Switching control of a rotary inverted pendulum

Boglárka Sütő
Technical University of Cluj-Napoca
Department of Automation
Memorandumului 28
400114 Cluj-Napoca, Romania
suto.an.boglarka@student.utcluj.ro

Zsófia Lendek
Technical University of Cluj-Napoca
Department of Automation
Memorandumului 28
400114 Cluj-Napoca, Romania
zsofia.lendek@aut.utcluj.ro

***Abstract*—In this paper we consider the problem of swinging-up and stabilization of a rotary inverted pendulum. We design switching controllers and analyse whether the performance can be improved using more controllers. We optimize both the control gains and the switching points. All the controllers are tested experimentally on a rotary inverted pendulum. Our results indicate that having a large number of switching controllers does not necessarily lead to a performance increase.**

***Index Terms*—switching, control, parametric optimization, swing-up control, nonlinear system, rotary inverted pendulum.**

I. Introduction

For linear models there are many methods to design controllers. Therefore we usually linearize the nonlinear model about an operating point and then we design the controller for the linear model. The problem with this approach is that when we apply the controller on the nonlinear model it will work only in a small domain. A possible solution for this problem is to linearize the model in several points, design several controllers and then to combine them. These type of controllers are called switching controllers. In this paper we analyze whether having more switching points and controllers leads to better performance, or there is a maximum number of switching points and controllers over which we can not obtain better performance.

The application we consider is the rotary inverted pendulum (RIP). The RIP was introduced by Furuta [1] and it is often called as Furuta pendulum. It has been used as a benchmark in modern control studies [2]. For example, because the dynamics of the system is similar to the dynamics of a RIP, it was used to design the balance control for rockets launching [3].

To swing-up and to stabilize the RIP it needs at least two controllers: one to swing-up the pendulum and another one to stabilize it. In this paper we test whether more optimized controllers can improve the performance.

Many types of controllers were proposed throughout the years for the swing-up of the pendulum and for stabilizing it in the upright vertical position. A bang-bang control which was robust with respect to parameter uncertainties, was proposed by Furuta [4]. Other researchers designed energy based controllers [5], sliding mode controllers [6], linear quadratic regulators (LQR) [2], combination of LQR and refined PID control [7], fractional PID [8], controller using model-free backstepping technique [9], robust control based on adaptive neural network [10], constrained nonlinear feedforward control [11] and many others.

Many controllers are very complex and the performance increase brought by their complexity in the practical application is not always clear. In this paper we study whether more complex switching controllers result in increased performance. Therefore, we design for the RIP an optimized controller, which is able to stabilize the pendulum in its upright position. We start from a working control law, then we optimize the gains and the switching points. Next we design several controllers with several switching points. First, we optimize only the switching points and then both gains and switching points. We test the resulting controllers in simulation and experimentally. We compare the results to see in which case we obtain better performance indices. Our primary goal is to minimize a given performance index, while the control effort remains in the allowed range.

The structure of the paper is the following. In Section 2 we describe our hypothesis in general. In Section 3 we test the hypothesis for the RIP. In Section 4 we present the obtained results. Section 5 concludes the paper.

II. General problem statement

The general problem we consider is the following. We consider a nonlinear system $f : \mathbb{R}^{n_x \times n_u} \to \mathbb{R}^{n_u}, \dot{x} = f(x, u)$, where $x \in \mathbb{R}^{n_x}$ is the state vector, $u \in \mathbb{R}^{n_u}$ is the input vector of the system. The system has an unstable equilibrium point in zero: $x_{eq} = 0_{n_x \times 1}$ and maybe several stable equilibrium points. Our goal is to stabilize the system in this unstable equilibrium point. In order to do this, we linearize the system in several points and design simple linear state feedback controllers. This means that we adapt the control law to the states and use a switching law:

$$u = -K_i x, \quad \text{if } x \in \mathcal{D}_i \tag{1}$$

where $\mathcal{D}_i$ is a predefined domain. If the state vector x is in the domain $\mathcal{D}_i$, then we use the control gain K_i.

