + Q & D_{zz_\tau}P - B_{zz_\tau}N_{z_\tau} & B_{zz_\tau}GM + PH^T & P \\ * & -(1-d)Q & -PH^T & 0 \\ * & * & \nu(M) & 0 \\ * & * & * & -\epsilon I \end{bmatrix} \leq 0. \quad (34)$$

The LMI conditions in (20) are obtained by applying Lemma 2 on (34).

4. EXAMPLES

In the following, first we illustrate the use of the conditions of Theorem 1, then we compare our results with the results obtained using the conditions presented by Moodi and Kazemy (2019).

4.1 Numerical example

For simplicity we omit the time dependency of the states when there are no delays. Consider the following nonlinear system:

$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \begin{bmatrix} -2 & 0 \\ 0 & -6 + \sin(x_1) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\ &+ \begin{bmatrix} 2 & 1 \\ 0.9 + 0.1\sin(x_1) & 4 + \sin(x_1(t-\tau)) \end{bmatrix} \begin{bmatrix} x_1(t-\tau) \\ x_2(t-\tau) \end{bmatrix} \\ &+ \begin{bmatrix} 0 \\ 0.75 + 0.25\sin(x_1) \end{bmatrix} (u(t-\tau) + \alpha_1(x_1) + \alpha_2(x_2)), \end{aligned} \quad (35)$$

where $\alpha_1(x_1)$ and $\alpha_2(x_2)$ are two nonlinear functions which satisfy Assumption 1. For the simulations we consider

$$\alpha_1(v) = \alpha_2(v) = \cos(v) + v, \quad (36)$$

and the constants that satisfy Assumption 1 are $b_1 = b_2 = 2$, but the obtained results are valid for any other

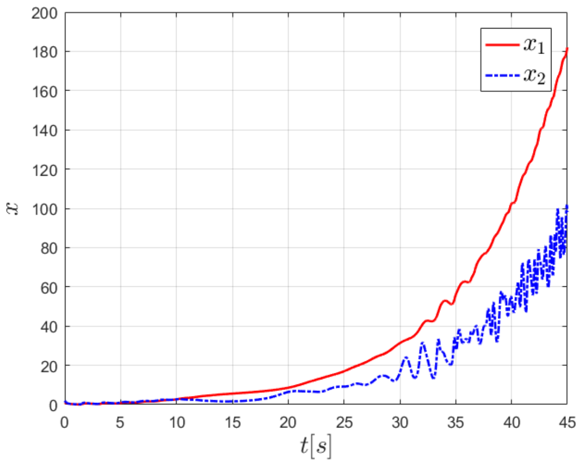

Fig. 1. Unstable open-loop system

nonlinear functions which satisfy Assumption 1 with b_1 and b_2. The time-delay function, τ, is slowly varying with $\dot{\tau}(t) \leq d = 0.01$, and has the form: $\tau(t) = 1 + 0.5\cos(0.01t)$. For the rest of the nonlinearities we consider the sector nonlinearity approach by (Ohtake et al., 2001) to obtain the local matrices:

$$\begin{aligned} &A_{11} = A_{12} = \begin{bmatrix} -2 & 0 \\ 0 & -5 \end{bmatrix}, \quad A_{21} = A_{22} = \begin{bmatrix} -2 & 0 \\ 0 & -7 \end{bmatrix} \\ &D_{11} = \begin{bmatrix} 2 & 1 \\ 0.8 & 3 \end{bmatrix}, D_{12} = \begin{bmatrix} 2 & 1 \\ 0.8 & 5 \end{bmatrix}, G = [1\ \ 1], \\ &D_{21} = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}, D_{22} = \begin{bmatrix} 2 & 1 \\ 1 & 5 \end{bmatrix}, H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &B_{11} = B_{12} = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}, B_{21} = B_{22} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \\ &h_1(z) = \frac{1 - sin(z)}{2}, h_2(z) = 1 - h_1(z), z = x_1. \end{aligned} \quad (37)$$

The initial conditions for the state vector is $x_0 = [1\ \ 2]^T$. The open-loop system without the control is unstable. This can be seen on Fig 1. Applying Theorem 1 the obtained control gains are the following:

$$K_1 = [5.62\ \ 6.99], \quad K_2 = [2.38\ \ 5.06]. \quad (38)$$

The obtained results can be seen on Fig. 2, i.e, this control stabilizes the system.

