Observer based guaranteed cost control for time-delay TS fuzzy systems

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***Abstract*—This paper considers the design of observer based guaranteed cost control of time-delay nonlinear systems represented by TS fuzzy models. We consider that both the states and the inputs are affected by time varying delay, which is assumed to be known. We propose conditions for observer and controller design with the aim that the closed-loop is asymptotically stable and the cost is minimized. The conditions are bilinear and we solve them in two steps. We also give different possibilities for minimizing the cost function along with a performance comparison between them. The results are validated on a numerical example.**

***Index Terms*—observer design, controller design, fuzzy model, guaranteed cost**

I. Introduction

Most of the systems that are found in the environment surrounding us are nonlinear. Conventional control approaches that are widely applied use a linear approximation of them. Linearized systems present the drawback of failing to completely reproduce highly nonlinear plants. Based on linear control theory, a control law is computed for the system linearized around an equilibrium point. However, this is a disadvantage for recent applications that require control over a large operating range. This led to an increased interest research in control of nonlinear systems.

Regarding nonlinear systems, [1] describes procedures to analyze stability and design a control law. Nonlinear systems may also present a time delay when the distance between the sensor and the actuator is significant. This delay was observed in biological and chemical systems [2], management of the air-fuel ratio [3], networked control systems [4], etc. Time delay has a large impact on the stability of the system. Neglecting delays when designing the control law can lead to instability. Also, the closed-loop system performance is affected when delays appear at the input of a networked control system [5]. An increased use of distributed network systems [6] lea to a rise of researches regarding stability analysis and control of systems with time delay.

In order to reduce the energy consumption needed to stabilize a nonlinear system, the guaranteed cost control strategy [7] can be used. Guaranteed cost control of linear systems with uncertain discrete delays was investigated in [8] where the authors considered all the states being available. This research was extended in [9], where a linear system subject to uncertain delays have been investigated. These delays have a known nominal part and a perturbation. The added terms vanish when the perturbations in the delay approach zero. [10] studied the case of a nonlinear system with mixed time-delays in states and assumed that all the states are measured. [11] also considered the case of nonlinear systems with time-delay and used a Lyapunov-Krasovskii functional with adjustable parameters to design the controller. The approach in [7] uses a model that predicts the upcoming trajectory of the plant across a prediction horizon. The control signal is computed by solving an optimization problem at each step during the prediction horizon.

In some cases, states needed to compute the control input are not available to be measured, thus, they must be estimated. This leads to an observer based controller. An introduction into designing observer-based linear controllers for network control systems is presented by [12]. [13] conducted a research on developing an observer based controller to improve the air-fuel ratio of engines. They also compared the results with a sliding mode control concluding that the proposed method reduces the chattering considerably. [14] modeled the delays as Markov chains and developed a predictive observer-based controller.

In this paper we consider the problem of designing observer-based guaranteed cost control for Takagi-Sugeno (TS) fuzzy systems where both the states and the inputs are affected by delay. Although several results regarding linear systems exists, to our best knowledge, the problem of observer-based control of TS system with both inputs and states affected by delay has not been investigated in the literature. A minimal order control for linear uncertain time-delay systems using LMIs has been developed in [15]. Non-fragile control for Lipschitz nonlinear systems with the delay affecting the states has been investigated in [16]. [17] considered uncertain linear neutral systems with time-varying delays and computed delay-dependent conditions for stabilization, but observer-based control has not been studied. [18] computed a decentralized control for uncertain large-scale interconnected systems where the time-delay only affects the states.

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In our previous research, [19] and [20], we have investigated stabilization and estimation problems for a specific class of time-delay TS fuzzy models. In this paper we propose a two step method for designing an observer based controller while guaranteeing a minimal cost for time-delay nonlinear systems represented by Takagi-Sugeno fuzzy models. We organize the paper as follows: Section 2 presents the problem statement and assumptions. Section 3 describes the structure of the proposed controller, observer and their design. Section 4 presents the simulation results. Conclusions and future work are given in Section 5.

