# Discrete-time Takagi-Sugeno descriptor models: controller design

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*Abstract*—**Many physical systems are naturally represented by descriptor models. This paper is concerned with stabilization of discrete-time descriptor systems represented by Takagi-Sugeno fuzzy models. Two different approaches are presented based on non-quadratic Lyapunov functions. The results are expressed in terms of linear matrix inequalities. Numerical examples validate the proposed methods.**

## I. Introduction

IN the last thirty years, nonlinear models have been to be studied via the so-called Takagi-Sugeno (TS) models [1].

A TS model is a collection of local linear models blended together by membership functions (MFs) [2]. The sector nonlinearity approach [3] provides a systematic way to construct a TS model. The resulting TS model represents the considered nonlinear models in a compact set of the state space [4].

The direct Lyapunov method has been used to derive conditions for the stability and stabilization of TS models. Several Lyapunov functions have been proposed: quadratic Lyapunov functions [4], piecewise Lyapunov functions [5], [6], line-integral Lyapunov functions [7], and more recently non-quadratic Lyapunov functions [8]–[13]. The conditions are formulated as linear matrix inequalities (LMIs), which can be solved via convex optimization techniques [14], [15].

A drawback of using the sector nonlinearity approach is that an increase in the number of nonlinear terms implies an exponential increase of the number of LMI conditions.

The behavior of many physical systems is naturally described by nonlinear descriptor models [16]. The TS descriptor model was introduced in [17]; this representation reduces the number of LMI constraints because it conserves nonlinearities in the left-hand side and keeps the original structure of the nonlinear model [18]–[23].

The works [18]–[23] develop conditions for TS descriptor models in continuous-time. However, for the discrete-time case there are few results [24], [25]. Therefore, this paper presents approaches in discrete-time via two Lyapunov functions, thus filling a gap in the literature. Moreover, the use of the Finsler's Lemma [26]–[29] is helpful for "decoupling" the control law from the Lyapunov function.

The paper is organized as follows: Section 2 provides some useful notation and properties, it also introduces the TS descriptor model; Section 3 presents the main results for controller design for discrete-time TS descriptor models; Section 4 illustrates the effectiveness of the proposal approaches via examples.

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## II. Notations and problem statement

Given a set of nonlinear functions $h_i(\cdot) \geq 0$, $i \in \{1,\ldots,r\}$ having the convex sum property $\sum_{i=1}^{r} h_i(\cdot) = 1$, a shorthand notation will be used in the sequel to represent convex sums of matrix expressions: $\Upsilon_h = \sum_{i=1}^{r} h_i(z(\kappa))\Upsilon_i$ and $\Upsilon_v = \sum_{k=1}^{r} v_k(z(\kappa))\Upsilon_k$ for single convex sums; $\Upsilon_{h+} = \sum_{l=1}^{r} h_l(z(\kappa+1))\Upsilon_l$ for a delayed convex sum; $\Upsilon_h^{-1} = \left(\sum_{i=1}^{r} h_i(z(\kappa))\Upsilon_i\right)^{-1}$ for the inverse of a convex sum, and $\Upsilon_{hh} = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(\kappa))h_j(z(\kappa))\Upsilon_{ij}$ for a double nested convex sum. An asterisk $(*)$ will be used in matrix expressions to denote the transpose of the symmetric element; for in-line expressions it will denote the transpose of the terms on its left side. Arguments will be omitted when their meaning is direct.

Consider the following discrete-time TS model in the descriptor form.

$$E_v x(\kappa+1) = A_h x(\kappa) + B_h u(\kappa), \qquad (1)$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the control input vector, $\kappa$ is the current sample. Matrices $A_i$ and $B_i$, $i \in \{1,\ldots,r\}$ represent the *i*-th linear right-hand side model and $E_k$, $k \in \{1,\ldots,r_e\}$ represent the *k*-th linear left-hand side model of the TS descriptor model. In this work we assume that $E_v$ is regular matrix. This is motivated by mechanical systems, where $E_v$ contains the inertia matrix and therefore it is a regular matrix. In what follows, $x_{\kappa+}$ and $x_\kappa$ stand for $x(\kappa+1)$ and $x(\kappa)$ respectively.