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The overall control has to stabilize the system in the equilibrium point x_{eq} starting from a given initial position. To enforce this, we solve the optimization problem:

$$\min_u h(u, x_0) = \int_0^\infty \sqrt{x^T(t)Wx(t)}\, dt \tag{2}$$

subject to

$$\dot{x} = f(x, u)$$
$$x(0) = x_0$$
$$u = -K_i x, \quad \text{if } x \in \mathcal{D}_i \quad i = 1, \ldots, n$$

where x is the state vector, x_0 is the initial state vector, u is the control law and has the structure defined in (1) and W is a diagonal matrix with positive diagonal entries. We assume that it is possible to design a working controller, that stabilizes the system in the desired position. Our hypothesis is that by minimizing the function $h(u, x_0)$, we will obtain a faster and smoother behaviour. In the above problem, we assume that x_0 is given.

To obtain robust controllers and ensure stabilization from different initial conditions, we also consider the optimization problem:

$$\min_u k(u) = \frac{1}{N} \sum_{x_0 \in \mathcal{X}_0} h(u, x_0) \tag{3}$$

where $\mathcal{X}_0$ is the set of initial conditions and N is the cardinality of $\mathcal{X}_0$.

Next, we test our hypothesis by optimizing controllers designed for a rotary inverted pendulum.

III. Application: Rotary inverted pendulum

In this paper we use the rotary inverted pendulum (RIP) from Quanser [12], operated by the rotary servo plant SRV02. The RIP is a standard benchmark to study control approaches. It is a simple robot with two joints and two links, see Figure 1. The main parameters of the system are described in Table I. The general coordinates are $q = [\theta \quad \alpha]^T$. The robot is attached to a servo motor that actuates the rotary arm. The angles θ and α increase in counter clockwise direction if a positive voltage, $V_m > 0$, is applied on the servo motor.

The maximum input that can be applied to the equipment is $|V_m| \leq 10V$, which is not enough to move the pendulum in one go from the downward position to the upright vertical position. Therefore, a destabilizing control, that swings up the pendulum, is also needed.

Next, we describe the mathematical model of the RIP.

A. Mathematical model

The mathematical model of the RIP is deduced using the Euler-Lagrange equations:

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = Q(\dot{q}) \tag{4}$$

where q are the generalized coordinates: $q = \begin{bmatrix}\theta & \alpha\end{bmatrix}^T$, $\dot{q}$ are the angular velocities: $\dot{q} = \begin{bmatrix}\dot{\theta} & \dot{\alpha}\end{bmatrix}^T$, and $\ddot{q}$ are the angular accelerations: $\ddot{q} = \begin{bmatrix}\ddot{\theta} & \ddot{\alpha}\end{bmatrix}^T$. D is the inertia matrix, C has

