Stability analysis of switching TS models using α-samples approach ⋆

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Abstract: In this paper we consider stability analysis of discrete-time switching systems represented by Takagi-Sugeno fuzzy models. For the analysis we use a switching Lyapunov function defined on the switches. In order to develop stability conditions, we consider the variation of this function along switching paths of length α, i.e., an α sample variation of the Lyapunov function. This approach is able to determine stability of a switching system containing unstable local models. The developed conditions can be formulated as LMIs and they are illustrated on a numerical example.

Keywords: switching systems, TS fuzzy systems, stability analysis, nonquadratic Lyapunov functions, sample variation

1. INTRODUCTION

Switched systems are systems usually described by continuous and discrete dynamics, and their interactions. For such systems, one can (Daafouz et al., 2002) analyze the stability of the whole system assuming that the switching sequence is not known, find a stabilizing switching system, design switching observers and controllers, etc. In this paper we analyze the stability of switching nonlinear systems represented by Takagi-Sugeno (TS) fuzzy systems.

Takagi-Sugeno (TS) fuzzy systems (Takagi and Sugeno, 1985) are nonlinear, convex combinations of local linear models, which are able to exactly represent a large class of nonlinear systems (Lendek et al., 2010). In general, for the analysis and synthesis of TS systems, the direct Lyapunov approach has been used. Stability conditions have been derived using quadratic Lyapunov functions (Tanaka et al., 1998; Tanaka and Wang, 2001; Sala et al., 2005), piecewise continuous Lyapunov functions (Johansson et al., 1999; Feng, 2004a), and nonquadratic Lyapunov functions (Guerra and Vermeiren, 2004; Kruszewski et al., 2008; Mozelli et al., 2009). The stability or design conditions are generally derived in the form of linear matrix inequalities (LMIs), which can be solved using available algorithms.

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For discrete-time TS systems, non-quadratic Lyapunov functions have shown a real improvement in the design conditions (Guerra and Vermeiren, 2004; Ding et al., 2006; Dong and Yang, 2009a; Lee et al., 2011). They have also been extended to double-sum Lyapunov functions by (Ding et al., 2006) and later on to polynomial Lyapunov functions by (Sala and Ariño, 2007; Ding, 2010; Lee et al., 2010). A different type of improvement in the discrete case has been developed in (Kruszewski et al., 2008), conditions being obtained by replacing the classical one sample variation of the Lyapunov function by its variation over several samples (α-sample variation).

Switching TS systems have been investigated in the last decades mainly in the continuous case where the stability is based on the use of a quadratic Lyapunov function (Tanaka et al., 2001; Lam et al., 2002, 2004; Ohtake et al., 2006) or a piecewise one (Feng, 2003, 2004b). Although results are available for discrete-time linear switching systems (Daafouz et al., 2002), for discrete-time TS models, few results exist (Doo et al., 2003; Dong and Yang, 2009b).

In this paper, we derive relaxed LMI conditions for the stability analysis of switching TS systems. For this, we use a switching nonquadratic Lyapunov function and make use of its variation over several samples. We assume that although the exact switching sequence is not known, the set of all the admissible switches is known. Furthermore, once a subsystem is activated, it will remain active for a number of samples, for which bounds are known.

The structure of the paper is as follows. Section 2 presents the notations used in this paper. The stability analysis of switching systems is presented in Section 3. Section 4 discusses the developed conditions and illustrates their use on a numerical example. Section 5 concludes the paper.

2. PRELIMINARIES

In this paper we analyze the stability of discrete-time switching TS systems. We consider subsystems of the form

$$\begin{aligned}\boldsymbol{x}(k+1) &= \sum_{i=1}^{r_j} h_{j,i}(\boldsymbol{z}_j(k))A_{j,i}\boldsymbol{x}(k)\\ &= A_{jz}\boldsymbol{x}(k)\end{aligned} \tag{1}$$

where j is the number of the subsystem, $j = 1, 2, \ldots, n_s$, n_s being the number of the subsystems, $\boldsymbol{x}$ denotes the state vector, r_j is the number of rules in the jth subsystem, $\boldsymbol{z}_j$ is the scheduling vector, $h_{j,i}$, $i = 1, 2, \ldots, r_j$ are normalized membership functions, and $A_{j,i}$, $i = 1, 2, \ldots, r_j$, $j = 1, 2, \ldots, n_s$, are the local models.

