Improvements on Non-Quadratic Stabilization of Continuous-Time Takagi-Sugeno Descriptor Models

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***Abstract*—This paper presents a relaxed approach for stabilization and H_∞ disturbance rejection of continuous-time Takagi-Sugeno models in descriptor form. Based on Finsler's Lemma, the control law can be conveniently decoupled from a non-quadratic Lyapunov function. These developments include and outperform previous results on the same subject while preserving the advantage of being expressed as linear matrix inequalities. Two examples are presented to illustrate the improvements.**

Keywords*— *Descriptors; Linear Matrix Inequality; H_∞ criterion; Finsler's Lemma.

I. Introduction

Takagi-Sugeno (TS) models [1] have been intensively used for analysis and controller synthesis because of their convex structure which facilitates dealing with highly nonlinear characteristics while allowing the direct Lyapunov method to be employed for controller design [2]. Via the sector nonlinearity approach [3], TS models can exactly represent a nonlinear system in a compact set of the state space as a collection of linear models blended together by nonlinear membership functions (MFs) which hold the convex-sum property [2]. This structure permits stability analysis of TS models via common quadratic Lyapunov functions [4] and controller synthesis via parallel distributed compensation (PDC) control laws [5]. Since results are expressed as linear matrix inequalities (LMIs), many performance requirements can be naturally incorporated [6] and efficiently solved via interior-point methods [7]).

The aforementioned framework has been extended to the wider class of TS models in the descriptor form [8, 9]. This structure is particularly suitable to deal with mechanical models whose equations are naturally written with left-hand side nonlinearities [10]. Moreover, the descriptor form may lead to a significant reduction in the number of MFs involved in the TS model, thus alleviating the computational burden derived from the number of LMI conditions arising in analysis and controller design.

The use of common quadratic Lyapunov functions produces only sufficient conditions in the TS framework; thus, a number of efforts have been made in the last decade to overcome this source of conservativeness. Different types of Lyapunov functions have been tried: piecewise [11], path-independent integral [12] and fuzzy-like [13,14]. All of these choices have been combined with different ways to obtain pure LMI expressions; they are usually referred to as sum relaxations [15, 16, 17]. Both the choice of the Lyapunov function and sum relaxations have been used for TS models in the descriptor form to obtain progressively better and more general results [18, 19, 20].

In recent works, the well-known Finsler's Lemma has been used to obtain more relaxed conditions in the form of parameter-dependent LMIs, thus producing important generalizations even within the quadratic framework [21, 22]. This paper proposes a way to outperform existing non-quadratic results on stabilization for TS descriptor models [3, 18] with the help of Finsler's Lemma which decouples the control law from the Lyapunov function.

The contents are organized as follows: Section 2 introduces the TS descriptor model, the Lyapunov function and the control law this work is based on; Section 3 provides the main result on stabilization and H_∞ performance design based on Finsler's Lemma; Section 4 illustrates the improvements with two examples; Section 5 provides some concluding remarks to close this paper.

II. TS Descritor Form

Consider the following nonlinear model in the descriptor form:

$$\begin{aligned} E(x)\dot{x}(t) &= A(x)x(t)+B(x)u(t)+D(x)w(t) \\ y(t) &= C(x)x(t) \end{aligned}, \qquad (1)$$

where $x(t)\in\mathbb{R}^n$ is the state vector, $u(t)\in\mathbb{R}^m$ the control

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input, $y(t)\in\mathbb{R}^o$ the output vector, and $w(t)\in\mathbb{R}^q$ a vector of external disturbances; $A(x(t))$, $B(x(t))$, $C(x(t))$, $D(x(t))$ are matrices of appropriate size while $E(x(t))$ is a **regular** matrix of appropriate size.