Notations. We use standard notations, same as in [19]. Consider a real symmetric matrix $F = F^T \in R^{n\times n}$; $F > 0$ or $F < 0$ denotes that F is positive or negative definite, respectively. We denote with I the identity matrix, and with 0 the zero matrix of appropriate dimensions. A transposed quantity in the symmetric position in a matrix is denoted with the symbol $*$, for example $\begin{pmatrix} Q & * \\ B & R \end{pmatrix} = \begin{pmatrix} Q & B^T \\ B & R \end{pmatrix}$, and $B + * = B + B^T$. The Euclidean norm of x is denoted with $\|x\|$, $\forall\, x \in R^{n_x}$.

II. Preliminaries and problem statement

We consider TS fuzzy models with time-delay of the form:

$$\begin{aligned} \dot{x}(t) =& \sum_{i=1}^{s}\sum_{j=1}^{s} h_i(z(t))h_j(z(t-\tau)) \\ & (A_{ij}x(t) + D_{ij}x(t-\tau)) + B_{ij}u(t-\tau(t)) \\ y(t) =& \sum_{i=1}^{s}\sum_{j=1}^{s} h_i(z(t))h_j(z(t-\tau))C_{ij}x(t) \\ x(t) =& \phi(t), t \le \tau \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^{n_x}$ is the state vector, $u \in \mathbb{R}^{n_u}$ is the control input, A_{ij}, B_{ij}, C_{ij} and D_{ij}, are local models matrices. $\phi(t)$ represents the initial condition of the states. $\phi(t)$ may be unknown but it is bounded with a known bound and z denotes the premise variables. The time-delay is denoted with τ, the number of rules is denoted by s and h_i, $i = 1,..,s$, are nonlinear membership functions with the property $h_i(z) \in [0, 1], i = 1, ..., s, \sum_{i=1}^{s} h_i(z) = 1$.

The goal is to develop a fuzzy observer based controller that stabilises the system (1) while guaranteeing a minimal cost.

Assumption 1. $\tau(t)$ is the time varying delay which is supposed to be exactly known. Also, τ is differentiable, $\dot{\tau} \le d$, $d \in [0,\, 1)$, where d is the maximum speed change of the delay. In this paper we assume a single delay, multiple delays are left for future work.

To simplify the notation, in what follows, all the sums present in the equations are denoted by the matrix name present in the sum and subscripts that denote the dependence on the current state or delayed one. For instance, $F_{zz_\tau} = \sum_{i=1}^{s} h_i(z(t)) \sum_{j=1}^{s} h_j(z(t-\tau))F_{ij}$.

Using such notations, (1) can be written as:

$$\begin{aligned} \dot{x}(t) =& A_{zz_\tau}x(t) + D_{zz_\tau}x(t-\tau(t)) + B_{zz_\tau}u(t-\tau(t)) \\ y(t) =& C_{zz_\tau}x(t) \end{aligned} \tag{2}$$

Results are developed using the following lemma and property:

Lemma 1. (Congruence) Having the matrix $P = P^T$ and a full column rank matrix Q, it holds that:

$$P > 0 \implies QPQ^T > 0$$

Property 1. (Schur complement) Let $M = M^T = \begin{pmatrix} M_{11} & M_{12} \\ M_{12}^T & M_{22} \end{pmatrix}$ with M_{11} and M_{22} square matrices of appropriate dimensions. Then:

$$\begin{aligned} M < 0 \iff & \begin{cases} M_{11} < 0 \\ M_{22} - M_{12}^T M_{11}^{-1} M_{12} < 0 \end{cases} \\ \iff & \begin{cases} M_{22} < 0 \\ M_{11} - M_{12} M_{22}^{-1} M_{12}^T < 0 \end{cases} \end{aligned} \tag{3}$$

We consider a state-feedback controller

$$u(t) = -K_z x(t) \tag{4}$$

where $K_z = \sum_{k=1}^{s} h_k(z(t))K_k$ contains the controller gains. Due to the fact that not all states are measured, we use the control law:

$$u(t) = -K_z \hat{x}(t) \tag{5}$$

where $\hat{x}$ represents the estimated state vector. The observer has the form:

$$\begin{aligned} \dot{\hat{x}}(t) =& A_{zz_\tau}\hat{x}(t) + D_{zz_\tau}\hat{x}(t-\tau(t)) \\ & + B_{zz_\tau}u(t-\tau(t)) + L_{zz_\tau}(y - \hat{y}) \\ \hat{y}(t) =& C_{zz_\tau}\hat{x}(t) \end{aligned} \tag{6}$$

where L_{zz_τ} is the observer gain. In order to implement this observer, we consider the assumption – generally used in observer design –:

Assumption 2. The scheduling variables are exactly known at all time.

