Local stabilization of discrete-time TS descriptor systems

Zsófia Lendek[a], Zoltán Nagy[a], Jimmy Lauber[b]

[a] *Department of Automation, Technical University of Cluj-Napoca, Romania ({zsofia.lendek, zoltan.nagy}@aut.utcluj.ro)*

[b] *University of Valenciennes and Hainaut-Cambresis, LAMIH, Le Mont Houy, 59313 Valenciennes Cedex 9, France, (jlauber@univ-valenciennes.fr)*

Abstract

Descriptor models are naturally obtained from the Euler-Lagrange modeling approach to mechanical systems. Since the underlying system is nonlinear, global stabilization and/or tracking is possible only in a limited number of cases. Therefore, we develop conditions for local stabilization and tracking of discrete-time descriptor systems represented by Takagi-Sugeno fuzzy models, using both quadratic and nonquadratic Lyapunov functions. An estimate of the region of attraction is also obtained. The conditions are illustrated on a numerical example and in tracking control for a robot arm.

Keywords: local stabilization, TS models, descriptor form, robot arm.

1. Introduction

Mechanical systems [1, 2, 3] are frequently used in transportation, handling, assembly; security, surveillance, quality inspection, exploration can be done by mobile robots [4]; and healthcare problems may be alleviated via bio-mechanic devices [5, 6]. The dynamic models of such mechanical systems are generally determined using the Euler-Lagrange equations [1, 3], which give second-order vector differential equations. Once the Euler-Lagrange equations are obtained, the state-space representation is naturally in a descriptor form [1, 7, 8], i.e., it has the mass matrix on the left-hand side. Since the model is nonlinear, nonlinear tools should be used for the analysis and design of such systems.

In the last two decades, Takagi-Sugeno (TS) fuzzy models [9] have attracted considerable interest in the automatic stability analysis and controller design of nonlinear systems. TS models represent the nonlinear system considered as a convex combination of local linear models blended together by nonlinear membership functions (MFs), which share the convex-sum property [10, 11]. When using the sector nonlinearity approach [10] to obtain it, the TS model will be an exact representation of the system in the considered compact set of the state-space. A shortcoming of this approach is that the number of local models in the TS model is exponentially related to the number of nonlinearities in the original nonlinear model [12], leading to increased computational costs. If descriptor models are represented in this classical form (via matrix inversion), this number grows very large.

Instead, the goal of this paper is to develop conditions that ensure local asymptotic stability of discrete-time TS models directly in the descriptor form, i.e., we study the case when the considered equilibrium point is only locally stable. To our best knowledge, this is the first time that this topic has been addressed. We consider the problem of local stabilization and estimating a domain of attraction of the equilibrium point. We combine the tools existing in the discrete TS framework with the determination of a non-quadratic domain of attraction using an easy procedure that requires only the knowledge of the membership functions. We also extend this procedure to input-to-state stability (ISS).

In order to obtain an exact TS representation of the original nonlinear model, we use the sector nonlinearity approach. A TS descriptor model [13] generalizes the standard one and is obtained by applying the approach to both sides of the nonlinear differential equation. In this way, the nonlinear terms are separated and thus a smaller number of local models [14, 15, 16] and a reduced number of LMI constraints [17, 18] may be achieved.

We develop conditions for local asymptotic stability and tracking. Since TS models are nonlinear, for their analysis and control synthesis the direct Lyapunov approach has been employed, using initially quadratic Lyapunov functions [19, 12, 20], then piecewise continuous Lyapunov functions [21, 22], and recently, nonquadratic Lyapunov functions [23, 24, 25], that have significantly improved existing results [23, 26, 27, 28]. The conditions for stability analysis or controller and observer design are developed in general in form of linear matrix inequalities (LMIs) that can be solved using available convex optimization methods [29, 30].