Let the p nonlinearities in (1) be bounded as $nl_j(\cdot)\in\left[\underline{nl_j},\overline{nl_j}\right]$, $j\in\{1,\dots p\}$ in a compact set of the state space. Applying the sector nonlinearity approach [2], the described nonlinearities are captured via the following weighting functions

$$\omega_0^j(\cdot)=\frac{nl_j(\cdot)-\underline{nl_j}}{\overline{nl_j}-\underline{nl_j}},\quad \omega_1^j(\cdot)=1-\omega_0^j(\cdot),\quad j\in\{1,\dots,p\} \tag{2}$$

and the MFs as

$$h_i=\prod_{j=1}^{p}\omega_{i_j}^j(z_j)\ . \tag{3}$$

The MFs hold the convex-sum property $\sum_{i=1}^{r}h_i(\cdot)=1$, $h_i(\cdot)\geq 0$ in a compact set of the state variables and $r=2^p$ is the number of rules. It is assumed that the MFs depend on the premise variables $z(t)=\left[z_1(x(t))\ \cdots\ z_p(x(t))\right]$, which are known.

A shorthand notation will be used in the sequel to represent convex sums of matrix expressions: $\Upsilon_h=\sum_{i=1}^{r}h_i\Upsilon_i$ for single convex sums, $\Upsilon_h^{-1}=\left(\sum_{i=1}^{r}h_i\Upsilon_i\right)^{-1}$ for the inverse of a convex sum, and $\Upsilon_{hh}=\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\Upsilon_{ij}$ for a double nested convex sum. Indices may change to v if the respective MF is v_k. An asterisk $(*)$ will be used in matrix expressions to denote the transpose of the symmetric element; for in-line expressions it will denote the transpose of the terms on its left side. Arguments will be omitted when convenient.

The nonlinear system (1) can be rewritten as the following TS descriptor model:

$$\begin{aligned}E_v\dot{x}(t)&=A_hx(t)+B_hu(t)+D_hw(t)\\ y(t)&=C_hx(t)\end{aligned}\ , \tag{4}$$

where $Y_h=\sum_{i=1}^{r}h_i(z(t))Y_i$ and $E_v=\sum_{k=1}^{r_e}v_k(z(t))E_k$, with $h_i(z(t))\geq 0$, $i\in\{1,\dots,r\}$ and $v_k(z(t))\geq 0$, $k\in\{1,\dots,r_e\}$ being MFs. The sets (A_i,B_i,C_i,D_i) and E_k represent the i-th linear right-side model and the k-th linear left-side model of the TS descriptor model.

Let $\bar{x}(t)=\begin{bmatrix}x(t)\\ \dot{x}(t)\end{bmatrix}$ so (4) can be written as:

$$\begin{aligned}\bar{E}\dot{\bar{x}}(t)&=\bar{A}_{hv}\bar{x}(t)+\bar{B}_hu(t)+\bar{D}_hw(t)\\ y(t)&=\bar{C}_h\bar{x}(t)\end{aligned}\ , \tag{5}$$

with $\bar{E}=\begin{bmatrix}I&0\\0&0\end{bmatrix}$, $\bar{A}_{hv}=\begin{bmatrix}0&I\\A_h&-E_v\end{bmatrix}$, $\bar{B}_h=\begin{bmatrix}0\\B_h\end{bmatrix}$, $\bar{D}_h=\begin{bmatrix}0\\D_h\end{bmatrix}$, and $\bar{C}_h=\begin{bmatrix}C_h&0\end{bmatrix}$.

Consider the following control law:

$$u(t)=\begin{bmatrix}K_{hv}&0\end{bmatrix}Y_{hhv}^{-1}\bar{x}(t)=\bar{K}_{hv}Y_{hhv}^{-1}\bar{x}(t)\ , \tag{6}$$

with K_{jk}, Y_{ijk}, $i,j\in\{1,\dots,r\}$, $k\in\{1,\dots,r_e\}$ being gain matrices of proper size. Moreover Y_{hhv} has a special structure $Y_{hhv}=\begin{bmatrix}Y_{1hv}&Y_{2hv}\\Y_{3hh}&Y_{4hh}\end{bmatrix}$.

Remark 1: (6) corresponds to a new control since classically the matrix inverted is always the one used for the Lyapunov function. Therefore, (6) allows new structure of control law that is more flexible and less conservative. It corresponds to “decouple” the Lyapunov function from the control. In this context, the special structure of Y_{hhv} is explained through a non increase of LMI constraints, see (14) for example.