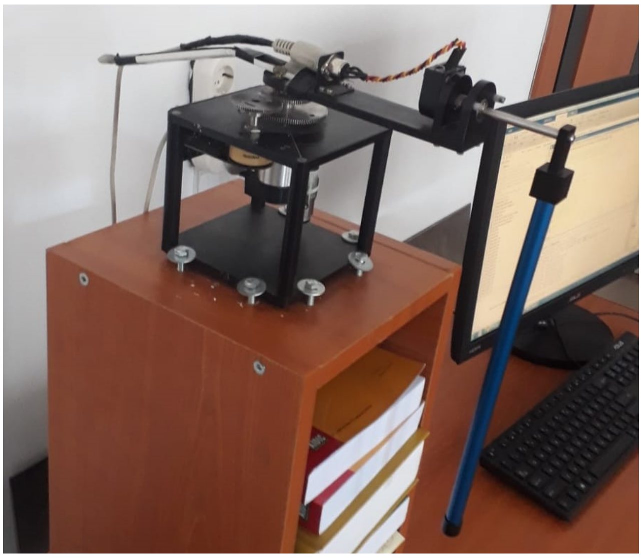


Fig. 1. The rotary inverted pendulum

TABLE I
RIP parameters

Symbol	Name	Value	Units
g	gravitational acceleration	9.81	m/s^2
k_t	motor torque constant	0.00767	$N \cdot m$
k_m	back EMF constant	0.00767	$V/(rd/s)$
K_g	total gear ratio	70	$-$
R_m	armature resistance	2.6	Ω
η_g	gear efficiency	0.9	$-$
η_m	motor efficiency	0.69	$-$
L_p	pendulum true length	0.3365	m
m_p	pendulum mass	0.127	kg
J_p	pendulum inertia	0.0012	$kg \cdot m^2$
B_p	pendulum damping coefficient	0.0024	$-$
α	pendulum angle	-	rad
L_r	rotary arm true length	0.2159	m
m_r	rotary arm mass	0.257	kg
J_r	rotary arm inertia	$9.9829e^{-4}$	$kg \cdot m^2$
B_r	rotary arm damping coefficient	0.0024	$-$
θ	rotary arm angle	-	rad

elements related to centrifugal and Coriolis forces, and G contains the effect of the gravity. The matrices are defined as [12], [8]:

$$D = \begin{bmatrix} J_r + \frac{1}{4}L_p^2 m_p s_2^2 + L_r^2 m_p + \frac{1}{4}L_r^2 m_r & -\frac{1}{2}L_p L_r m_p c_2 \\ -\frac{1}{2}L_p L_r m_p c_2 & J_p + \frac{1}{4}L_p^2 m_p \end{bmatrix}, \tag{5}$$

$$C = \begin{bmatrix} \frac{1}{4}L_p^2 \dot{\alpha} m_p s_2 c_2 & L_p m_p (\frac{1}{4}L_p \dot{\theta} c_2 + \frac{1}{2}L_r \dot{\alpha}) s_2 \\ -\frac{1}{4}L_p^2 \dot{\theta} m_p s_2 c_2 & 0 \end{bmatrix}, \tag{6}$$

$$G = \begin{bmatrix} 0 \\ -\frac{1}{2}L_p g m_p s_2 \end{bmatrix} \tag{7}$$

$$Q = \begin{bmatrix} \frac{\eta_g K_g \eta_m k_t (V_m - K_g k_m \dot{\theta})}{R_m} - B_r \dot{\theta} \\ -B_p \dot{\alpha} \end{bmatrix} \tag{8}$$

where we denote $s_1 := \sin(\theta)$, $s_2 := \sin(\alpha)$, $c_1 := \cos(\theta)$, $c_2 := \cos(\alpha)$. The model and the parameters in Table I have been validated on the actual pendulum from Quanser.

Our goal is to swing-up the pendulum and to stabilize it in its upright position as fast as possible. This can be done using two controllers:

1) a stabilizing one that keeps the pendulum at zero, and
2) a destabilizing one that brings the pendulum close to the upright position.

Next, we describe our approach for optimizing the controller designed for the RIP.

B. Performance optimization for the RIP

In Section II we defined the objective function $h(u, x_0)$ in (2), which we want to minimize. In the case of the RIP this function has the form:

$$h(u, x_0) = \int_0^5 \sqrt{\theta(t)^2 + \alpha(t)^2 + \dot{\theta}(t)^2 + \dot{\alpha}(t)^2}\, dt \tag{9}$$

since the state vector contains the angles and the angular velocities: $x = [\theta \quad \alpha \quad \dot{\theta} \quad \dot{\alpha}]$, which depend on time.

We consider that we start the pendulum from the hanging down position: $\alpha(0) = -\pi[rad]$ and all the other states are zero. Accordingly the initial condition is $x_0 = [0 \quad -\pi \quad 0 \quad 0]$. We choose the integration from $0[s]$ to $5[s]$, instead of having the final time ∞, because we noticed that the system can be stabilized in less than 5 seconds. The same weight is used for the angles and for the angular velocities, thus the weighting matrix W is the identity matrix. We compute $h(u, x_0)$ using the nonlinear model.

