Takagi–Sugeno Fuzzy Payload Estimation and Adaptive Control

Selami Beyhan ∗ Farnaz Eghbal Sarabi ∗∗ Zsófia Lendek ∗∗∗
Robert Babuška ∗∗

∗ Department of Electrical and Electronics Engineering, Pamukkale University, Kinikli Campus, 20070 Denizli, Turkey, sbeyhan@pau.edu.tr
∗∗ Delft Center for Systems and Control, Delft University of Technology, Delft, The Netherlands, r.babuska@tudelft.nl
∗∗∗ Department of Automation, Technical University of Cluj-Napoca, Memorandumului nr. 28, 400114 Cluj-Napoca, Romania, zsofia.lendek@aaut.utcluj.ro

Abstract: In this paper, a novel adaptive Takagi-Sugeno (TS) fuzzy observer-based controller is proposed. The closed-loop stability and the boundedness of all the signals are proven by Lyapunov stability analysis. The proposed controller is applied to a flexible-transmission experimental setup. The performance for constant payload in the presence of noisy measurements is compared to a controller based on a classical extended Luenberger observer. Simulation and real-time results show that the proposed observer-based feedback controller provides accurate position tracking under constant and varying payloads.

Keywords: TS fuzzy modeling, adaptive payload estimation, flexible-transmission system, observer-based control, stability.

1. INTRODUCTION

Robust control is essential for current industrial automation systems and will become even more important in applications like robotics, where future robots will be adopted for tasks in unstructured environments. In such settings, control algorithms will have to deal with a large degree of uncertainty and unexpected disturbances, which can lead to reduced performance and even to instability. Variable payload causes uncertainty in the dynamics and consequently the deterioration of performance. In this paper, we address the design of an observer that estimates the payload mass and so it enables the controller to perform adequately even under large variations of the payload.

Many methods have been proposed for the control of robotic manipulators and similar mechatronic systems. However, most of them assume constant payload and there are only a few techniques addressing the case of varying payload. They can be classified into three broad groups: adaptive controllers (Jin, 1998; Chien and Huang, 2007; Wai and Yang, 2008; Hashemi et al., 2012; Li et al., 2013), robust control methods (Su and Leung, 1993; Rojko and Jezernik, 2004; Liang et al., 2008; Pi and Wang, 2011; Corradini et al., 2012) and observer-based controllers (Leaky et al., 1991; Nho and Meckl, 2003; Savia and Koivo, 2004). In adaptive control, the uncertainty in the system is addressed through online adjustment of the control parameters. A common drawback of all adaptive schemes is that when fast variations in the system dynamics occurs, the convergence of the controlled variable to the reference signal is slow. In robust control, the uncertainty caused by the payload variation is compensated by a suitable choice of fixed control parameters. This always involves a tradeoff between robustness and performance. Observer-based approaches employ payload estimators, typically based on nonlinear models, such as fuzzy systems or neural networks (Leaky et al., 1991; Nho and Meckl, 2003; Savia and Koivo, 2004; Abiko and Yoshida, 2004), trained offline using measurement data. Once the payload mass is accurately estimated, standard control methods can be applied that make use of this estimate.

In this paper, we propose a novel nonlinear observer for the simultaneous velocity and payload estimation. We prove that under specific conditions, the estimates converge exponentially to the true velocity and payload. Observer-based output-feedback controller is employed. Compared to results from the literature, the proposed observer-based control scheme has the following advantages, which also constitute the main contributions of our paper: (i) the use of the mechanistic plant model for estimation, without the need to train a model through experiments; (ii) velocity measurements are not needed for payload estimation; (iii) exponentially stable adaptive velocity and payload estimation; (iv) estimation of the payload connected to the drive via a flexible link. The effectiveness of the
control scheme is demonstrated using real-time control experiments with a lab-scale flexible-transmission system.

The rest of the paper is organized as follows. Takagi-Sugeno fuzzy modeling and observer design are explained in Section 2.1. Feedback linearizing control for the estimated system is described in Section 2.2 and experimental results are provided in Section 3. Section 4 concludes the paper.

2. METHODS

2.1 Takagi-Sugeno Fuzzy Observer Design

A large class of nonlinear systems can be exactly represented by Takagi-Sugeno (TS) fuzzy models (Takagi and Sugeno, 1985) on a compact subset of the state-space. Several types of observers have been developed for TS fuzzy systems, among which fuzzy Thau-Luenberger observers (Tanaka et al., 1998; Tanaka and Wang, 1997), reduced-order observers (Bergsten et al., 2001), (Bergsten et al., 2002), and sliding-mode observers (Palm and Bergsten, 2000). Most design methods for TS observers rely on solving a linear matrix inequality (LMI) feasibility problem.