III. Main result

Our target is to design K_z and L_{zz_τ} and minimize the cost function:

$$J(t) = \int_0^\infty \Big[x^T(t)R_1x(t) + u^T(t)R_2u(t)\Big]dt \tag{7}$$

where $R_1 = R_1^T > 0$ and $R_2 = R_2^T > 0$ are given symmetric positive matrices of appropriate dimensions.

We start by defining the estimation error $e(t) = x(t) - \hat{x}(t)$ and the error dynamics:

$$\begin{aligned} \dot{e}(t) =& \dot{x}(t) - \dot{\hat{x}}(t) \\ =& \big(A_{zz_\tau} - L_{zz_\tau}C_{zz_\tau}\big)e(t) + D_{zz_\tau}e\big(t-\tau(t)\big) \end{aligned} \tag{8}$$

Substituting (5) in (6) results in:

$$\begin{aligned} \dot{\hat{x}}(t) =& A_{zz_\tau}\hat{x}(t) + (D_{zz_\tau} - B_{zz_\tau}K_{z_\tau})\hat{x}(t-\tau) \\ & + L_{zz_\tau}C_{zz_\tau}e(t). \end{aligned} \tag{9}$$

Substituting (5) in (2) gives us the closed loop system:

$$\dot{x}(t) = A_{zz_\tau} x(t) + \big(D_{zz_\tau} - B_{zz_\tau} K_{z_\tau}\big) x(t-\tau(t)) - B_{zz_\tau} K_{z_\tau} e(t-\tau) \tag{10}$$

To develop the design conditions, we consider the dynamics of $\hat{x}$ and e that yield the augmented dynamics as:

$$\begin{bmatrix}\dot{\hat{x}}(t)\\ \dot{e}(t)\end{bmatrix} = \begin{bmatrix}A_{zz_\tau} & L_{zz_\tau} C_{zz_\tau}\\ 0 & A_{zz_\tau} - L_{zz_\tau} C_{zz_\tau}\end{bmatrix}\begin{bmatrix}\hat{x}(t)\\ e(t)\end{bmatrix} + \begin{bmatrix}D_{zz_\tau} - B_{zz_\tau} K_{z_\tau} & 0\\ 0 & D_{zz_\tau}\end{bmatrix}\begin{bmatrix}\hat{x}(t-\tau)\\ e(t-\tau)\end{bmatrix} \tag{11}$$

Note that if $\begin{bmatrix}\hat{x}\\ e\end{bmatrix} \to 0$ as $t \to \infty$ then $\begin{bmatrix}x\\ e\end{bmatrix} \to 0$ as $t \to \infty$. We make the following notations:

$$\begin{aligned}\tilde{x}(t) &= \begin{bmatrix}\hat{x}(t)\\ e(t)\end{bmatrix}\\ \tilde{A} &= \begin{bmatrix}A_{zz_\tau} & L_{zz_\tau} C_{zz_\tau}\\ 0 & A_{zz_\tau} - L_{zz_\tau} C_{zz_\tau}\end{bmatrix}\\ \tilde{D} &= \begin{bmatrix}D_{zz_\tau} - B_{zz_\tau} K_{z_\tau} & 0\\ 0 & D_{zz_\tau}\end{bmatrix}\end{aligned} \tag{12}$$

thus, (11) can be rewritten as:

$$\dot{\tilde{x}}(t) = \tilde{A}\tilde{x}(t) + \tilde{D}\tilde{x}(t-\tau) \tag{13}$$

We use the following candidate Lyapunov–Krasovskii functional:

$$V(t) = \tilde{x}^T(t)\tilde{P}\tilde{x}(t) + \int_{t-\tau}^{t} \tilde{x}^T(t)\tilde{Q}\tilde{x}(t)ds \tag{14}$$

where $\tilde{P} = \begin{bmatrix}P_1 & 0\\ 0 & P_2\end{bmatrix}$, $\tilde{Q} = \begin{bmatrix}Q_1 & 0\\ 0 & Q_2\end{bmatrix}$, $P_i = P_i^T > 0$ and $Q_i = Q_i^T > 0$ for $i \in \{1,2\}$ and denote:

$$V_1(t) = \tilde{x}^T(t)\tilde{P}\tilde{x}(t) \qquad V_2(t) = \int_{t-\tau}^{t} \tilde{x}^T(s)\tilde{Q}\tilde{x}(s)ds \tag{15}$$