Substituting (6) in (5) and properly grouping terms yields the following closed-loop model

$$\begin{bmatrix}\bar{A}_{hv}+\bar{B}_h\bar{K}_{hv}Y_{hhv}^{-1}&-I&\bar{D}_h\end{bmatrix}\begin{bmatrix}\bar{x}\\ \bar{E}\dot{\bar{x}}\\ w\end{bmatrix}=0;\quad y(t)=\bar{C}_h\bar{x}(t)\ . \tag{7}$$

Consider the following Lyapunov function candidate [9, 18]:

$$V(\bar{x})=\bar{x}^T\bar{E}P_{hhv}^{-1}\bar{x};\quad \bar{E}P_{hhv}^{-1}=P_{hhv}^{-T}\bar{E}\ , \tag{8}$$

with $P_{hhv}=\begin{bmatrix}P_1&0\\P_{3hhv}&P_{4hhv}\end{bmatrix}$, $P_{hhv}^{-1}=X_{hhv}=\begin{bmatrix}X_1&0\\X_{3hhv}&X_{4hhv}\end{bmatrix}$, $X_1=P_1^{-1}$, $X_{4hhv}=P_{4hhv}^{-1}$, $X_{3hhv}=-P_{4hhv}^{-1}P_{3hhv}P_1^{-1}$, and $P_1=P_1^T>0$. As it will become apparent later, P_1 is chosen as a constant matrix to prevent the time-derivatives of the MFs from emerging in the following developments [18].

In order to drop off the MFs of double convex sums to obtain LMI conditions, the following relaxation scheme will be employed. This is a good compromise between effectiveness and computational complexity:

Relaxation Lemma [17]: Let Υ_{ij}^{k} be matrices of proper dimensions. Then $\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r_e} h_i h_j v_k \Upsilon_{ij}^{k} < 0$ holds if

$$\begin{aligned} &\Upsilon_{ii}^{k} < 0 \quad \forall i \\ &\frac{2}{r-1}\Upsilon_{ii}^{k} + \Upsilon_{ij}^{k} + \Upsilon_{ji}^{k} < 0, \quad i \neq j \end{aligned} \tag{9}$$

for $i, j \in \{1,\ldots,r\}$, $k \in \{1,\ldots,r_e\}$.

Finsler's Lemma [23]: Let $x \in \mathbb{R}^n$, $Q = Q^T \in \mathbb{R}^{n\times n}$, and $R \in \mathbb{R}^{m\times n}$ such that $rank(R) < n$; the following expressions are equivalent:

a) $x^T Q x < 0$, $\forall x \in \{x \in \mathbb{R}^n : x \neq 0, Rx = 0\}$

b) $\exists X \in \mathbb{R}^{n\times m} : Q + XR + R^T X^T < 0$.

III. Main Result

A. Stabilization

Theorem 1: The TS descriptor model (4) with $w(t) = 0$ under control law (6) is asymptotically stable if, for a given $\varepsilon > 0$, there exist matrices P_{ijk}, Y_{ijk}, K_{jk}, $i, j \in \{1,\ldots,r\}$, $k \in \{1,\ldots,r_e\}$ as defined in (6) and (8), such that (9) holds with

$$\Upsilon_{ij}^{k} = \begin{bmatrix} \bar{A}_{ik}Y_{ijk} + \bar{B}_i\bar{K}_{jk} + (*) & (*) \\ Y_{ijk} + \varepsilon\left(\bar{A}_{ik}Y_{ijk} + \bar{B}_i\bar{K}_{jk}\right) - P_{ijk} & -\varepsilon\left(P_{ijk} + P_{ijk}^T\right) \end{bmatrix}. \tag{10}$$