In order to design a TS fuzzy observer, the nonlinear system model must first be transformed to a TS fuzzy model by using the sector nonlinearity approach (Ohtake et al., 2001). Scheduling variables $z_j \in [z^j_{\min}, z^j_{\max}], j = 1, \ldots, p$ are chosen as the variables that appear in the nonlinearities of the system model. Then, for each $z_j$, two membership (weighting) functions are constructed:

$$h_1(z_j) = \frac{z_j - z^j_{\min}}{z^j_{\max} - z^j_{\min}}, \quad h_2(z_j) = 1 - h_1(z_j),$$  \hspace{1cm} (1)

with $h_1, h_2 \in [0, 1]$ and $h_1(z_j) + h_2(z_j) = 1$. Note that the following equation holds: $z_j = h_1(z_j)z^j_{\min} + h_2(z_j)z^j_{\max}$. Consequently, the TS fuzzy system consists of $M = 2^p$ rules. The degree of fulfillment of each rule is computed as the product of the membership functions in the antecedent of that rule, i.e., $h_i(z) = \prod_{j=1}^{p} h_i(z_j)$, where $h_i(z)$ is either $h_1(z_j)$ or $h_2(z_j)$, depending on which weighting function is used in the rule. This approach yields an exact representation of the nonlinear model in the following form:

$$\dot{x} = \sum_{i=1}^{M} h_i(z)(A_i x + B_i u + a_i)$$  \hspace{1cm} (2)

where $x$ is the state, $y$ the output, and $u$ the control input. The constant matrices and vectors $A_i, B_i, C_i$, and $a_i$ are constructed by substituting the elements corresponding to the weighting function used in rule $i$ into the nonlinear system matrix and vector functions. In the sequel, we assume that $z$ is measured.

For the above TS model, the Thau-Luenberger fuzzy observer (Palm and Driankov, 1999) can be derived (Oudghiri et al., 2007; Herrera et al., 2007; Lendek et al., 2010) in the following form:

$$\dot{x} = \sum_{i=1}^{M} h_i(z)(A_i x + B_i u + a_i + L_i(y - \hat{y}))$$  \hspace{1cm} (3)

$$\dot{y} = Cx.$$  \hspace{1cm} (4)
Function $g(\chi)$ in (8) is assumed to be $g(\chi) \neq 0$ for all $\chi \in U_c$, where $U_c$ denotes the controllability region. By substituting (9) into (8) we obtain the closed-loop system governed by

$$
eq \lambda_1 e^{(n-1)} + \ldots + \lambda_n e = 0$$

(10)

with the constants $\lambda_i$, $i = 1, 2, \ldots, n$ appropriately chosen such that the roots of the polynomial $s^n + \lambda_1 s^{(n-1)} + \ldots + \lambda_n = 0$ are in the open left-half of the complex plane. This implies that $\lim_{t \to \infty} e(t) = 0$, which means that the output converges asymptotically to the desired reference.

For the feedback linearizing control based on the estimated states, the control law is written as

$$u = \frac{1}{g(\chi)} [-f(\chi) + g(\chi) + \lambda^T e].$$

(11)

By substituting (11) into (8), we obtain the error dynamics

$$\dot{\epsilon} = \Lambda \epsilon + B \delta + \tilde{\Lambda} \dot{\epsilon}$$

(12)

with $\delta$ defined as $\delta = f(\chi) - f(\dot{\chi}) + (g(\chi) - g(\dot{\chi})) u$, $\tilde{\epsilon} = \chi - \dot{\chi}$ the state estimation error, and $\Lambda \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times 1}$, $\tilde{\Lambda} \in \mathbb{R}^{n \times n}$ defined as

$$\Lambda = \begin{bmatrix}
0 & 1 & 0 & 0 & \ldots & 0 & 0 \\
0 & 0 & 1 & 0 & \ldots & 0 & 0 \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
0 & 0 & 0 & 0 & \ldots & 0 & 0 \\
-\lambda_n & -\lambda_n-1 & \ldots & \ldots & \ldots & \ldots & -\lambda_1
\end{bmatrix}$$

(13)

$$\tilde{\Lambda} = \begin{bmatrix}
\lambda_1 & 0 & 0 & \ldots & 0 & 0 \\
0 & \lambda_2 & 0 & \ldots & 0 & 0 \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
0 & 0 & \ldots & \ldots & \ldots & \ldots & \lambda_n
\end{bmatrix}$$

where $B = B \Lambda + \tilde{\Lambda}$ and $||B|| \leq \nu$ for some $\nu > 0$.