The membership functions (MFs) hold the convex sum property in a compact set of the state:

$$h_i\left(z(\kappa)\right)\geq 0,\quad i\in\left\{1,\dots,2^p\right\},\quad \sum_{i=1}^{r}h_i\left(z(\kappa)\right)=1,$$

$$v_k\left(z(\kappa)\right)\geq 0,\quad k\in\left\{1,\dots,2^{p_e}\right\},\quad \sum_{k=1}^{r_e}v_k\left(z(\kappa)\right)=1,$$

where $p$ and $p_e$ represent the number of nonlinear terms in the right-hand side and left-hand side, respectively, of (1). The MFs depend on the premise variables grouped in the vector $z(\kappa)$ which is known and usually depends on the state vector.

In order to obtain LMI conditions, the following relaxation scheme will be employed due to its good compromise between effectiveness and computational complexity.

**Relaxation Lemma** [30]: Let $\Upsilon_{ij}^k$ be matrices of appropriate dimensions. Then $\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r_e}h_i\left(z(\kappa)\right)h_j\left(z(\kappa)\right)v_k\left(z(\kappa)\right)\Upsilon_{ij}^k<0$, holds if

$$\begin{aligned}&\Upsilon_{ii}^k<0,\\&\frac{2}{r-1}\Upsilon_{ii}^k+\Upsilon_{ij}^k+\Upsilon_{ji}^k<0,\quad i\neq j,\end{aligned}\tag{2}$$

for $i,j\in\{1,\dots,r\}$, $k\in\{1,\dots,r_e\}$.

**Finsler's Lemma** [26]: Let $x\in\mathbb{R}^n$, $Q=Q^T\in\mathbb{R}^{n\times n}$, and $R\in\mathbb{R}^{m\times n}$ such that $rank(R)<n$; the following expressions are equivalent:

a) $x^TQx<0$, $\forall x\in\left\{x\in\mathbb{R}^n : x\neq 0, Rx=0\right\}$.

b) $\exists M\in\mathbb{R}^{n\times m}: Q+MR+R^TM^T<0$.

**Property 1**. Let $X=X^T>0$ and $Y$ matrices of appropriate size. The following expression holds: $(Y-X)^T X^{-1}(Y-X)\geq 0 \Leftrightarrow Y^TX^{-1}Y\geq Y+Y^T-X$.

**Property 2** [14] (Schur complement). Consider a matrix $Q=\begin{bmatrix}Q_{11} & Q_{12}\\ Q_{21} & Q_{22}\end{bmatrix}$, with $Q_{11}$ and $Q_{22}$ being square matrices. Then:

$$Q<0\ \Leftrightarrow\ \begin{cases}Q_{11}<0\\ Q_{22}-Q_{12}^TQ_{11}^{-1}Q_{12}<0\end{cases}\ \Leftrightarrow\ \begin{cases}Q_{22}<0\\ Q_{11}-Q_{12}Q_{22}^{-1}Q_{12}^T<0.\end{cases}$$

The following example exhibits the motivation for the TS descriptor form vs. a TS form: $x_{\kappa+}=A(x)x_\kappa+B(x)u_\kappa$.