The decision variables of the optimization problem are the parameters of the control law: K_i and $\mathcal{D}_i$, $i = 1, \ldots, n_d$. For this application, the main state variable is the pendulum's angle. We determine the domains $\mathcal{D}_i$ based on the angle α, i.e. $\mathcal{D}_i = \{x \in \mathbb{R}^{n_x} | s_{i-1} \le |\alpha| \le s_i\}$, $s_0 = 0$, $s_{n_d} = \pi$, where s_i, $i = 0, 1, \ldots, n_d$, are the switching points.

To ensure stability from different initial conditions, we actually minimize the function $k(u)$ given in (3) based on simulated trajectories. Since we are really interested in the swinging up the pendulum and stabilizing i,t we give a much higher relative importance to simulations where the initial conditions are far from $\alpha = 0°$. We decided to test the controller for different initial positions of the pendulum. We only modify the starting angle α with $\frac{\pi}{180}[rad]$. In total we have 180 simulations, we start from $-\pi[rad]$ and we finish with $-\frac{\pi}{180}[rad]$. For simplicity, in all cases we consider zero initial velocities. Thus, the actual objective function is defined as:

$$k(u) = \frac{1}{180} \sum_{\alpha(0)=-\pi}^{\alpha(0)=-\frac{\pi}{180}} h(u, [0 \quad \alpha(0) \quad 0 \quad 0)]^T \tag{10}$$

The resulting optimized controllers are then applied on the RIP. The Quanser experimental setup is able to measure the angles of the pendulum and of the rotary arm. To apply a state feedback control, the angular velocities are numerically computed and filtered using a low pass filter with cut-off frequency $\omega_c = 62[rad/s]$. To avoid the possible estimation errors, we introduce the following performance index for experimental results:

$$g(u) = \int_0^5 \left(\sqrt{\theta(t)^2 + \alpha(t)^2}\right) dt \tag{11}$$

We use the $Matlab$ function $fminsearch$ to minimize the function $k(u)$. This function uses Nelder-Mead method, starts from an initial point and attempts to find a local minimum. The decision variables are the domains $\mathcal{D}_i$ and/or the gains K_i. Besides that, we have the constraint that the maximum input $|V_m|$ cannot be greater than $10V$. We use a working baseline controller as initial condition for the optimization problem. This will be presented next.

C. Initial control design

The system (4) can be written as $\dot{x} = f(x, u)$ with $x = [q \quad \dot{q}]^T$ and $u = V_m$. As a first step, we compute local controllers using linear models valid only around the equilibrium points $\alpha = 0$ and $\alpha = \pi$.

First we design a baseline stabilizing controller. To design this controller, we linearize the model around $x_{eq} = \begin{bmatrix} c & 0 & 0 & 0 \end{bmatrix}^T$, where c is any constant value. The linearized model has the form: $\dot{x} = A_0 x + B_0 u$. The gains are computed by placing the closed-loop poles in -20, -15, $-1.5 \pm 0.5i$, resulting $K_1 = [-0.7 \quad 22.8 \quad -1.5 \quad 2.7]$. We have also designed an LQR for stabilization, under the assumption that all the states have the same weight and the input is relatively negligible, i.e., $Q = \text{diag}(1, 1, 1, 1)$ and $R = \text{diag}(10^{-3}, 10^{-3})$ obtaining $K_{\text{LQR}} = [-31.62 \quad 581.16 \quad -43.03 \quad 90.22]$. Note that the LQR does not minimize the same performance index.

Next, we design a destabilizing controller for the linear model around: $x_{eq} = \begin{bmatrix} c & -\pi & 0 & 0 \end{bmatrix}^T$, where c is any constant value. The linearized model using the process variables has the form: $\dot{x} = A_\pi x + B_\pi u + a_\pi$. To destabilize the model, we place the poles in $2.5 \pm 10i$, $-1.5 \pm i$, resulting $K_2 = [0.3 \quad -0.2 \quad -0.3 \quad 0.4]$.