Assume that the derivative of the Lyapunov function in respect with time has an upper limit such that $\dot{V}(t) \le -J_0$ or

$$\dot{V}(t) + J_0 \le 0 \tag{16}$$

where $J_0 = x^T(t)R_1x(t) + u^T(t)R_2u(t)$. To obtain $J(t)$, equation (16) is integrated from t to ∞:

$$\int_0^\infty x^T(t)R_1x(t) + u^T(t)R_2u(t)dt \le -\int_0^\infty \dot{V}(t)dt = V(0) - V(\infty) \tag{17}$$

If the closed loop system is stable, $V(\infty) = 0$. Then, the cost function is bounded by $V(0)$ so, to minimize that, we consider the following:

$$J_0 = J_1 + J_2 \tag{18}$$

and impose

$$V_1(0) = \tilde{x}^T(0)\tilde{P}\tilde{x}(0) \le J_1 \tag{19}$$

$$V_2(0) = \int_{-\tau}^{0} \tilde{x}^T(s)\tilde{Q}\tilde{x}(s)ds \le J_2 \tag{20}$$

Imposing $\hat{x}(t) = 0$ for $t \in [-\tau,\ 0]$ the error of estimation is:

$$e(t) = x(t) - \hat{x}(t) = x(t), \qquad \forall t \in [-\tau,\ 0] \tag{21}$$

Replacing $\tilde{x}$ in $V_1(0)$ and using (21) gives:

$$\tilde{x}^T(0)\tilde{P}\tilde{x}(0) = x^T(0)P_2x(0) \quad \forall t \in [-\tau,\ 0] \tag{22}$$

We assume that:

$$\|x(t)\|^2 \le n_{max}, \quad \forall t \in [-\tau,\ 0] \tag{23}$$

and n_{max} is known. This is a reasonable assumption, as in practical cases, the state variables are bounded. Then, (19) holds if:

$$P_2 \le \frac{J_1}{n_{max}} I; \tag{24}$$

Replacing $\tilde{x}$ in (20) and using (21) gives:

$$\tilde{x}^T(t)\tilde{Q}\tilde{x}(t) = x^T(t)Q_2x(t) \quad \forall t \in [-\tau,\ 0] \tag{25}$$

For the integral we consider:

$$\int_{-\tau}^{0} x^T(s)Q_2x(s)ds \le \int_{-\tau}^{0} \frac{J_2}{\tau_{max}} ds \tag{26}$$

where $\tau(t) \le \tau_{max}$. (26) is satisfied if:

$$x^T(t)Q_2x(t) \le \frac{J_2}{\tau_{max}}, \quad \forall t \in [-\tau,\ 0] \tag{27}$$

It is known that:

$$x^T(t)Q_2x(t) \le \|x(t)\|^2 \lambda_{max}(Q_2), \quad \forall t \in \mathbb{R} \tag{28}$$

where $\lambda_{max}(Q_2)$ is the largest eigenvalue of Q_2.

Using (23) results in

$$x^T(t)Q_2x(t) \le n_{max}\lambda_{max}(Q_2) \tag{29}$$

Imposing :

$$n_{max}\lambda_{max}(Q_2) \le \frac{J_2}{\tau_{max}} \tag{30}$$

gives:

$$\lambda_{max}(Q_2) \le \frac{J_2}{\tau_{max} n_{max}} \tag{31}$$

which is satisfied if

$$Q_2 \le \frac{J_2}{\tau_{max} n_{max}} I \tag{32}$$

Next we compute the derivative of $V_1(t)$:

$$\dot{V}_1(t) = \tilde{x}^T(t)\tilde{A}^T\tilde{P}\tilde{x}(t) + \tilde{x}^T(t-\tau)\tilde{D}^T\tilde{P}\tilde{x}(t) + (*) \tag{33}$$

Denoting: $\tilde{\chi} = \begin{bmatrix}\tilde{x}(t)\\ \tilde{x}(t-\tau)\end{bmatrix}$, we obtain:

$$\dot{V}_1(t) = \tilde{\chi}^T \begin{bmatrix}\tilde{P}\tilde{A} + (*) & \tilde{P}\tilde{D}\\ (*) & 0\end{bmatrix}\tilde{\chi} \tag{34}$$