4.2 Comparison with (Moodi and Kazemy, 2019)

In this section we compare our approach to that of Moodi and Kazemy (2019). First of all, our approach considers slowly varying delays, which satisfy $\dot{\tau} \leq d$, while in (Moodi and Kazemy, 2019) only fixed delays can be handled. On the other hand, our conditions are valid for any magnitude of the delay, while the conditions of Moodi and Kazemy (2019) are delay-dependent. To further see the differences we consider the following simple example:

$$\dot{x}(t) = Ax(t) + Dx(t-\tau) + Bu(t) + BG\psi(Hx(t)) \quad (39)$$

where

$$\begin{aligned} &A = \begin{bmatrix} -2 & 0 \\ 0 & -5 \end{bmatrix}, \quad D = \begin{bmatrix} 2 & a_1 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \\ &G = a_2, \quad H = [0\ \ 1], \end{aligned} \quad (40)$$

$\psi(v)$ is a nonlinear function which satisfies Assumption 1 with $b = 2$, and a_1 and a_2 are two parameters. Because of

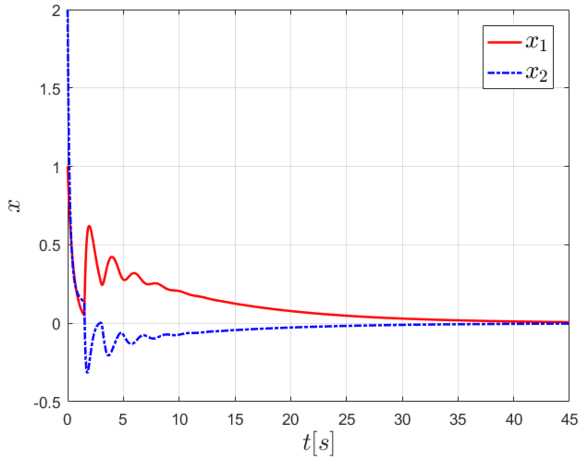

Fig. 2. Convergence of closed loop states

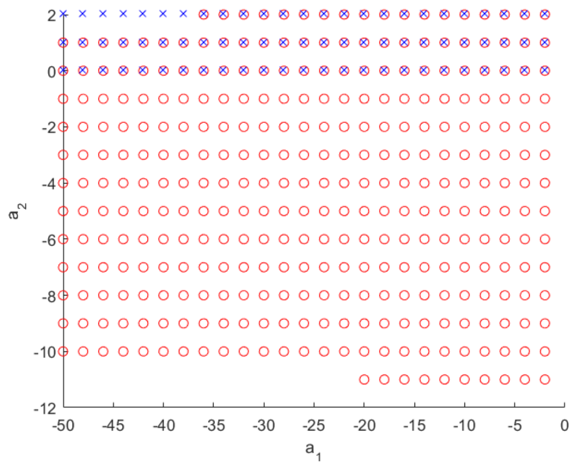

Fig. 3. Feasible solutions, 'o'-Theorem 1, 'x'-Corollary 1 from (Moodi and Kazemy, 2019)

condition (11) we can handle nonlinearities that are slope-bounded but not sector-bounded. For the sake of example we assume that $\psi(v)$ is also sector-bounded with the same constant, to be able to apply the conditions of Corollary 1 in (Moodi and Kazemy, 2019). Note that in the feasibility analysis we do not need the exact form of the nonlinearity.

The maximum τ for (39), for which Corollary 1 in (Moodi and Kazemy, 2019) gives feasible solution is $\tau = 0.6$.

Fig. 3 shows a map of feasible solutions. Although we obtain a feasible solution for a larger number of (a_1, a_2) pairs, it can be seen that the two results are complementary, and they do not include in each other.

5. CONCLUSIONS AND FUTURE WORK

This paper considered stabilization of time-delay Takagi-Sugeno fuzzy systems with nonlinear consequents. A general model in which the membership functions may depend on both current and delayed states has been chosen, and the local nonlinearities were assumed to be slope-bounded. Sufficient conditions for stabilization of the TS system have been formulated in terms of linear matrix inequalities. The design of the proposed controller has been illustrated on a numerical example. In our future work we will consider more complex Lyapunov-Krasovskii functionals and transport delay, together with uncertainties in the model.

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