**Theorem 3:** The nonlinear system (8) is asymptotically stabilized by the control law (11) under Assumption 1, if there exist $P_1 = P_1^T > 0$, $P_2 = P_2^T > 0$, $Q_1 = Q_1^T > 0$ and $L_i$, $i = 1, \ldots, M$, such that

$$P_2(A_i - L_i C) + (P_2(A_i - L_i C))^T + 2 \sigma P_2 + \nu^2 I_n < 0$$

(16)

$$\Lambda^T P_1 + P_1 \Lambda < -Q_1.$$ 

$\alpha > 0$, $\nu > 0$, and $L_i$ is the $n \times n$ identity matrix.

**Proof:** The Lyapunov function is chosen as

$$V = \frac{1}{2} e^T P_1 e + \frac{1}{2} \dot{\epsilon}^T P_2 \dot{\epsilon}$$

(17)

where $e$ and $\dot{\epsilon}$ are the tracking and estimation error vectors and $P_1, Q_1 \in \mathbb{R}^{n \times n}$ and $\Lambda$ in (13) satisfy

$$\Lambda^T P_1 + P_1 \Lambda < -Q_1.$$ 

(18)

Using (12)-(18) and the completion of squares $X^T Y + Y^T X \leq X^T X + Y^T Y$, the derivative of the Lyapunov function is obtained as

$$\dot{V} = \frac{1}{2} e^T P_1 e + \frac{1}{2} \dot{\epsilon}^T P_2 \dot{\epsilon} + \frac{1}{2} \dot{\epsilon}^T P_2 \dot{\epsilon}$$

$$\leq -e^T Q_1 e + \frac{1}{2} \left( e^T P_1 B \dot{\epsilon} + \dot{\epsilon}^T B^T P_1 e \right) +$$

$$\sum_{i=1}^M \frac{1}{2} \omega_i (\dot{\epsilon}^T [P_2(A_i - L_i C)] \dot{\epsilon} +$$

$$\dot{\epsilon}^T [(A_i - L_i C)^T P_2] \dot{\epsilon}]$$

$$\leq -e^T Q_1 e + \frac{1}{2} e^T P_1 e + \frac{1}{2} \dot{\epsilon}^T B^T B \dot{\epsilon} +$$

(19)

$$\sum_{i=1}^M \omega_i (\dot{\epsilon}^T [P_2(A_i - L_i C)] \dot{\epsilon})$$

Given that (16) holds, the terms in the final inequality are always negative definite, meaning that the closed-loop control system is asymptotically stable.

**3. REAL-TIME CONTROL OF A FLEXIBLE-TRANSMISSION SYSTEM**

The flexible transmission system, shown in Figure 1, is controlled to illustrate the real-time performance of the observer-based controller under varying payload.

Fig. 1. Flexible-link system. On the left is the driving motor and on the right the payload.

A DC motor drives a disk, which is connected by an elastic belt to another disk with a payload. Due to the low stiffness of the belt, the tracking of the payload is very difficult compared to a rigid manipulator. The belt stretches when moving the payload, which makes the system difficult to control. The complete model of the system, shown in Figure 1, is

$$\dot{\alpha} = \frac{-K^2 - b R \alpha - k (\alpha - \beta)}{R J} u,$$

$$\dot{\beta} = \frac{k}{J} (\alpha - \beta) + \frac{g L}{J} \sin(\beta),$$

(20)

with $\alpha$ the angle of the motor, $\dot{\alpha}$ the angular velocity of the motor, $\beta$ the angle of the payload, $\dot{\beta}$ the angular velocity of the payload, and $m$ the unknown payload. The definitions of the constants in (20) and their approximate values are given in Table 1. The TS observer is constructed by using $\alpha$
and \( \beta \) as measurements, while \( \dot{\beta} \) and \( m \) are to be estimated. The flexible system can be made feedback linearizable via a simple nonlinear coordinate transformation (Spong and Vidyasagar, 1989) as follows.

\[
\begin{align*}
\dot{\chi}_1 &= \chi_2, \\
\dot{\chi}_2 &= \chi_3, \\
\dot{\chi}_3 &= \chi_4, \\
\dot{\chi}_4 &= \psi(\chi) + \phi u,
\end{align*}
\]  

(21)

with

\[
\psi(\chi) = -\frac{mgL}{J} \sin(\chi_1) \left( \frac{\chi_2^2}{J} + \frac{k}{J} \cos(\chi_1) \right) + \frac{k}{J} (\chi_3 - \chi_1) \left( \frac{mgL}{J} \cos(\chi_1) - \frac{2k}{J} \right) + \frac{-K^2 - bR}{RJ} \chi_2 + \frac{k^2}{J^2 R} u,
\]  