Proof: Consider the following inequality which guarantees $\dot{V}(\bar{x}(t)) < 0$:

$$\begin{bmatrix} \bar{x} \\ \bar{E}\dot{\bar{x}} \end{bmatrix}^T \begin{bmatrix} 0 & P_{hhv}^{-T} \\ P_{hhv}^{-1} & 0 \end{bmatrix} \begin{bmatrix} \bar{x} \\ \bar{E}\dot{\bar{x}} \end{bmatrix} < 0, \tag{11}$$

which already takes into account the fact that $\bar{E}\dot{P}_{hhv}^{-1} = 0$, together with equation (7) with $w(t) = 0$. Via Finsler's Lemma the following inequality guarantees $\dot{V}(\bar{x}(t)) < 0$ under restriction (7):

$$\begin{bmatrix} 0 & X_{hhv}^T \\ X_{hhv} & 0 \end{bmatrix} + \begin{bmatrix} U_{hhv} \\ V_{hhv} \end{bmatrix} \begin{bmatrix} \bar{A}_{hv} + \bar{B}_h\bar{K}_{hv}Y_{hhv}^{-1} & -I \end{bmatrix} + (*) < 0, \tag{12}$$

where U_{ijk}, V_{ijk}, $i, j \in \{1,\ldots,r\}$, $k \in \{1,\ldots,r_e\}$ are new matrix variables of adequate size to be chosen later.

Multiplying by $\begin{bmatrix} Y_{hhv}^T & 0 \\ 0 & P_{hhv} \end{bmatrix}$ on the left-hand side and by its transpose $\begin{bmatrix} Y_{hhv} & 0 \\ 0 & P_{hhv}^T \end{bmatrix}$ on the right-hand side of (12), gives

$$\begin{bmatrix} 0 & Y_{hhv}^T \\ Y_{hhv} & 0 \end{bmatrix} + \begin{bmatrix} Y_{hhv}^T U_{hhv} \\ P_{hhv}V_{hhv} \end{bmatrix} \begin{bmatrix} \bar{A}_{hv}Y_{hhv} + \bar{B}_h\bar{K}_{hv} & -P_{hhv}^T \end{bmatrix} + (*) < 0, \tag{13}$$

which after the suitable choice of $U_{hhv} = Y_{hhv}^{-T}$ and $V_{hhv} = \varepsilon P_{hhv}^{-1}$, $\varepsilon > 0$ renders (13) as:

$$\begin{bmatrix} \bar{A}_{hv}Y_{hhv} + \bar{B}_h\bar{K}_{hv} + (*) & (*) \\ Y_{hhv} + \varepsilon\left(\bar{A}_{hv}Y_{hhv} + \bar{B}_h\bar{K}_{hv}\right) - P_{hhv} & -\varepsilon\left(P_{hhv} + P_{hhv}^T\right) \end{bmatrix} < 0 \tag{14}$$

Applying the Relaxation Lemma to (14) ends the proof. □

Remark 2: Conditions in (10) are LMIs up to the selection of ε. Prefixing this sort of parameter has been a common practice among the LPV community in recent years [21, 22, 23] since it allows searching for a feasible solution in a logarithmically spaced family of values $\varepsilon \in \{10^{-6}, 10^{-5}, \ldots, 10^{6}\}$ which avoids an exhaustive linear search.

B. H_∞ Performance

A system performs disturbance attenuation $\gamma > 0$ if

$$\sup_{\|w(t)\|_2 \neq 0} \frac{\|y(t)\|_2}{\|w(t)\|_2} \leq \gamma, \tag{15}$$

where $\|\bullet\|_2$ stands for the ℓ_2 norm [2]. The following well-known condition guarantees (15):

$$\dot{V}(\bar{x}(t)) + y^T(t)y(t) - \gamma^2 w^T(t)w(t) \leq 0, \quad \forall \bar{x} \tag{16}$$