Classic conditions, particularly in the discrete-time case, have been developed to establish global asymptotic stability of the (closed-loop) model, i.e., if the conditions are satisfied, then the model is globally asymptotically stable, but if they are unfeasible, no conclusions may be drawn. Thus, when the model is only locally stable or stabilizable, these conditions will not be satisfied. If the studied equilibrium point is only locally stable, the existing conditions become unusable, in the sense that the LMI conditions become unfeasible. In our previous works [31, 32] we have developed sufficient conditions for local stability and stabilization of classic discrete-time TS models. However, these results are not directly applicable to descriptor models, due to the descriptor matrix appearing on the left-hand side. In this paper we specifically address local stability and set point tracking for descriptor models.

Next to developing local conditions, we also estimate the region of attraction of the considered equilibrium point. Results on this topic are scarce even for linear models. References [33, 34] consider linear discrete-time systems with actuator and/or state saturations and the stability analysis is performed using quadratic Lyapunov functions, while trying to find the maximum admissible quadratic domain of attraction. Others, considering the same problem of actuator saturation still using quadratic Lyapunov functions, proposed ways to design the domain of attraction based on convex set as polyhedrons [35] or as being saturation dependent [36]. Here, we determine instead a non-quadratic domain of attraction based simply on the system's dynamics.

The paper is organized as follows. Section 2 presents the notations to be used throughout the paper and states the preliminaries. Section 3 concerns local stabilization of descriptor TS models. The conditions for local tracking are developed in Section 4. The conditions are discussed and illustrated on a numerical example and in tracking control of a robot arm in Section 5. Section 6 concludes the paper.

2. Notation and preliminaries

The dynamics of mechanical systems is usually represented in a state-space form, which can be obtained from the Euler-Lagrange equations [1]:

$$M(\boldsymbol{q})\ddot{\boldsymbol{q}} + C_o(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + F_r(\dot{\boldsymbol{q}}) + G_r(\boldsymbol{q})\boldsymbol{q} = \boldsymbol{\tau} \tag{1}$$

where $\boldsymbol{q}$ is the generalized position of the joints, $\dot{\boldsymbol{q}}$ is the velocity, $\ddot{\boldsymbol{q}}$ is the acceleration, $M(\boldsymbol{q})$ is the mass matrix, $C_o(\boldsymbol{q}, \dot{\boldsymbol{q}})$ is the Coriolis/centrifugal matrix, $F_r(\dot{\boldsymbol{q}})$ is the friction, $G_r(\boldsymbol{q})$ is the gravity matrix, $\boldsymbol{\tau}$ is the input vector. Note that the mass matrix is positive definite and thus in what follows we consider regular descriptor models.

Rewriting (1) as

$$\begin{pmatrix} I & 0 \\ 0 & M(\boldsymbol{q}) \end{pmatrix} \begin{pmatrix} \dot{\boldsymbol{q}} \\ \ddot{\boldsymbol{q}} \end{pmatrix} = \begin{pmatrix} \dot{\boldsymbol{q}} \\ -C_o(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} - F_r(\dot{\boldsymbol{q}}) - G_r(\boldsymbol{q})\boldsymbol{q} \end{pmatrix} + \begin{pmatrix} 0 \\ I \end{pmatrix} \boldsymbol{\tau} \tag{2}$$

a state-space representation of (2) may be obtained by defining $\boldsymbol{x} = \begin{pmatrix} \boldsymbol{q} \\ \dot{\boldsymbol{q}} \end{pmatrix}$ as the state vector and $\boldsymbol{u} = \boldsymbol{\tau}$ as the input and can be written in the general form [1]

$$E(\boldsymbol{x})\dot{\boldsymbol{x}} = f(\boldsymbol{x}, \boldsymbol{u}) \tag{3}$$

with $E(x)$ being a positive definite matrix. Equation (3) is in the general form of the so-called descriptor model [7]. When discretizing (3), still a descriptor form is obtained:

$$E(\boldsymbol{x}(k))\boldsymbol{x}(k+1) = f(\boldsymbol{x}(k), \boldsymbol{u}(k)) \tag{4}$$

In what follows, we will consider this discrete-time descriptor model.