(22)

and

\[
\phi = \frac{k^2}{J^2 R}.
\]  

(23)

The new state variables correspond to the payload angle, velocity, acceleration and jerk, respectively. The control law is chosen as

\[
u = \frac{1}{\phi} (-\psi(\chi) + \xi)
\]  

(24)

where \( \xi = \chi^*_4 + \lambda_1 (\chi^*_1 - \chi_1) + \lambda_2 (\chi^*_2 - \chi_2) + \lambda_3 (\chi^*_3 - \chi_3) + \lambda_4 (\chi^*_4 - \chi_4) \). Using the control input and the tracking error vector \( e = \chi^* - \chi \), the error dynamics can be derived as in (12).

Table 1. Parameters of the flexible link

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>Back EMF constant</td>
</tr>
<tr>
<td>b</td>
<td>Mechanical damping</td>
</tr>
<tr>
<td>R</td>
<td>Electric resistance</td>
</tr>
<tr>
<td>J</td>
<td>Rotor moment of inertia</td>
</tr>
<tr>
<td>k</td>
<td>Belt stiffness</td>
</tr>
<tr>
<td>g</td>
<td>Gravitational acceleration</td>
</tr>
<tr>
<td>L</td>
<td>Payload distance to center of disk</td>
</tr>
</tbody>
</table>

Constant Payload Case. The angle of the constant unknown payload is controlled to desired reference angles. We have designed conventional augmented-Luenberger observer-based controller (ELFC) and the proposed TS fuzzy observer-based controller (TSFC). Using equations (16) and (21), and the parameters in Table 1, the feedback linearizing control parameters are determined as \( \lambda_1 = 50 \), \( \lambda_2 = 15 \), \( \lambda_3 = 300 \), and \( \lambda_4 = 40 \), respectively.

Table 2. Obtained performances.

<table>
<thead>
<tr>
<th>Control Method</th>
<th>IAE</th>
<th>IAU</th>
</tr>
</thead>
<tbody>
<tr>
<td>TSFC</td>
<td>1.587</td>
<td>42.452</td>
</tr>
<tr>
<td>ELFC</td>
<td>2.281</td>
<td>54.792</td>
</tr>
</tbody>
</table>

Figure 2(a) shows the real-time tracking results of a 30 gram payload. Using TSFC, the payload can be positioned in two seconds with steady-state tracking error less than 0.01 radians. The ELFC provides faster transient response, but with larger steady-state errors. Note that in both approaches, the control accuracy depends on the velocity and payload estimation. The estimated velocities for TSFC and ELFC are shown in Figure 2(b). When the reference changes, the velocity estimated by the ELFC is much
larger. The control voltage for the two cases is shown in Figure 2(c). Finally, Figure 2(d) presents the estimated payload. When the reference changes, the estimate of the payload has errors which influence the tracking. Table 2 presents two comparison results for this experiment. The first criterion is the integral of absolute tracking error \( IAE = \int_0^T |e|dt \) and second one is the integral of the required absolute control effort \( IAU = \int_0^T |u|dt \). For both criteria, TSFC provides better experimental results.

**Varying Payload Case.** The transmission system is physically not suitable to pick and place different payloads. Therefore, to be able to show the estimation of different payloads, a constant payload is tracked to the reference \( \pi \), then different payloads are hung up on the opposite side to change the payload value. We use 10, 30 and 68 grams payloads for testing the system. The 68 gram payload is tracked to the constant reference by the controller while other payloads are used to reduce 68 gram payload randomly in time. Due not being able to provide fair comparisons for this experiment, no comparisons are made on the controllers.

The varying payload is presented in Figure 3(d). Although the payload was changed by hand randomly, the proposed TSFC controls the angle of the payload to be held in \( \pi \) by producing the required control effort given in Figure 3(c). In less than two seconds after the change of the payload the tracking error is reduced to small values. In Figure 3(a) and Figure 3(b), respectively, the reference tracking of varying payloads and estimated velocities are shown.

4. CONCLUSION

In this paper, a novel TS fuzzy observer-based feedback linearizing controller was introduced for nonlinear SISO systems. The proposed controller was applied to a flexible mechanical transmission system. In the observer-based control, payload and velocity estimates were used and accurate position tracking results have been obtained.

The proposed observer-based controller was compared to an output feedback controller based on an extended Luenberger observer and better estimation and control results have been obtained illustrated by the experiments, the proposed observer-based controller is also robust with respect to varying payload and noisy measurements. The proposed observer can also be applied to the general fault estimation of industrial processes.

REFERENCES