**Example 1.** Consider the following system in nonlinear descriptor form with $E(x)=\begin{bmatrix}1 & x_1\\ -x_1 & 1\end{bmatrix}$, $A(x)=\begin{bmatrix}-x_2 & -1\\ 0.5 & \cos(x_2)+2\end{bmatrix}$, and $B(x)=\begin{bmatrix}0\\ 1\end{bmatrix}$. The representation in the form (1) gives $r_e=2$ and $r=4$ due to the number of nonlinearities on the left-hand side and right-hand side. To rewrite the original nonlinear descriptor system into the classical TS one it is necessary to compute $\left(E(x)\right)^{-1}$, resulting in $x_{\kappa+}=\left(E(x)\right)^{-1}\left(A(x)x_\kappa+B(x)u_\kappa\right)$. This means that four different nonlinearities have to be considered, which results in $r=16$ since all the nonlinear terms are on the right-hand side. Considering the relaxation lemma above and the quadratic framework, the number of LMI conditions to be verified for a 'classical' TS representation is $r^2+1=257$ whereas for the TS descriptor model, it is $r_er^2+1=33$. ◊

## III. Main Results

For the controller design purpose, the following non-PDC control law is used

$$u_\kappa=F_{hv}H_{(\bullet)}^{-1}x_\kappa,\tag{3}$$

where $F_{hv}=\sum_{j=1}^{r}\sum_{k=1}^{r_e}h_j\left(z(\kappa)\right)v_k\left(z(\kappa)\right)F_{jk}$ and matrix $H_{(\bullet)}$ will be defined afterward.

The TS descriptor model (1) together with the control law (3) yields:

$$E_vx_{\kappa+}=A_hx_\kappa+B_hF_{hv}H_{(\bullet)}^{-1}x_\kappa.\tag{4}$$

Expression (4) can be rewritten as an equality constraint:

$$\begin{bmatrix}A_h+B_hF_{hv}H_{(\bullet)}^{-1} & -E_v\end{bmatrix}\begin{bmatrix}x_\kappa\\ x_{\kappa+}\end{bmatrix}=0.\tag{5}$$

Thereinafter two different Lyapunov functions will be considered:

**Case 1:** $V(x_\kappa)=x_\kappa^TP_h^{-1}x_\kappa$, with $P_h=P_h^T>0$, $P_h^{-1}=X_h$.

**Case 2:** $V(x_\kappa)=x_\kappa^TH_h^{-T}P_hH_h^{-1}x_\kappa$, with $P_h=P_h^T>0$.

### A. Case 1.

The variation of the Lyapunov function in Case 1 is

$$\Delta V(x_\kappa)=x_{\kappa+}^TX_{h+}x_{\kappa+}-x_\kappa^TX_hx_\kappa<0.\tag{6}$$

The expression $\Delta V(x_\kappa)$ can be written as

$$\Delta V(x_\kappa)=\begin{bmatrix}x_\kappa\\ x_{\kappa+}\end{bmatrix}^T\begin{bmatrix}-X_h & 0\\ 0 & X_{h+}\end{bmatrix}\begin{bmatrix}x_\kappa\\ x_{\kappa+}\end{bmatrix}<0.\tag{7}$$

Via Finsler's Lemma, equality (5) and inequality (7) results in

$$\begin{bmatrix}-X_h & 0\\ 0 & X_{h+}\end{bmatrix}+\begin{bmatrix}M\\ N\end{bmatrix}\begin{bmatrix}A_h+B_hF_{hv}H_{(\bullet)}^{-1} & -E_v\end{bmatrix}+(*)<0,\tag{8}$$

where matrices $M\in\mathbb{R}^{n\times n}$ and $N\in\mathbb{R}^{n\times n}$ are free matrices fixed later on.

Let us select $H_{(\bullet)}=H_{hv}$. At this point two results can be stated depending on different congruence transformations of (8). The first one is stated in the following Lemma.