We consider $s_1 = 0.72[rad]$ as the switching point, because this is the maximum angle from where the pendulum can reach the zero position with zero initial velocities. However, the LQR control gain has the order 10^2, and does not stabilize the system, even for an initial condition of $\alpha = 0.08[rad]$. Therefore, we will use in what follows the baseline control law:

$$u = \begin{cases} -[-0.7 \quad 22.8 \quad -1.5 \quad 2.7]x, & 0 \le |\alpha| \le 0.72 \\ -[0.30 \quad -0.2 \quad -0.3 \quad 0.4]x, & 0.72 \le |\alpha| \le \pi \end{cases} \tag{12}$$

IV. RESULTS

In what follows, we present the obtained controllers and the simulation and experimental results in three cases:

1) the baseline controller: presented in Section III-C
2) optimized two-rule controller: we optimize the gains and the switching points of the baseline controller
3) optimized complex controller: we design several linear controllers and we optimize both the switching points and the gains

All the results are summarized in Table II.

A. Baseline controller

We start by testing the initial controller (12). This controller in simulation is able to swing-up and stabilize the pendulum from $\alpha = -\pi[rad]$, see Figure 2. The pendulum angle at $t = 2.1[s]$ converges to $-2\pi[rad]$, equivalent to $0[rad]$. The settling time is approximately $t_s = 3[s]$. The average performance is $k(u) = 5883$. The performance of the swing-up is $h(u, x_0) = 8819$. In the actual experiments the baseline controller is not able to stabilize the system, and is therefore not shown.

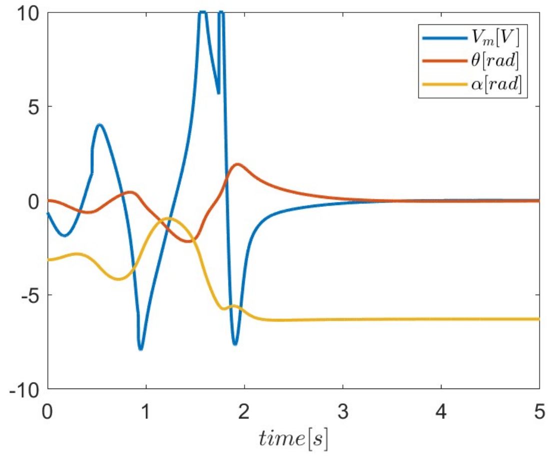


Fig. 2. Baseline controller, simulation results

B. Optimized two-rule controller

Next, we consider the baseline controller given in (12), and optimize both the controller gains and the switching point with the objective to minimize f given in (10). The optimized controller is:

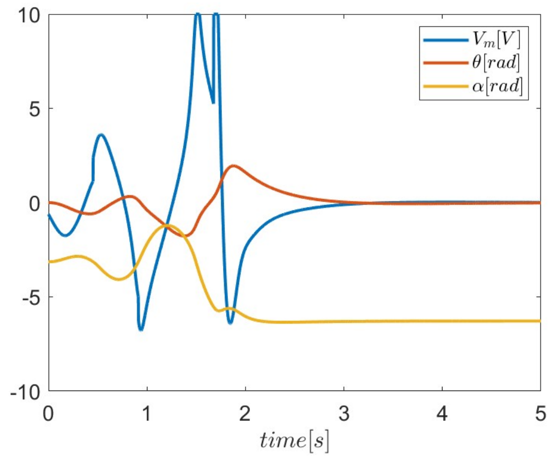
$$u = \begin{cases} -[-0.78 \quad 21.78 \quad -1.48 \quad 2.81]x, & 0 \leq |\alpha| \leq 0.75 \\ -[0.30 \quad -0.20 \quad -0.30 \quad 0.38]x, & 0.75 \leq |\alpha| \leq \pi \end{cases} \tag{13}$$

In simulation, the optimized swing-up controller (13) is able to stabilize the pendulum in zero for any initial condition. The average performance index is $k(u) = 5585$. The swing-up performance is $h(u, x_0) = 8198$. The pendulum's angle at $t = 2[s]$ is zero and the settling time is $t_s = 3[s]$, see Figure 3(a).