Theorem 2: The TS descriptor model (4) under control law (6) is asymptotically stable and ensures disturbance attenuation $\gamma > 0$ for the H_∞ criterion (16) if, for a given $\varepsilon > 0$, there exist matrices P_{ijk}, Y_{ijk}, K_{jk}, $i, j \in \{1,\ldots,r\}$, $k \in \{1,\ldots,r_e\}$ as defined in (6) and (8), such that (9) holds with

$$\Upsilon_{ij}^{k}=\begin{bmatrix}\bar{A}_{ik}Y_{ijk}+\bar{B}_i\bar{K}_{jk}+(*) & (*) & (*) & (*)\\ Y_{ijk}+\varepsilon\left(\bar{A}_{ik}Y_{ijk}+\bar{B}_i\bar{K}_{jk}\right)-P_{ijk} & -\varepsilon\left(P_{ijk}+P_{ijk}^{T}\right) & (*) & (*)\\ \bar{D}_i^{T} & \varepsilon\bar{D}_i^{T} & -\gamma^2 I & 0\\ \bar{C}_i Y_{ijk} & 0 & 0 & -I\end{bmatrix} \tag{17}$$

Proof: From the definition of the Lyapunov function candidate (7) it is clear that $V\left(\bar{x}(t)\right)>0$ everywhere, except possibly in the origin. Then condition (16) can be rewritten as

$$\begin{aligned}&\dot{V}\left(\bar{x}\right)+y^{T}y-\gamma^{2}w^{T}w\\ &=\dot{\bar{x}}^{T}\bar{E}X_{hhv}\bar{x}+\bar{x}^{T}X_{hhv}^{T}\bar{E}\dot{\bar{x}}+\bar{x}^{T}\bar{C}_h^{T}\bar{C}_h\bar{x}-\gamma^{2}w^{T}w\\ &=\begin{bmatrix}\bar{x}\\ \bar{E}\dot{\bar{x}}\\ w\end{bmatrix}^{T}\begin{bmatrix}\bar{C}_h^{T}\bar{C}_h & X_{hhv}^{T} & 0\\ X_{hhv} & 0 & 0\\ 0 & 0 & -\gamma^{2}I\end{bmatrix}\begin{bmatrix}\bar{x}\\ \bar{E}\dot{\bar{x}}\\ w\end{bmatrix}\le 0\end{aligned} \tag{18}$$

Via Finsler's Lemma the following inequality guarantees (18) under restriction (7):

$$\begin{bmatrix}\bar{C}_h^{T}\bar{C}_h & X_{hhv}^{T} & 0\\ X_{hhv} & 0 & 0\\ 0 & 0 & -\gamma^{2}I\end{bmatrix}+\begin{bmatrix}U_{hhv}\\ V_{hhv}\\ W_{hhv}\end{bmatrix}\begin{bmatrix}\bar{A}_{hv}+\bar{B}_h\bar{K}_{hv}Y_{hhv}^{-1} & -I & \bar{D}_h\end{bmatrix}+(*)<0 \tag{19}$$

where U_{ijk}, V_{ijk}, W_{ijk}, $i,j\in\{1,\ldots,r\}$, $k\in\{1,\ldots,r_e\}$ are matrix variables to be chosen later.

Multiplying (19) by $\begin{bmatrix}Y_{hhv}^{T} & 0 & 0\\ 0 & P_{hhv} & 0\\ 0 & 0 & I\end{bmatrix}$ on the left-hand side and by its transpose on the right-hand side, gives

$$\begin{bmatrix}Y_h^{T}\bar{C}_h^{T}\bar{C}_hY_h & Y_{hhv}^{T} & 0\\ Y_{hhv} & 0 & 0\\ 0 & 0 & -\gamma^{2}I\end{bmatrix}+\begin{bmatrix}Y_{hhv}^{T}U_{hhv}\\ P_{hhv}V_{hhv}\\ IW_{hhv}\end{bmatrix}\begin{bmatrix}\bar{A}_{hv}Y_{hhv}+\bar{B}_h\bar{K}_{hv} & -P_{hhv}^{T} & \bar{D}_h\end{bmatrix}+(*)<0, \tag{20}$$

which after choosing $U_{hhv}=Y_{hhv}^{-T}$ and $V_{hhv}=\varepsilon P_{hhv}^{-1}$, $W_{hhv}=0$, $\varepsilon>0$ renders (20) as:

$$\begin{bmatrix}Y_h^{T}\bar{C}_h^{T}\bar{C}_hY_h & Y_{hhv}^{T} & 0\\ Y_{hhv} & 0 & 0\\ 0 & 0 & -\gamma^{2}I\end{bmatrix}+\begin{bmatrix}\bar{A}_{hv}Y_{hhv}+\bar{B}_h\bar{K}_{hv}+(*) & (*) & (*)\\ \varepsilon\left(\bar{A}_{hv}Y_{hhv}+\bar{B}_h\bar{K}_{hv}\right)-P_{hhv} & -\varepsilon\left(P_{hhv}+P_{hhv}^{T}\right) & (*)\\ \bar{D}_h^{T} & \varepsilon\bar{D}_h^{T} & 0\end{bmatrix}<0 \tag{21}$$

Finally, applying Schur complement yields

$$\begin{bmatrix}\bar{A}_{hv}Y_{hhv}+\bar{B}_h\bar{K}_{hv}+(*) & (*) & (*) & (*)\\ Y_{hhv}+\varepsilon\left(\bar{A}_{hv}Y_{hhv}+\bar{B}_h\bar{K}_{hv}\right)-P_{hhv} & -\varepsilon\left(P_{hhv}+P_{hhv}^{T}\right) & (*) & (*)\\ \bar{D}_h^{T} & \varepsilon\bar{D}_h^{T} & -\gamma^{2}I & 0\\ \bar{C}_hY_{hhv} & 0 & 0 & -I\end{bmatrix}<0 \tag{22}$$

Applying the Relaxation Lemma to (22) ends the proof. □

IV. Examples

A. Example 1.

Consider a TS fuzzy model (4) with $r=r_e=2$ and matrices $A_1=\begin{bmatrix}-4.3 & 4.8\\ -1.7 & 1\end{bmatrix}$, $A_2=\begin{bmatrix}a & 2\\ -4.7 & -2.8\end{bmatrix}$, $B_1=\begin{bmatrix}5.6\\ 0.9\end{bmatrix}$, $B_2=\begin{bmatrix}8.1\\ b\end{bmatrix}$, $E_1=\begin{bmatrix}0.8+a & 0.0\\ 0.21+b & 0.03\end{bmatrix}$, and $E_2=\begin{bmatrix}0.8 & 0.7\\ 0.5 & 0.68\end{bmatrix}$ with $a\in[-7\quad 4]$ and $b\in[0.4\quad 2]$ parameters.

Fig. 1 shows the feasibility region for several values of a and b employing Theorem 1 in [18] (O) and Theorem 1 in this work (x).

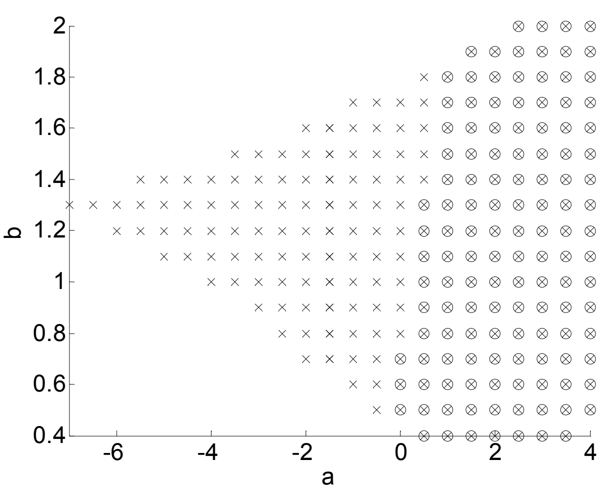

Fig. 1. Feasibility region in Example 1.