**Lemma 1**. The closed-loop TS descriptor model (4) is asymptotically stable if there exist matrices $P_j=P_j^T>0$, $H_{jk}$, and $F_{jk}$, for $i,j,l\in\{1,\dots,r\}$, $k\in\{1,\dots,r_e\}$ such that conditions (2) are satisfied with

$$\Upsilon_{ijl}^k=\begin{bmatrix}-H_{jk}^T-H_{jk}+P_j & (*)\\ A_iH_{jk}+B_iF_{jk} & -E_kP_l-P_lE_k^T+P_l\end{bmatrix}<0.\tag{9}$$

*Proof:* By using the congruence property with the full-rank matrix $\begin{bmatrix} H_{hv}^T & 0 \\ 0 & P_{h+} \end{bmatrix}$, (8) yields

$$\begin{bmatrix} -H_{hv}^T X_h H_{hv} & 0 \\ 0 & P_{h+} X_{h+} P_{h+} \end{bmatrix} + \begin{bmatrix} H_{hv}^T M \\ P_{h+} N \end{bmatrix} \begin{bmatrix} A_h H_{hv} + B_h F_{hv} & -E_v P_{h+} \end{bmatrix} + (*) < 0. \quad (10)$$

In order to obtain an LMI problem, a good choice is $M = 0$ and $N = X_{h+}$. Then (10) yields if

$$\begin{bmatrix} -H_{hv}^T X_h H_{hv} & (*) \\ A_h H_{hv} + B_h F_{hv} & -E_v P_{h+} - P_{h+} E_v^T + P_{h+} \end{bmatrix} < 0 \quad (11)$$

Finally, applying Property 1 and the relaxation lemma to (11) ends the proof. □

A more general result can be reached: by multiplying by $\begin{bmatrix} H_{hv}^T & 0 \\ 0 & G_{hhh+}^T \end{bmatrix}$ on the left-hand side and by its transpose on the right-hand side of (8), gives

$$\begin{bmatrix} -H_{hv}^T X_h H_{hv} & 0 \\ 0 & G_{hhh+}^T X_{h+} G_{hhh+} \end{bmatrix} + \begin{bmatrix} H_{hv}^T M \\ G_{hhh+}^T N \end{bmatrix} \begin{bmatrix} A_h H_{hv} + B_h F_{hv} & -E_v G_{hhh+} \end{bmatrix} + (*) < 0. \quad (12)$$

Note that a new matrix $G_{hhh+}$ is introduced, thus adding extra degrees of freedom to the inequality. Therefore, the following theorem can be stated:

**Theorem 1:** The closed-loop TS descriptor model (4) is asymptotically stable if there exist matrices $P_j = P_j^T > 0$, $H_{jk}$, $G_{ijl}$, and $F_{jk}$, for $i, j, l \in \{1, \ldots, r\}$, $k \in \{1, \ldots, r_e\}$ such that conditions (2) are satisfied with

$$\Upsilon_{ijl}^k = \begin{bmatrix} -H_{jk}^T - H_{jk} + P_j & (*) & (*) \\ A_i H_{jk} + B_i F_{jk} & -E_k G_{ijl} - G_{ijl}^T E_k^T & (*) \\ 0 & G_{ijl} & -P_l \end{bmatrix}. \quad (13)$$

*Proof:* Recall (12). Choosing the matrices $M = 0$ and $N = G_{hhh+}^{-T}$ yields:

$$\begin{bmatrix} -H_{hv}^T X_h H_{hv} & (*) \\ A_h H_{hv} + B_h F_{hv} & -E_v G_{hhh+} - G_{hhh+}^T E_v^T + G_{hhh+}^T X_{h+} G_{hhh+} \end{bmatrix} < 0.$$

Employing Property 1 and the Schur complement, it gives

$$\begin{bmatrix} -H_{hv}^T - H_{hv} + P_h & (*) & (*) \\ A_h H_{hv} + B_h F_{hv} & -E_v G_{hhh+} - G_{hhh+}^T E_v^T & (*) \\ 0 & G_{hhh+} & -P_{h+} \end{bmatrix} < 0, \quad (14)$$

or

$$\Upsilon_{hhh+}^v = \sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{l=1}^{r} \sum_{k=1}^{r_e} h_i(z(\kappa)) h_j(z(\kappa)) h_l(z(\kappa+1)) v_k(z(\kappa)) \cdot \cdot \begin{bmatrix} -H_{jk}^T - H_{jk} + P_j & (*) & (*) \\ A_i H_{jk} + B_i F_{jk} & -E_k G_{ijl} - G_{ijl}^T E_k^T & (*) \\ 0 & G_{ijl} & -P_l \end{bmatrix} < 0.$$