For the easier notation, we use a directed graph representation of the switching system (1). The graph associated to (1) is $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V}$ denotes the set of vertices or subsystems and $\mathcal{E}$ denotes the set of admissible switches. As such, $(v_i, v_j) \in \mathcal{E}$ if a switch from subsystem i to subsystem j is possible. Note that we assume that self-transitions are also possible: these correspond to the subsystem being active for more than one sample. We also assume that none of the subsystems have a finite escape time, i.e., the states cannot become infinite in a finite number of samples.

A path $\mathcal{P}(v_i, v_j)$ between two vertices v_i and v_j in the graph $\mathcal{G}$ is a sequence of vertices $\mathcal{P}(v_i, v_j) = [v_{p_1}, v_{p_2}, \ldots, v_{p_{n_p}}]$ so that $v_i = v_{p_1}$, $v_j = v_{p_{n_p}}$, and $(v_{p_k}, v_{p_{k+1}}) \in \mathcal{E}$, $p_k = 1, 2, \ldots, n_p - 1$. A path in a graph associated to a switching system corresponds to a switching law. The length of a path is given by the number of edges it contains.

Let us illustrate the notations above on an example.

Example 1. Consider a switching system composed of 4 subsystem:

$$\boldsymbol{x}(k+1) = A_{i,z}\boldsymbol{x}(k)$$

for $i = 1, 2, 3, 4$, and with admissible switches $(1,2)$, $(2,1)$, $(2,3)$, $(3,1)$, $(4,3)$, $(1,4)$. Each subsystem can be active for more than one sample. The corresponding graph representation is illustrated in Figure 1.

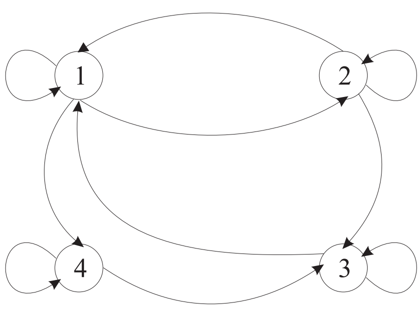

Fig. 1. Graph representation of the switching system in Example 1.

The graph is $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, with $\mathcal{V} = \{1, 2, 3, 4\}$ and

$$\begin{aligned}\mathcal{E} = \{&(1,1), (1,2), (1,4), (2,1), (2,2), (2,3), (3,1),\\ &(3,3), (4,3), (4,4)\}\end{aligned}$$

A path $\mathcal{P}(1,3)$ is given by $\mathcal{P}(1,3) = [1, 2, 3]$. The length of the path $\mathcal{P}(1,3) = [1, 2, 3]$ is 2.

Once activated, a subsystem may be active continuously for at least $p_i^m \in \mathbb{N}^+$ and at most $p_i^M \in \mathbb{N}^+$ samples, that are assumed known. Our goal is to develop conditions under which the switching system is stable, with any admissible switching law.

0 and I denote the zero and identity matrices of appropriate dimensions, and a $(*)$ denotes the term induced by symmetry. The subscript $z + m$ (as in A_{1z+m}) stands for the scheduling vector being evaluated at the current sample plus mth instant, i.e., $\boldsymbol{z}(k + m)$.

In what follows, we will make use of the following results:

Lemma 1. (Skelton et al., 1998) Consider a vector $\boldsymbol{x} \in \mathbb{R}^{n_x}$ and two matrices $Q = Q^T \in \mathbb{R}^{n_x \times n_x}$ and $R \in \mathbb{R}^{m \times n_x}$ such that $\text{rank}(R) < n_x$. The two following expressions are equivalent:

(1) $\boldsymbol{x}^T Q \boldsymbol{x} < 0$, $\boldsymbol{x} \in \{\boldsymbol{x} \in \mathbb{R}^{n_x}, \boldsymbol{x} \neq 0, R\boldsymbol{x} = 0\}$
(2) $\exists M \in \mathbb{R}^{n_x \times m}$ such that $Q + MR + R^T M^T < 0$

Analysis and design for TS models often lead to double-sum negativity problems of the form

$$\boldsymbol{x}^T \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\boldsymbol{z}(k))h_j(\boldsymbol{z}(k))\Gamma_{ij}\boldsymbol{x} < 0 \tag{2}$$

where Γ_{ij}, $i, j = 1, 2, \ldots, r$ are matrices of appropriate dimensions.