Takagi-Sugeno (TS) fuzzy models are convex combinations of local linear models, with the descriptor TS model [18] having the form

$$\sum_{j=1}^{r_e} v_j(\boldsymbol{z}(k))E_j\boldsymbol{x}(k+1) = \sum_{i=1}^{r} h_i(\boldsymbol{z}(k))(A_i\boldsymbol{x}(k) + B_i\boldsymbol{u}(k))$$

where (E_j, A_i, B_i) are local linear models, $\boldsymbol{x}$ is the state vector, $\boldsymbol{u}$ is the input vector, $\boldsymbol{z}$ is the scheduling vector, r and r_e are the number of local linear models, $v_j(\boldsymbol{z})$ and $h_i(\boldsymbol{z})$ are the membership functions that hold the convex sum property, i.e., $v_j(\boldsymbol{z}) \in [0,\, 1]$, $\sum_{j=1}^{r_e} v_j(\boldsymbol{z}) = 1$, $h_i(\boldsymbol{z}) \in [0,\, 1]$, $\sum_{i=1}^{r} h_i(\boldsymbol{z}) = 1$.

TS models may be either an approximation of the nonlinear model (obtained e.g., by local linearization [12] or by substitution [37]) or an exact representation (when applying the sector nonlinearity approach [10]) in a compact set of the state-space. In this paper, the sector nonlinearity approach [10] is used, thus the resulting TS model will exactly represent the dynamic system in the considered compact set.

The main idea of obtaining a fuzzy model using the sector nonlinearity approach [10] is as follows. Consider the nonlinear descriptor system

$$\begin{aligned} E(\boldsymbol{z}(k))\boldsymbol{x}(k+1) &= \boldsymbol{f}(\boldsymbol{z}(k))\boldsymbol{x}(k) + \boldsymbol{g}(\boldsymbol{z}(k))\boldsymbol{u}(k) \\ \boldsymbol{y}(k) &= C\boldsymbol{x}(k) \end{aligned} \tag{5}$$

with E, $\boldsymbol{f}$ and $\boldsymbol{g}$ smooth nonlinear matrix functions, $\boldsymbol{x} \in \mathcal{R}^n$ the state vector, $\boldsymbol{u} \in \mathcal{R}^{n_u}$ the input vector, and $\boldsymbol{y} \in \mathcal{R}^{n_y}$ the measurement vector, $\boldsymbol{z}$ some vector function of $\boldsymbol{x}$, $\boldsymbol{y}$, all variables assumed to be bounded on a compact set $\mathcal{D}$.

Let $\mathrm{nl}_j(\cdot) \in [\underline{\mathrm{nl}}_j, \overline{\mathrm{nl}}_j]$, $j = 1, 2, \ldots, p$ be the set of bounded nonlinearities on the right-hand-side, i.e., components of either $\boldsymbol{f}$ or $\boldsymbol{g}$ and $\mathrm{nl}^e_j(\cdot) \in [\underline{\mathrm{nl}}^e_j, \overline{\mathrm{nl}}^e_j]$, $j = 1, 2, \ldots, p_e$ those on the left-hand-side, i.e., components of E. An exact TS fuzzy representation of (5) can be obtained by constructing first the weighting functions

$$w^j_0(\cdot) = \frac{\overline{\mathrm{nl}}_j - \mathrm{nl}_j(\cdot)}{\overline{\mathrm{nl}}_j - \underline{\mathrm{nl}}_j} \quad w^j_1(\cdot) = 1 - w^j_0(\cdot) \quad j = 1, 2, \ldots, p$$
$$w^j_{e0}(\cdot) = \frac{\overline{\mathrm{nl}}^e_j - \mathrm{nl}^e_j(\cdot)}{\overline{\mathrm{nl}}^e_j - \underline{\mathrm{nl}}^e_j} \quad w^j_{e1}(\cdot) = 1 - w^j_{e0}(\cdot) \quad j = 1, 2, \ldots, p_e$$