Applying the relaxation lemma the proof is ended. □

**Remark 1:** The best choice for matrix $G_{hhh+}$ allows obtaining extra degrees of freedom without increasing the number of LMIs to be satisfied. The number of extra matrices is $r^3$.

**Remark 2:** Consider the quadratic case for the classical TS models [4] $x_{\kappa+} = A_h x_\kappa + B_h u_\kappa$ with the classical stabilization condition:

$$\begin{bmatrix} -P & (*) \\ A_h P + B_h F_h & -P \end{bmatrix} < 0. \quad (15)$$

Results in Theorem 1 always include those from (15). To see that, consider inequality (14) with $G_{hhh+} = H_{hv} = P_h = P$. Employing the Schur complement, we have

$$\begin{bmatrix} -P & (*) \\ A_h P + B_h F_{hv} & -E_v P - P E_v^T + P \end{bmatrix} < 0. \quad (16)$$

Note that the classical TS model is a special case of the TS descriptor one when $E_v = I$, therefore inequality (16) yields expression (15).

## B. Case 2.

Consider $H_{(\cdot)} = H_h$ in (3). Then the variation of the Lyapunov function in Case 2 is

$$\Delta V(x_\kappa) = x_{\kappa+}^T H_{h+}^{-T} P_{h+} H_{h+}^{-1} x_{\kappa+} - x_\kappa^T H_h^{-T} P_h H_h^{-1} x_\kappa < 0. \quad (17)$$

The expression $\Delta V(x_\kappa)$ can be written as

$$\begin{bmatrix} x_\kappa \\ x_{\kappa+} \end{bmatrix}^T \begin{bmatrix} -H_h^{-T} P_h H_h^{-1} & 0 \\ 0 & H_{h+}^{-T} P_{h+} H_{h+}^{-1} \end{bmatrix} \begin{bmatrix} x_\kappa \\ x_{\kappa+} \end{bmatrix} < 0. \quad (18)$$

Through Finsler's Lemma, expressions (5) and (18) result in

$$\begin{bmatrix} -H_h^{-T} P_h H_h^{-1} & 0 \\ 0 & H_{h+}^{-T} P_{h+} H_{h+}^{-1} \end{bmatrix} + \begin{bmatrix} M \\ N \end{bmatrix} \begin{bmatrix} A_h + B_h F_{hv} H_h^{-1} & -E_v \end{bmatrix} + (*) < 0. \quad (19)$$

Using the property of congruence with $\begin{bmatrix} H_h^T & 0 \\ 0 & H_{h+}^T \end{bmatrix}$, (19) yields

$$\begin{bmatrix} -P_h & 0 \\ 0 & P_{h+} \end{bmatrix} + \begin{bmatrix} H_h^T M \\ H_{h+}^T N \end{bmatrix} \begin{bmatrix} A_h H_h + B_h F_{hv} & -E_v H_{h+} \end{bmatrix} + (*) < 0. \quad (20)$$

The following result can be stated.