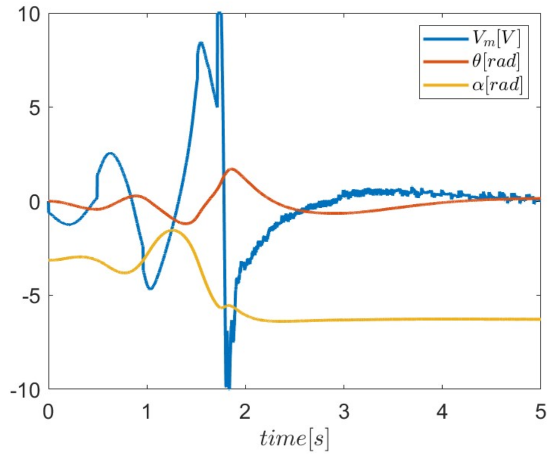
In experiments, the optimized controller (13) swings up and stabilizes the pendulum, see Figure 3(b), and we obtained the experimental performance index $g(u) = 2908$. The settling time is $t_s = 4.5[s]$, comparable to the simulation results.

C. Optimized complex controller

Next we tested whether using more controllers lead to better results. For this reason we compute four more controllers based on the linear approximation of the model around the points: $\alpha = 0.52[rad]$, $\alpha = 0.70[rad]$, $\alpha = 0.87[rad]$, $\alpha = 1.05[rad]$ and all the other states are zero. We obtained the following control law:



(a) Simulation



(b) Experiment

Fig. 3. Results using the optimized two-rule controller

$$u = \begin{cases} -[-0.78 \quad 21.78 \quad -1.48 \quad 2.81]x, & 0 \leq |\alpha| \leq 0.17 \\ -[-1.2 \quad 34.6 \quad -2.1 \quad 4.5]x, & 0.17 \leq |\alpha| \leq 0.61 \\ -[-1.5 \quad 45.3 \quad -2.6 \quad 6.2]x, & 0.61 \leq |\alpha| \leq 0.79 \\ -[-1 \quad 18.7 \quad -1.4 \quad 1.1]x, & 0.79 \leq |\alpha| \leq 0.96 \\ -[-1.4 \quad 26.3 \quad -1.8 \quad 2.2]x, & 0.96 \leq |\alpha| \leq 1.13 \\ -[0.30 \quad -0.20 \quad -0.30 \quad 0.38]x, & 1.13 \leq |\alpha| \leq \pi \end{cases} \tag{14}$$

In simulation the control law works for any initial condition and the average performance is $k(u) = 6353$, larger than the previous cases. However, the swing-up performance is $h(u, x_0) = 4732$. It is almost the half of the previous values. This controller works much better when we start from the downward position, but in other cases it is slower.

Experimental results, see Figure 4, using the controller (14) are much worse than the simulated ones. It starts with smaller swings and it takes more time to stabilize. The control effort is

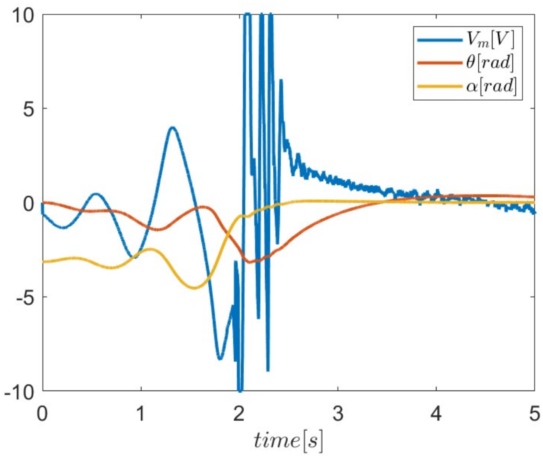


Fig. 4. Several controllers, experimental results

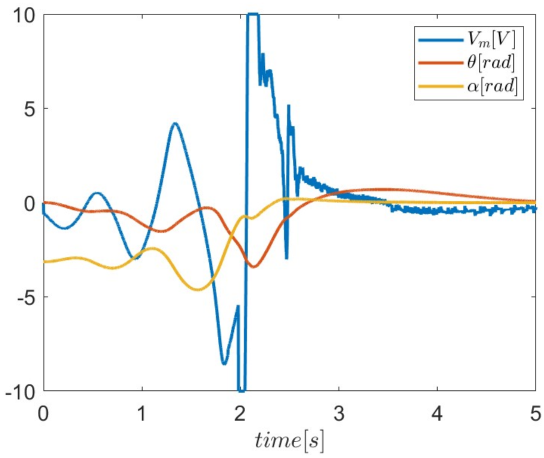


Fig. 5. Optimized controllers, experimental results

also much higher. There are spikes in the input signal, exactly in the switching points. This control law does not work better than (13), and the experimental performance is $g(u) = 4073$.