Lemma 2. (Wang et al., 1996) The double-sum (2) is negative, if

$$\begin{aligned}&\Gamma_{ii} < 0\\ &\Gamma_{ij} + \Gamma_{ji} < 0, \quad i, j = 1, 2, \ldots, r,\ i < j\end{aligned}$$

Lemma 3. (Tuan et al., 2001) The double-sum (2) is negative, if

$$\begin{aligned}&\Gamma_{ii} < 0\\ &\frac{2}{r-1}\Gamma_{ii} + \Gamma_{ij} + \Gamma_{ji} < 0, \quad i, j = 1, 2, \ldots, r,\ i \neq j\end{aligned}$$

3. STABILITY CONDITIONS

Lets us now consider the stability of the origin of the switching system

$$\boldsymbol{x}(k+1) = A_{j,z}\boldsymbol{x}(k) \tag{3}$$

Example 2. As a motivating example, consider the system depicted in Figure 2. Switches are possible between subsystems 1 and 2 (in both directions), from 2 to 3, from 3 to 1, and from 3 to 3, i.e., subsystem 3 can remain continuously active. Consequently, $p_1^m = p_1^M = p_2^m = p_2^m = 1$, $p_3^m = 1$, $p_3^M = \infty$.

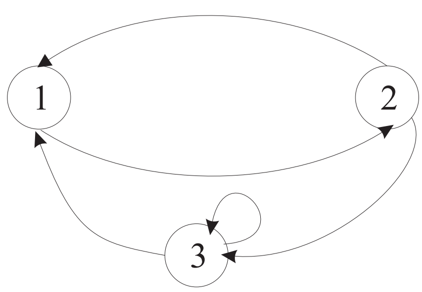

Fig. 2. Switching system for Example 2.

Due to the switching between the subsystems, a possibility to establish stability is to consider a switching Lyapunov function, and verify that it decreases during each subsystem and during each switch. This is in effect an extension to TS models of the results of (Daafouz et al., 2002), that was established for linear systems.

However, since subsystems 1 and 2 can only be active for one time instance, it can be seen that to guarantee the systems stability it is enough if the (switching) Lyapunov function decreases while subsystem 3 is active (since it can be continuously active), and it also decreases during the cycles [1, 2, 1] and $[1, 2, 3, 1]$.

To derive stability conditions, consider the switching Lyapunov function

$$V(x_k) = \boldsymbol{x}_k^T P_{i,j,z} \boldsymbol{x}_k \tag{4}$$

defined during the switches, i.e., on the edges of the associated graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, with $(v_i, v_j) \in \mathcal{E}$.

Remark: If a subsystem i may be active for several number of samples, the edge (v_i, v_i) is also considered.

In a first step, with this, the following result can be formulated:

Theorem 1. The origin of the switching system (3) is asymptotically stable, if there exist $P_{i,j,k} = P_{i,j,k}^T > 0$, $M_{i,j,k}$, $(v_i, v_j) \in \mathcal{E}$, $k = 1, 2, \ldots, r$, such that

$$\begin{pmatrix} -P_{i,j,z} & (*) \\ M_{j,l,z}A_{j,z} & -M_{j,l,z} - M_{j,l,z}^T + P_{j,l,z+1} \end{pmatrix} < 0 \tag{5}$$

for all admissible paths $\mathcal{P}(v_i, v_l) = [v_i, v_j, v_l]$, $v_i \in \mathcal{V}$.

Proof. Consider the switching Lyapunov function (4), defined on the edges of the associated graph, with $\boldsymbol{x}_k^T P_{i,j,z} \boldsymbol{x}_k$ being active during the transition from vertex i to vertex j. The difference in the Lyapunov function for two consecutive samples is

$$\begin{aligned} \Delta V &= \boldsymbol{x}_{k+1}^T P_{j,l,z+1} \boldsymbol{x}_{k+1} - \boldsymbol{x}_k^T P_{i,j,z} \boldsymbol{x}_k \\ &= \begin{pmatrix} \boldsymbol{x}_k \\ \boldsymbol{x}_{k+1} \end{pmatrix}^T \begin{pmatrix} -P_{i,j,z} & 0 \\ 0 & P_{j,l,z+1} \end{pmatrix} \begin{pmatrix} \boldsymbol{x}_k \\ \boldsymbol{x}_{k+1} \end{pmatrix} \end{aligned}$$

where $[v_i, v_j, v_l]$ is an admissible path.