The derivative of V_2 is:

$$\dot{V}_2(t) = \tilde{x}^T(t)\tilde{Q}\tilde{x}(t) - (1-\dot{\tau}(t))\tilde{x}^T(t-\tau)\tilde{Q}\tilde{x}(t-\tau) \tag{35}$$

Based on Assumption 2, (35) becomes:

$$\dot{V_2}(t) \leq \tilde{\chi}^T \begin{bmatrix} \tilde{Q} & 0 \\ (*) & -(1-d)\tilde{Q} \end{bmatrix} \tilde{\chi} \quad (36)$$

Substituting the delayed control law (5) in J_0 gives

$$\begin{aligned} J_0 =& \big(e(t)+\hat{x}(t)\big)^T R_1 \big(e(t)+\hat{x}(t)\big) \\ &+ \hat{x}^T(t-\tau) K_{z_\tau}^T R_2 K_{z_\tau} \hat{x}(t-\tau) \end{aligned} \quad (37)$$

Using (12) we have:

$$J_0 = \tilde{x}^T(t)\tilde{R}_1\tilde{x}(t) + \tilde{x}^T(t-\tau)\tilde{R}_2\tilde{x}(t-\tau) \quad (38)$$

with $\tilde{R}_1 = \begin{bmatrix} R_1 & R_1 \\ R_1 & R_1 \end{bmatrix}$, $\tilde{R}_2 = \begin{bmatrix} K_{z_\tau}^T R_2 K_{z_\tau} & 0 \\ 0 & 0 \end{bmatrix}$ and (38) can be rewritten as $J_0 = \tilde{\chi}^T \begin{bmatrix} \tilde{R}_1 & 0 \\ 0 & \tilde{R}_2 \end{bmatrix} \tilde{\chi}$.

Substituting $\dot{V}(t,x)$ and J_0 in (16) gives:

$$\tilde{\chi}^T \begin{bmatrix} \tilde{P}\tilde{A} + (*) + \tilde{Q} + \tilde{R}_1 & \tilde{P}\tilde{D} \\ (*) & -(1-d)\tilde{Q} + \tilde{R}_2 \end{bmatrix} \tilde{\chi} \leq 0 \quad (39)$$

which is satisfied if (40) (on the next page) holds.

The main result of this work is summarised in the following theorem:

Theorem 1. Consider the augmented dynamics (11) and assume that τ is bounded, $\tau \leq \tau_{max}$ with $\tau_{max} > 0$, and differentiable, $\dot{\tau} \leq d$ and (23) holds, with $d \in [0, 1)$ and n_{max} a given constant. If there exist matrices $\tilde{P} = \tilde{P}^T > 0$, $\tilde{Q} = \tilde{Q}^T > 0$, K_i, L_{ij}, $i, j = 1, ..., s$, such that (40), (24), (32) are satisfied then the augmented dynamics (11) is asymptotically stable. The upper bound of the cost function is minimised if $J_0 = J_1 + J_2$ is minimised.

It can be observed that equation (40) is not a LMI, and direct sufficient LMI conditions can not be stated, thus we will solve it in two steps. First we will compute an observer and then the controller.

We start by computing the observer for model (2). The observer can be designed by solving the matrix inequality:

$$\begin{bmatrix} P_2\big(A_{zz_\tau} - L_{zz_\tau}C_{zz_\tau}\big) + (*) + Q_2 + R_1 & P_2 D_{zz_\tau} \\ (*) & -(1-d)Q_2 \end{bmatrix} \leq 0 \quad (41)$$

Denote $N_{zz_\tau} = P_2 L_{zz_\tau}$ and rewrite (41) as:

$$\begin{bmatrix} \big(P_2 A_{zz_\tau} - N_{zz_\tau}C_{zz_\tau}\big) + (*) + Q_2 + R_1 & P_2 D_{zz_\tau} \\ (*) & -(1-d)Q_2 \end{bmatrix} \leq 0 \quad (42)$$