**Theorem 2:** The closed-loop TS descriptor systems (4) is asymptotically stable if there exist matrices $P_j = P_j^T > 0$, $H_j$, and $F_{jk}$, for $i, j \in \{1, \ldots, r\}$, $k \in \{1, \ldots, r_e\}$ such that the LMI conditions (2) are satisfied with

$$\Upsilon_{ijl}^k = \begin{bmatrix} -P_j & (*) \\ A_i H_j + B_i F_{jk} & -E_k H_l - H_l^T E_v^T + P_l \end{bmatrix}. \quad (21)$$

*Proof:* Recall (20). Assigning $M=0$ and $N=H_{h+}^{-T}$ yields

$$\begin{bmatrix} -P_h & (*) \\ A_h H_h + B_h F_{hv} & -E_v H_{h+} - H_{h+}^T E_v^T + P_{h+} \end{bmatrix} < 0, \tag{22}$$

or

$$\Upsilon_{hhh+}^{v} = \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r}\sum_{k=1}^{r_e} h_i(z(\kappa)) h_j(z(\kappa)) h_l(z(\kappa+1)) v_k(z(\kappa)) \cdot$$
$$\cdot \begin{bmatrix} -P_j & (*) \\ A_i H_j + B_i F_{jk} & -E_k H_l - H_l^T E_k^T + P_l \end{bmatrix} < 0.$$

Through the relaxation lemma the proof is concluded. □

**Remark 3:** The approaches presented in Theorem 1 and Theorem 2 are not equivalent [29]. Also note that matrix $H_{(\bullet)}$ has a different structure in each case. This fact allows keeping the same number of LMI conditions, which is $r^3 r_e$.

## IV. Examples

The proposed results are illustrated via the following two numerical examples.

**Example 2.** Consider a TS descriptor model (1), with $r=r_e=2$, $A_1=\begin{bmatrix} -2.34 & -1.93 \\ 0.35 & 0.46 \end{bmatrix}$, $A_2=\begin{bmatrix} 1.47 & -1.26 \\ -0.17 & -0.93 \end{bmatrix}$, $B_1=\begin{bmatrix} 0.5 \\ -0.1 \end{bmatrix}$, $B_2=\begin{bmatrix} 0.5 \\ -0.34 \end{bmatrix}$, $E_1=\begin{bmatrix} 1.1 & 0 \\ 0 & 0.87 \end{bmatrix}$, and $E_2=\begin{bmatrix} 0.95 & 0 \\ 0 & 0.23 \end{bmatrix}$. The MFs are defined as follows: $v_{1\kappa}=\dfrac{x_{2\kappa}+2}{4}$, $v_{2\kappa}=1-v_{1\kappa}$, $h_{1\kappa}=\dfrac{x_{1\kappa}^2}{4}$, and $h_{2\kappa}=1-h_{1\kappa}$. The MFs hold the convex-sum property on the compact set $\Delta=\{x_\kappa : |x_{1\kappa}|\le 2, |x_{2\kappa}|\le 2\}$.

For this model, only the conditions of Theorem 1 are feasible, i.e., conditions in Theorem 2 are unfeasible. The following values were obtained:

$$P_1=\begin{bmatrix} 2.73 & -0.08 \\ -0.08 & 0.31 \end{bmatrix},\quad P_2=\begin{bmatrix} 2.16 & 0.42 \\ 0.42 & 0.20 \end{bmatrix},$$
$$H_{11}=\begin{bmatrix} 2.04 & 0.05 \\ -0.48 & 0.44 \end{bmatrix},\quad H_{12}=\begin{bmatrix} 2.19 & 0.14 \\ -0.47 & 0.23 \end{bmatrix},$$
$$H_{21}=\begin{bmatrix} 2.02 & 0.33 \\ 0.95 & 0.59 \end{bmatrix},\quad H_{22}=\begin{bmatrix} 2.05 & 0.35 \\ 1.39 & 0.67 \end{bmatrix},$$
$$F_{11}=\begin{bmatrix} 6.12 & 1.76 \end{bmatrix},\quad F_{12}=\begin{bmatrix} 5.97 & 1.46 \end{bmatrix},$$
$$F_{21}=\begin{bmatrix} -3.70 & -1.50 \end{bmatrix},\quad F_{22}=\begin{bmatrix} -4.71 & -1.85 \end{bmatrix}.$$

Simulation results with initial conditions $x(0)=\begin{bmatrix} 0.3 & -0.3 \end{bmatrix}^T$ are presented in Figure 1.