Starting from the above initial condition, we optimize the switching points. The obtained switching points are: $s = \begin{bmatrix} 0 & 0.19 & 0.79 & 1.01 & 0.91 & 0.69 & \pi \end{bmatrix}$. The optimized s_4 and s_5 switching points are smaller than s_3, so the two gains are not used. The obtained control law is:

$$u = \begin{cases} -[-0.78 \quad 21.78 \quad -1.48 \quad 2.81]x, & 0 \leq |\alpha| \leq 0.19 \\ -[-1.2 \quad 34.6 \quad -2.1 \quad 4.5]x, & 0.19 \leq |\alpha| \leq 0.79 \\ -[-1.5 \quad 45.3 \quad -2.6 \quad 6.2]x, & 0.79 \leq |\alpha| \leq 1.01 \\ -[0.30 \quad -0.20 \quad -0.30 \quad 0.38]x, & 1.01 \leq |\alpha| \leq \pi \end{cases} \tag{15}$$

Although we now have 4 control gains instead of 6, the simulation results did not change much compared to the previous case. The control effort is acceptable and the average performance index is $k(u) = 5820$. The swing-up performance is $h(u, x_0) = 5707$, which is not as good as (14), but the average performance is better.

Experimental results, see Figure 5, are somewhat worse than those obtained with the optimized two-rule controller. The settling time is $t_s = 5[s]$. The experimental performance index is $g(u) = 3908$.

Finally, we optimize all the gains and all switching points from the above control law. The objective function that we minimize is (10). The resulting control law is:

$$u = \begin{cases} -[-0.72 \quad 21.91 \quad -1.44 \quad 2.68]x, & 0 \leq |\alpha| \leq 0.20 \\ -[-1.23 \quad 34.32 \quad -2.31 \quad 4.53]x, & 0.20 \leq |\alpha| \leq 0.80 \\ -[-1.49 \quad 44.74 \quad -2.64 \quad 6.06]x, & 0.80 \leq |\alpha| \leq 1 \\ -[0.30 \quad -0.20 \quad -0.30 \quad 0.40]x, & 1 \leq |\alpha| \leq \pi \end{cases} \tag{16}$$

In simulation we obtained the results presented in Figure 6(a). The average performance index is $k(u) = 5632$. However, the swing-up performance is $h(u, x_0) = 6322$ and the settling time is $t_s = 4[s]$.

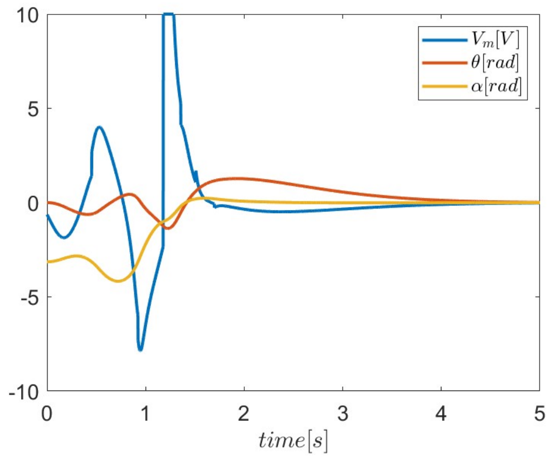
Results obtained experimentally using the control law (16) are presented in Figure 6(b). The experimental performance is $g(u) = 4146$, which is a little bit higher than the previous result, 3908. However, the control effort is much smaller. It does not reach the maximum input, it uses maximum $-7.16V$ and the settling time is $t_s = 4[s]$.