Inequality (42) holds if

$$\frac{2}{s-1}F_{ijil} + F_{ijkl} + F_{kjil} \leq 0 \quad \forall i, j, k, l = 1, ..., s \quad (43)$$

where:

$$F_{ijkl} = \begin{bmatrix} \Gamma_{11}^{44} & P_2 D_{ij} \\ (*) & -(1-d)Q_2 \end{bmatrix} \leq 0 \quad (44)$$

and

$$\Gamma_{11}^{44} = \big(P_2 A_{ij} - N_{ij}C_{kl}\big) + (*) + Q_2 + R_1 \quad (45)$$

We solve the LMI (43) and replace the obtained value for L_{zz_τ} in (40), thus having the decision variables K_{z_τ}, P_1, P_2, Q_1, Q_2. Then, congruence of (40) with diag(P_1^{-1}, I, P_1^{-1}, I) results in (46) where:

$$\begin{aligned} \Gamma_{11}^{46} =& A_{zz_\tau}P_1^{-1} + (*) + P_1^{-1}Q_1P_1^{-1} + P_1^{-1}R_1P_1^{-1} \\ \Gamma_{22}^{46} =& P_2\big(A_{zz_\tau} - L_{zz_\tau}C_{zz_\tau}\big) + (*) + Q_2 + R_1 \end{aligned}$$

Applying the Schur complement on (46) for $P_1^{-1}R_1P_1^{-1}$ and $P_1^{-1}K_{z_\tau}^T R_2 P_1^{-1} K_{z_\tau}$ and making the notations $S = P_1^{-1}, W = P_1^{-1}Q_1P_1^{-1}$ and $N_{z_\tau} = K_{z_\tau}S$ results in (47).

For (47), sufficient LMI conditions can be established as:

$$\frac{2}{s-1}F_{ijil} + F_{ijkl} + F_{kjil} \leq 0 \quad \forall i, j, k, l = 1, ..., s \quad (48)$$

where F_{ijkl} is defined in (49) and

$$\Gamma_{22}^{49} = P_2\big(A_{ij} - L_{ij}C_{kl}\big) + (*) + Q_2 + R_1$$

The conditions presented above can be summarised in the following algorithm as:

1) Solve equation (43), with the matrices defined in (44). Compute $L_{ij} = P_2^{-1}N_{ij}$.
2) Replace L_{ij} in (40)
3) Solve (48) with the decision variables K_{z_τ}, P_1, P_2, Q_1, Q_2.
4) Compute the controller gains as $K_i = N_i S^{-1}$

Regarding the minimization of the cost function J, since the conditions are solved in two steps, the result will be sub-optimal. There are several possibilities to minimize J_1 and J_2, respectively, in step (1) or in step (3) of the algorithm. Different possibilities will be illustrated in the next section on a numerical example.

IV. Numerical example

In this section we illustrate the performances of the proposed method on a numerical example. Consider the following nonlinear system:

$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} =& \begin{bmatrix} -2 & -1+\cos(x_1) \\ 0.5\big(\cos(x_1)+1\big) & -2) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\ &+ \begin{bmatrix} 0.6 & 0.9 \\ 1.5+0.3\cos(x_1) & 0.9+0.3\cos(x_1(t-\tau)) \end{bmatrix} \begin{bmatrix} x_1(t-\tau) \\ x_2(t-\tau) \end{bmatrix} \\ &+ \begin{bmatrix} 0 \\ 0.65+0.35\cos(x_1) \end{bmatrix} u(t-\tau), \\ y =& \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}. \end{aligned}$$

The states of the open-loop system do not converge to zero. A slowly varying time-delay, τ, with the form $\tau(t) = 0.35 + 0.35\cos(1.42t)$ and $\dot{\tau}(t) \leq d = 0.5$ is used for this example. Thus $\tau_{max} = 0.7$ and we also assume that $\|x(t)\|^2 \leq 13$ for $t \in [-\tau,\ 0]$.

To obtain the TS representation, we consider the premise variable x_1, which is measured and the nonlinear terms $cos(x_1(t))$ and $cos(x_1(t-\tau))$ are included in the membership functions. Thus $h_1(x_1(t)) = \frac{1-cos(x_1(t))}{2}$ and $h_1(x_1(t-\tau)) = \frac{1-cos(x_1(t-\tau))}{2}$ are obtained. The local matrices are computed