During the transition for j to l, the dynamics of the system are described by

$$\begin{pmatrix} A_{j,z} & -I \end{pmatrix} \begin{pmatrix} \boldsymbol{x}_k \\ \boldsymbol{x}_{k+1} \end{pmatrix} = 0$$

Using Lemma 1, the difference is the Lyapunov function is negative, if there exists M such that

$$\begin{pmatrix} -P_{i,j,z} & 0 \\ 0 & P_{j,l,z+1} \end{pmatrix} + M \begin{pmatrix} A_{j,z} & -I \end{pmatrix} + (*) < 0$$

By choosing

$$M = \begin{pmatrix} 0 \\ M_{j,l,z} \end{pmatrix}$$

we have directly (5).

Remark: It should be noted that in order to reduce the conservativeness of the solution and exploit available relaxations, a double sum can also be used in the matrix M, i.e., M can be chosen as

$$M = \begin{pmatrix} 0 \\ M_{j,l,z,z+1} \end{pmatrix}$$

leading to the following corollary:

Corollary 1. The origin of the switching system (3) is asymptotically stable, if there exist $P_{i,j,k} = P_{i,j,k}^T > 0$, $M_{i,j,k,\beta}$, $(v_i, v_j) \in \mathcal{E}$, $(v_j, v_l) \in \mathcal{E}$, $k, \beta = 1, 2, \ldots, r$, such that

$$\begin{pmatrix} -P_{i,j,z} & (*) \\ M_{j,l,z,z+1}A_{j,z} & -M_{j,l,z,z+1} - M_{j,l,z,z+1}^T + P_{j,l,z+1} \end{pmatrix} < 0 \tag{6}$$

for all admissible paths $\mathcal{P}(v_i, v_l) = [v_i, v_j, v_l]$, $v_i \in \mathcal{V}$.

In this case, using Lemma 3, we can formulate the conditions as LMIs, as follows:

Corollary 2. The origin of the switching system (3) is asymptotically stable, if there exist $P_{i,j,k} = P_{i,j,k}^T > 0$, $M_{i,j,k,l}$, $(v_i, v_j) \in \mathcal{E}$, $k, l = 1, 2, \ldots, r$, such that

$$\begin{aligned} &\Gamma_{kk}^{i,j,l} < 0 \\ &\frac{2}{r-1}\Gamma_{kk}^{i,j,l,\gamma} + \Gamma_{k\beta}^{i,j,l} + \Gamma_{\beta k}^{i,j,l} < 0, \quad k, \beta = 1, 2, \ldots, r \end{aligned}$$

with

$$\Gamma_{k\beta}^{i,j,l,\gamma} = \begin{pmatrix} -P_{i,j,k} & (*) \\ M_{j,l,k,\gamma}A_{j,\beta} & -M_{j,l,k,\gamma} + (*) + P_{j,l,\gamma} \end{pmatrix}$$

for all admissible paths $\mathcal{P}(v_i, v_l) = [v_i, v_j, v_l]$, $v_j \in \mathcal{V}$.

Let us now consider an α-sample variation of the Lyapunov function. As it has been proven by Kruszewski and Guerra (2007), it is not necessary that the Lyapunov function decreases every sample, but it is sufficient if it decreases every α samples, $\alpha > 1$. Considering the Lyapunov function (4), the following result can be formulated:

Theorem 2. The origin of the switching system (3) is asymptotically stable, if there exist $\alpha \in \mathbb{N}^+$, $P_{i,j,k} = P_{i,j,k}^T > 0$, $M_{i,j,k}$, $(v_i, v_j) \in \mathcal{E}$, $k = 1, 2, \ldots, r$, such that

$$\begin{pmatrix} -P_{i_1,i_2,z} & (*) & \ldots & (*) \\ \begin{pmatrix} M_{i_2,i_3,z} \\ \cdot A_{i_2,z} \end{pmatrix} & -M_{i_2,i_3,z} + (*) & \ldots & (*) \\ 0 & \begin{pmatrix} M_{i_3,i_4,z+1} \\ \cdot A_{i_3,z+1} \end{pmatrix} & \ldots & (*) \\ \vdots & \vdots & \ldots & \begin{pmatrix} M_{i_{\alpha+1},i_{\alpha+2},z+\alpha-1} + (*) \\ +P_{i_{\alpha+1},i_{\alpha+2},z+\alpha} \end{pmatrix} \end{pmatrix} < 0 \tag{7}$$

for all admissible paths $\mathcal{P}(v_{i_1}, v_{i_{\alpha+2}}) = [v_{i_1}, v_{i_2}, \ldots, v_{i_{\alpha+2}}]$.