TABLE II
RESULTS

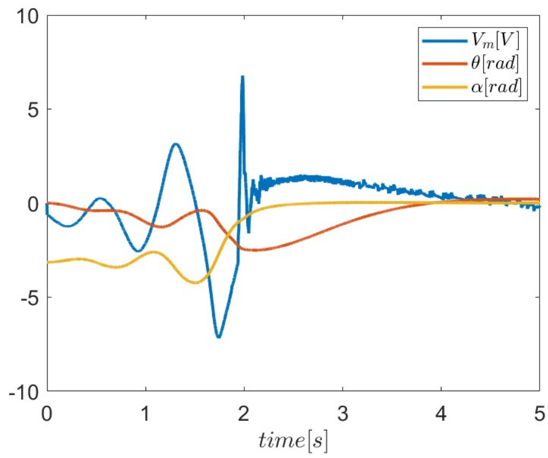
$\mathcal{D}_i$	$h(u, x_0)$	$k(u)$	$g(u)$	$t_s(exp.)$
$[0, \quad 0.72]$ $(0.72, \quad \pi]$	8819	5883	∞	∞
$[0 \quad 0.75]$ $(0.75 \quad \pi]$	8198	5585	2908	$4.5[s]$
$[0, \quad 0.17]$ $(0.17 \quad 0.61]$ $(0.61 \quad 0.79]$ $(0.79 \quad 0.96]$ $(0.96 \quad 1.13]$ $(1.13, \quad \pi]$	4732	6353	4073	$4[s]$
$[0 \quad 0.19]$ $(0.19 \quad 0.79]$ $(0.79, \quad 1.01]$ $(1.01, \quad \pi]$	5707	5820	3908	$5[s]$
$[0 \quad 0.2]$ $(0.2 \quad 0.8]$ $(0.8 \quad 1]$ $(1 \quad \pi]$	6322	5632	4146	$4[s]$

D. Comparison with results from the literature

In [13] the same rotary inverted pendulum from Quanser was used. They implemented an energy based method to swing-up the pendulum. For stabilizing the pendulum they designed a mixed H_2/H_∞ state feedback controller and compared with a classic state feedback controller and with an LQR. The best results were obtained with the mixed H_2/H_∞ controller that according to [13] stabilized the pendulum within $\pm 0.17[rad]$. Note that the same controller does not sta-



(a) Simulation



(b) Experiment

Fig. 6. Results using the optimized complex controller

bilize the system in the range $\pm 0.70[rad]$ that we considered. Furthermore the actual swinging takes several seconds.

As a textbook case, in the Quanser's Woorkbook, [12] an energy based controller is used for the swing-up, but it takes many swings until reaches close enough to the zero point. The settling time is more than 10 seconds. In our tests the pendulum reaches the zero point in 5 seconds.

V. Discussion and conclusions

The baseline control is able to stabilize the pendulum in the unstable equilibrium point in simulation, but in practice the control law does not work. Therefore, we used the approach described in Section II and we optimized the average norm of the states' trajectories of 180 simulation result.

We managed to decrease the value of the objective function from 5883 to 5585 by optimizing the baseline controller. This cotrol law also has the best performance in practice, $g(u) = 2908$.

We supposed that if we use more controllers we will have an even better performance, which was partially confirmed in simulation, but in practice due to the many switching points we had huge spikes in the input voltage. In simulation the best swing-up performance was $h(u, x_0) = 4732$. After optimizing the switching points the spikes decreased and the practical results were acceptable. Finally we optimized both the switching points and the gains. We managed to decrease the objective function f value from 6353 to 5632. The experimental performance index g varies between 3908 and 4146. The control effort does not reach the maximum input voltage, and the settling time remained around $t_s = 4[s]$.

Although the control gains did not change much after the optimization, the experimental results are quite different. A notable exception is the switching point optimization of (14), which actually resulted in reducing the number of local controllers. We see that the performance index is higher than in the case when we have only two local controllers. This may be due to the local nature of the Nelder-Mead method. In the future we will use global optimization methods and include explicit conditions on stabilization.

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