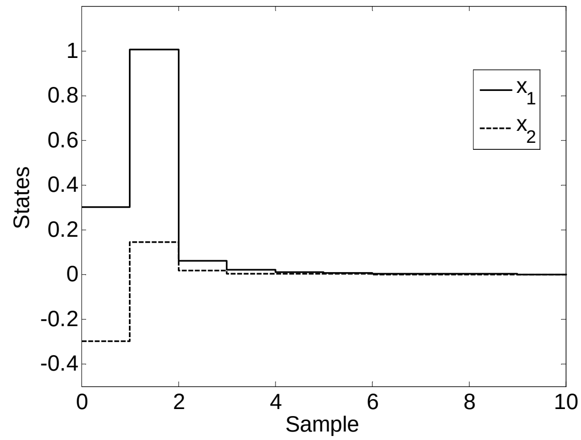

Fig. 1 States evolution in Example 2.

**Example 3.** Consider a TS descriptor model as in (1) with $r=r_e=2$, $A_1=\begin{bmatrix} 1.18 & -1.31 \\ -0.33 & 0.23 \end{bmatrix}$, $A_2=\begin{bmatrix} 0.69 & 1.41 \\ -1.17 & 1.43 \end{bmatrix}$, $B_1=\begin{bmatrix} 1 \\ -1.05 \end{bmatrix}$, $B_2=\begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $E_1=\begin{bmatrix} 1.1 & 0 \\ 0 & 0.36 \end{bmatrix}$, and $E_2=\begin{bmatrix} 0.95 & 0 \\ 0 & 1 \end{bmatrix}$. The MFs are defined the same as in Example 2.

Conditions in Theorem 1 are unfeasible, while Theorem 2 gives the following matrices:

$$P_1=\begin{bmatrix} 250.44 & 42.03 \\ 42.03 & 12.52 \end{bmatrix},\quad P_2=\begin{bmatrix} 4.33 & -1.73 \\ -1.73 & 3.17 \end{bmatrix},$$
$$H_1=\begin{bmatrix} 284.89 & 109.83 \\ 71.48 & 60.83 \end{bmatrix},\quad H_2=\begin{bmatrix} 114.77 & 44.59 \\ 88.09 & 42.52 \end{bmatrix},$$
$$F_{11}=\begin{bmatrix} 114.34 & -28.19 \end{bmatrix},\quad F_{12}=\begin{bmatrix} -136.15 & -31.59 \end{bmatrix},$$
$$F_{21}=\begin{bmatrix} -228.58 & -64.85 \end{bmatrix},\quad F_{22}=\begin{bmatrix} -217.15 & -76.16 \end{bmatrix}.$$

Simulation results with initial conditions $x(0)=\begin{bmatrix} 0.3 & -0.3 \end{bmatrix}^T$ are presented in Figure 2.

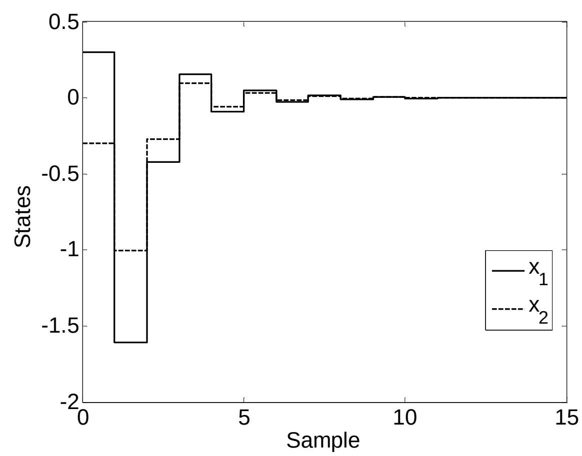

Fig. 2. States evolution in Example 3.

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