Proof. Consider the switching Lyapunov function (4), defined on the edges of the associated graph, with $P_{i,j,z}$ being active during the transition from vertex i to vertex j. The difference in the Lyapunov function for α consecutive samples is

$$\begin{aligned} \Delta V &= \boldsymbol{x}_{k+\alpha}^T P_{i_{\alpha+1},i_{\alpha+2},z+\alpha} \boldsymbol{x}_{k+\alpha} - \boldsymbol{x}_k^T P_{i_1,i_2,z} \boldsymbol{x}_k \\ &= \begin{pmatrix} \boldsymbol{x}_k \\ \boldsymbol{x}_{k+\alpha} \end{pmatrix}^T \begin{pmatrix} -P_{i_1,i_2,z} & 0 \\ 0 & P_{i_{\alpha+1},i_{\alpha+2},z+\alpha} \end{pmatrix} \\ &\quad \cdot \begin{pmatrix} \boldsymbol{x}_k \\ \boldsymbol{x}_{k+\alpha} \end{pmatrix} \end{aligned}$$

where $[v_{i_1}, v_{i_2}, \ldots, v_{i_{\alpha+2}}]$ is an admissible path.

Along the switching sequence $[v_{i_1}, v_{i_2}, \ldots, v_{i_{\alpha+2}}]$, the dynamics of the system are described by

$$\begin{pmatrix} A_{i_2,z} & -I & 0 & \dots & 0 \\ 0 & A_{i_3,z} & -I & \dots & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \dots & -I \end{pmatrix} \begin{pmatrix} \boldsymbol{x}_k \\ \boldsymbol{x}_{k+1} \\ \vdots \\ \boldsymbol{x}_{k+\alpha} \end{pmatrix} = 0$$

Similarly to the proof of Theorem 1, using Lemma 1, the difference is the Lyapunov function is negative, if there exists M such that

$$\begin{pmatrix} -P_{i_1,i_2,z} & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & P_{i_{\alpha+1},i_{\alpha+2},z+\alpha} \end{pmatrix} + M \begin{pmatrix} A_{i_2,z} & -I & 0 & \dots & 0 \\ 0 & A_{i_3,z} & -I & \dots & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \dots & -I \end{pmatrix} + (*) < 0$$

By choosing

$$M = \begin{pmatrix} 0 & 0 & \dots & 0 \\ M_{i_1,i_2,z} & 0 & \dots & 0 \\ 0 & M_{i_2,i_3,z+1} & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & M_{i_{\alpha+1},i_{\alpha+2},z+\alpha} \end{pmatrix}$$

we have directly (7).

4. DISCUSSION AND EXAMPLES

The assumption that we made on the system was that none of the subsystems have a finite escape time. Indeed, if any of the subsystems has a finite escape time, stability of the system can no longer be guaranteed without further assumptions on the switching law.

Let us now discuss the developed conditions, in particular those concerning the α-sample variation of the Lyapunov function, on an example.

Example 3. To illustrate the application of the conditions, let us revisit Example 2. The graph is $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, with $\mathcal{V} = \{1, 2, 3\}$ and

$$\mathcal{E} = \{(1,2), (2,1), (2,3), (3,1), (3,3)\}$$

The edge $(3,3)$ is introduced in order to take into account that subsystem 3 can be continuously active. The Lyapunov function is defined for every switch, i.e., we have $P_{i,2,z}$, $P_{2,1,z}$, $P_{2,3,z}$, etc. The conditions of Theorem 1 require that the Lyapunov function decreases with every switch/every sample, including here self-transitions, i.e., for all $(v_i, v_j) \in \mathcal{E}$. That is, we have the conditions:

$$\begin{pmatrix} -P_{1,2,z} & (*) \\ M_{2,1,z}A_{2,z} & -M_{2,1,z} + (*) + P_{2,1,z+1} \end{pmatrix} < 0$$
$$\begin{pmatrix} -P_{1,2,z} & (*) \\ M_{2,3,z}A_{2,z} & -M_{2,3,z} + (*) + P_{2,3,z+1} \end{pmatrix} < 0$$
$$\begin{pmatrix} -P_{2,3,z} & (*) \\ M_{3,3,z}A_{3,z} & -M_{3,3,z} + (*) + P_{3,3,z+1} \end{pmatrix} < 0$$
$$\begin{pmatrix} -P_{2,3,z} & (*) \\ M_{3,1,z}A_{3,z} & -M_{3,1,z} + (*) + P_{3,1,z+1} \end{pmatrix} < 0$$
$$\vdots$$

implying that each subsystem has to be stable. On the other hand, a 2-sample variation means that the Lyapunov function has to decrease along paths of lengths 3, i.e., we have the conditions:

$$\begin{pmatrix} -P_{1,2,z} & (*) & (*) \\ M_{2,1,z}A_{2,z} & -M_{2,1,z} + (*) & (*) \\ 0 & M_{1,2,z+1}A_{1,z+1} & -M_{1,2,z+1} + (*) + P_{1,2,z+2} \end{pmatrix} < 0$$
$$\begin{pmatrix} -P_{1,2,z} & (*) & (*) \\ M_{2,3,z}A_{2,z} & -M_{2,3,z} + (*) & (*) \\ 0 & M_{3,1,z+1}A_{3,z+1} & -M_{3,1,z+1} + (*) + P_{3,1,z+2} \end{pmatrix} < 0$$
$$\begin{pmatrix} -P_{1,2,z} & (*) & (*) \\ M_{2,3,z}A_{2,z} & -M_{2,3,z} + (*) & (*) \\ 0 & M_{3,3,z+1}A_{3,z+1} & -M_{3,3,z+1} + (*) + P_{3,3,z+2} \end{pmatrix} < 0$$
$$\vdots$$

Consider the following local models of the TS system above:

$$A_{1,1} = \begin{pmatrix} 0.23 & 0.2 \\ 0.03 & 0.9 \end{pmatrix} \qquad A_{1,2} = \begin{pmatrix} 0.47 & 0.47 \\ 0.23 & 0.16 \end{pmatrix}$$
$$A_{2,1} = A_{2,1} = \begin{pmatrix} 1.1 & 0 \\ 0.2 & 0.8 \end{pmatrix}$$
$$A_{3,1} = \begin{pmatrix} 0.11 & 0.09 \\ 0.09 & 0.07 \end{pmatrix} \qquad A_{3,2} = \begin{pmatrix} 0.32 & 0.30 \\ 0.09 & 0.38 \end{pmatrix}$$

The second subsystem is actually linear, but it is unstable. Due to this, available methods, including the extension of (Daafouz et al., 2002) to TS models yields unfeasible LMIs.

The LMIs given by Theorem 1 are also unfeasible. However, a 2-sample variation of the Lyapunov function (the conditions of Theorem 2 applied for $\alpha = 2$) results in a feasible set of LMIs. This is thanks to the fact that subsystem 2 can only be active for one sample, and therefore the Lyapunov function may decrease along a switching path of length at least 2.

Recall that we assume that the switching sequence is not known in advance and it cannot be directly influenced. With this assumption the stability conditions actually state that the switching system is stable if the Lyapunov function decreases along every path of length α. Naturally, this is the worst-case, i.e., all possible combinations on switches between the subsystems are taken into account. If the switching sequence can be chosen, or the goal is to find a stabilizing switching sequence, the conditions can be relaxed. The LMI conditions can also be relaxed by double sums in the Lyapunov function, e.g., using $P_{i,j,z,z}$ instead of $P_{i,j,z}$, or even several sums.

A shortcoming of the proposed conditions is the computational complexity of generating all the switching paths of length α, in particular for large α and large-scale switching systems and in consequence, the large number of LMIs that has to be solved. Unfortunately, reducing the conservativeness of the conditions by introducing additional sums in the Lyapunov function (eventually leading to ANS conditions) also increases the number of LMIs.

5. CONCLUSIONS

In this paper we developed relaxed stability conditions for discrete-time switching systems represented by Takagi-Sugeno fuzzy models, by using a switching Lyapunov function defined on the switches. We assumed that the switching sequence is not known in advance and it cannot be directly influenced. The developed conditions have been formulated as LMIs and are able to prove stability of switching systems where one or more subsystems are unstable.

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