$$\begin{bmatrix} P_1A_{zz_\tau}+(*)+Q_1+R_1 & P_1L_{zz_\tau}C_{zz_\tau}+R_1 & P_1\big(D_{zz_\tau}-B_{zz_\tau}K_{z_\tau}\big) & 0 \\ (*) & P_2\big(A_{zz_\tau}-L_{zz_\tau}C_{zz_\tau}\big)+(*)+Q_2+R_1 & 0 & P_2D_{zz_\tau} \\ (*) & (*) & K_{z_\tau}^TR_2K_{z_\tau}-(1-d)Q_1 & 0 \\ (*) & (*) & (*) & -(1-d)Q_2 \end{bmatrix} \leq 0 \quad (40)$$

$$\begin{bmatrix} \Gamma_{11}^{46} & L_{zz_\tau}C_{zz_\tau}+P_1^{-1}R_1 & \big(D_{zz_\tau}-B_{zz_\tau}K_{z_\tau}\big)P_1^{-1} & 0 \\ (*) & \Gamma_{22}^{46} & 0 & P_2D_{zz_\tau} \\ (*) & (*) & P_1^{-1}K_{z_\tau}^TR_2P_1^{-1}K_{z_\tau}-(1-d)P_1^{-1}Q_1P_1^{-1} & 0 \\ (*) & (*) & (*) & -(1-d)Q_2 \end{bmatrix} \leq 0 \quad (46)$$

$$\begin{bmatrix} A_{zz_\tau}S+(*)+W & L_{zz_\tau}C_{zz_\tau}+SR_1 & D_{zz_\tau}S-B_{zz_\tau}N_{z_\tau} & 0 & S & N_{z_\tau}^T \\ (*) & \Gamma_{22}^{46} & 0 & P_2D_{zz_\tau} & 0 & 0 \\ (*) & (*) & -(1-d)W & 0 & 0 & 0 \\ (*) & (*) & (*) & -(1-d)Q_2 & 0 & 0 \\ (*) & (*) & (*) & (*) & -R_1^{-1} & 0 \\ (*) & (*) & (*) & (*) & (*) & -R_2^{-1} \end{bmatrix} \leq 0 \quad (47)$$

$$F_{ijkl} = \begin{bmatrix} A_{ij}S+(*)+W & L_{ij}C_{kl}+SR_1 & D_{ij}S-B_{ji}N_k & 0 & S & N_i^T \\ (*) & \Gamma_{22}^{49} & 0 & P_2D_{ij} & 0 & 0 \\ (*) & (*) & -(1-d)W & 0 & 0 & 0 \\ (*) & (*) & (*) & -(1-d)Q_2 & 0 & 0 \\ (*) & (*) & (*) & (*) & -R_1^{-1} & 0 \\ (*) & (*) & (*) & (*) & (*) & -R_2^{-1} \end{bmatrix} \quad (49)$$

by replacing the limit values of $cos(x_1(t))$ and $cos(x_1(t-\tau))$ in the nonlinear matrices.

$$A_{11}=A_{12}=\begin{bmatrix}-2 & -2\\ 0 & -2\end{bmatrix},\quad A_{21}=A_{22}=\begin{bmatrix}-2 & 0\\ 1 & -2\end{bmatrix}$$
$$D_{11}=\begin{bmatrix}0.6 & 0.9\\ 1.8 & 1.2\end{bmatrix},\quad D_{12}=\begin{bmatrix}0.6 & 0.9\\ 1.8 & 0.6\end{bmatrix},$$
$$D_{21}=\begin{bmatrix}0.6 & 0.9\\ 1.2 & 1.2\end{bmatrix},\quad D_{22}=\begin{bmatrix}0.6 & 0.9\\ 1.2 & 0.6\end{bmatrix},$$
$$B_{11}=B_{12}=\begin{bmatrix}0\\ 1\end{bmatrix},\quad B_{21}=B_{22}=\begin{bmatrix}0\\ 0.3\end{bmatrix},$$
$$C_{11}=C_{12}=C_{21}=C_{22}=\begin{bmatrix}1 & 0\end{bmatrix},$$
$$h_1(z)=\frac{1-cos(z)}{2},\ h_2(z)=1-h_1(z),\ z=x_1.$$

We minimize (7) with $R_1=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, $R_2=1$.

The minimization can be applied in step (1) and step (3) of the algorithm. We tested all the possibilities and the following results were obtained:

1) Minimizing J_0 or J_1 in step (1) leads to an infeasible problem in step (3).
2) Feasible solutions are obtained if the minimization is performed for $J_1,\ J_2$ or J_0 in step (3).