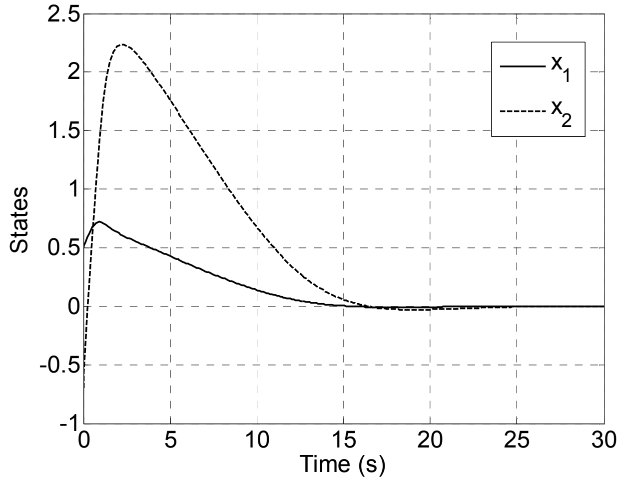

Fig. 2. Simulation results for Example 1.

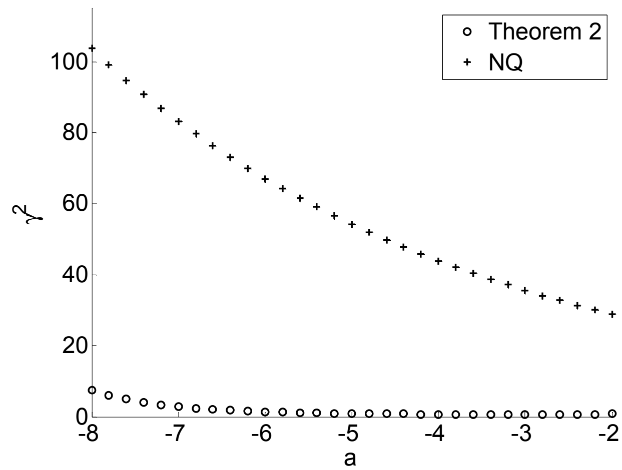

Fig. 3. H_∞ optimal values in Example 2.

Note that taking $a = -3$ and $b = 1$ there is no solution for Theorem 1 of [18], but employing Theorem 1 a stabilizing controller was found with $\varepsilon = 1$ and the following values: $P_1 = \begin{bmatrix} 1.89 & 6.93 \\ 6.93 & 30.02 \end{bmatrix}$, $K_{11} = \begin{bmatrix} -4.27 \\ -17.10 \end{bmatrix}^T$, $K_{12} = \begin{bmatrix} -4.57 \\ -19.38 \end{bmatrix}^T$, $K_{21} = \begin{bmatrix} 6.33 \\ 24.22 \end{bmatrix}^T$, and $K_{22} = \begin{bmatrix} 6.78 \\ 27.14 \end{bmatrix}^T$.

Fig. 2 shows the state evolution in Example 1 with initial conditions $x(0) = [0.5 \quad -0.7]^T$ for $a = -3$ and $b = 1$.

B. Example 2.

Consider a TS fuzzy model as in (4) with $r = r_e = 2$ and matrices as follows $A_1 = \begin{bmatrix} -4.4 & 2 \\ -4.1 & -3.5 \end{bmatrix}$, $A_2 = \begin{bmatrix} -4.2 & 2 \\ -4.7 & -2.8 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0.6 + a \\ 0.6 \end{bmatrix}$, $B_2 = \begin{bmatrix} 6.3 \\ 0.6 \end{bmatrix}$, $C_1 = [1 \quad 0]$, $C_2 = [0 \quad 1]$, $D_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $D_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $E_1 = \begin{bmatrix} 0.3 & 0.6 \\ 0.2 & 0.9 \end{bmatrix}$, and $E_2 = \begin{bmatrix} 0.9 & 0.4 \\ 0 & 0.1 \end{bmatrix}$, with the parameter $a \in [-8 \quad -2]$.

The optimal value for γ^2 is computed since γ^2 is not multiplied by any decision variables. In Fig. 3, it is shown results using Theorem 2 and [18].

V. CONCLUSIONS

In this work, a new scheme for non-quadratic stabilization for continuous-time TS descriptors models has been presented. This scheme is based on Finsler's Lemma, which allows decoupling the Lyapunov function and the controller, thus providing additional flexibility to the design. Moreover, a H_∞ performance based on the same lines is presented. Two examples are included to show the effectiveness of the present approach.

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