All the obtained costs are presented in Table 1. No optimization means that the controller and observer gains have been computed without optimization. The smallest one is achieved when the minimization is performed only in step (3) for J_0. The theoretical upper bound of the cost is quite large, 4131, as it is valid for $\forall\ x_0$ such that $\|x(0)\|^2 \leq 13,\ t\in[-\tau,\ 0]$. We obtain the following observer and controller gains:

TABLE I
GUARANTEED COST

Optimization type	J_0
No optimization	1499944
J1 in step (3)	250662
J2 in step (3)	234620
J2 in step (1) and step (3)	1493896
J0 in step (3)	4131
J2 in step (1) and J1 in step (3)	2884335

$$\begin{aligned} &K_1=\begin{bmatrix}2.23 & 1.56\end{bmatrix},\quad K_2=\begin{bmatrix}2.39 & 2.11\end{bmatrix},\\ &L_{11}=\begin{bmatrix}27.7\\ 5.34\end{bmatrix},\quad L_{12}=\begin{bmatrix}27.64\\ 5.3\end{bmatrix},\\ &L_{21}=\begin{bmatrix}27.99\\ 6.62\end{bmatrix},\quad L_{22}=\begin{bmatrix}27.95\\ 6.61\end{bmatrix}.\end{aligned} \quad (50)$$

Next we simulate the closed-loop system. The initial condition for the state vector is $x(t)=\begin{bmatrix}3 & 2\end{bmatrix}^T,\ t\in[-\tau,\ 0]$. Using the controller, the states converge to 0, thus the controller is stabilizing the system and the obtained results can be seen in Fig. 1. The control input is presented in Fig. 2 and the computed cost function in Fig. 3. As can be seen, the actual cost for this particular trajectory is much smaller than the theoretical one, 107.43.

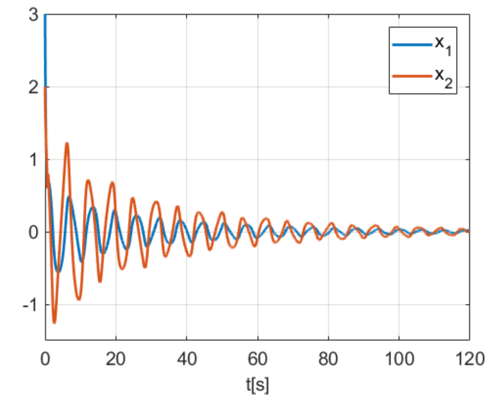

Fig. 1. Trajectories of the closed-loop system

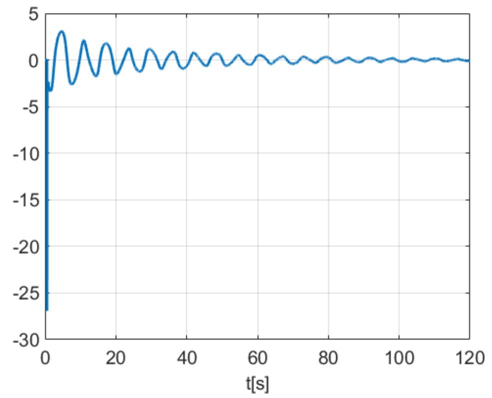

Fig. 2. Control input

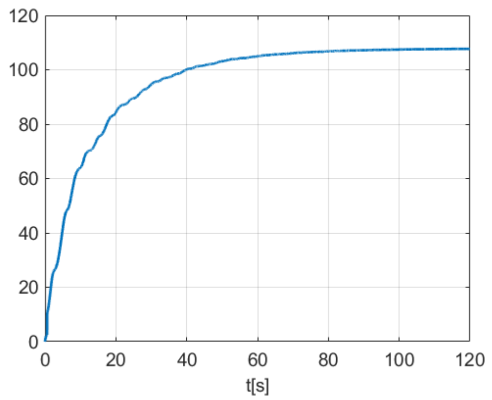

Fig. 3. Cost function

V. Conclusions and Future Work

This paper focused on observer based guaranteed cost controller of nonlinear systems with variable time delays that are described by Takagi-Sugeno fuzzy models. The delay is known at all time. A two-step algorithm is given to design the observer and the controller while minimizing the cost. As further development, we will investigate the case of unknown, multiple delays and finite-time convergence.

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