and defining the membership functions as

$$h_i(\boldsymbol{z}) = \prod_{j=1}^{p} w^j_{i_j}(\boldsymbol{z}_j) \qquad v_i(\boldsymbol{z}) = \prod_{j=1}^{p_e} w^j_{ei_j}(\boldsymbol{z}_j) \tag{6}$$

with $i = 1, 2, \cdots, 2^p$, $i_j \in \{0, 1\}$ and $i = 1, 2, \cdots, 2^{p_e}$, $i_j \in \{0, 1\}$, respectively. These membership functions are normal, i.e., $h_i(\boldsymbol{z}) \geq 0$ and $\sum_{i=1}^{r} h_i(\boldsymbol{z}) = 1$, $r = 2^p$, $v_i(\boldsymbol{z}) \geq 0$ and $\sum_{i=1}^{r_e} v_i(\boldsymbol{z}) = 1$, $r = 2^{p_e}$, where r and r_e are the number of rules. The local matrices for each rule are obtained by substituting the corresponding upper or lower bounds of the nonlinearities.

Using the membership functions defined in (6), an exact representation of (5) is given as:

$$\sum_{j=1}^{r_e} v_j(\boldsymbol{z}(k)) E_j \boldsymbol{x}(k+1) = \sum_{i=1}^{r} h_i(\boldsymbol{z}(k))(A_i \boldsymbol{x}(k) + B_i \boldsymbol{u}(k)) \tag{7}$$

where $\boldsymbol{x}$ denotes the state vector, r and r_e are the number of rules on the right and left-hand side, respectively, $\boldsymbol{z}$ is the scheduling vector, h_i, $i = 1, 2, \ldots, r$, v_j, $j = 1, 2, \ldots, r_e$ are normalized membership functions, and A_i, B_i, E_j, $i = 1, 2, \ldots, r$, $j = 1, 2, \ldots, r_e$, are the local models.

Remark: Since we assume that E is positive definite and thus it is invertible, by multiplying the nonlinear model with E^{-1} one can obtain a classic state-space model and continue with a classic TS modeling. However, as shown in [14, 17, 18], keeping the descriptor form presents several advantages, among which having a smaller number of local models and obtaining a reduced number of LMI constraints. This is why we consider descriptor TS models.

In this paper we develop sufficient conditions for the local stabilization of TS fuzzy models in descriptor form, i.e., of the form (7). Note that local results may also be obtained by simply linearizing the dynamics (4) around $\boldsymbol{x} = 0$ and computing a stabilizing state-feedback control law $\boldsymbol{u} = -F\boldsymbol{x}$. However, this linear control law in general will guarantee the stability of the closed-loop system only in a very small domain. By considering the TS descriptor representation (7) and developing local conditions, we aim to guarantee a much larger domain of attraction.

In what follows, we use the following shorthand notations for generic matrices X:

$$X_v \equiv \sum_{i=1}^{r_e} v_i(\boldsymbol{z}(k))X_i \qquad X_h \equiv \sum_{i=1}^{r} h_i(\boldsymbol{z}(k))X_i$$
$$X_{vh} \equiv \sum_{i=1}^{r_e}\sum_{j=1}^{r} v_i(\boldsymbol{z}(k))h_j(\boldsymbol{z}(k))X_{ij}$$
$$X_{vh}^{-1} \equiv \left(\sum_{i=1}^{r_e}\sum_{j=1}^{r} v_i(\boldsymbol{z}(k))h_j(\boldsymbol{z}(k))X_{ij}\right)^{-1}$$

Thus, (7) will be denoted as:

$$E_v\boldsymbol{x}(k+1) = A_h\boldsymbol{x}(k) + B_h\boldsymbol{u}(k)$$

Furthermore, 0 and I denote the zero and identity matrices of appropriate dimensions, and a $(*)$ denotes the term induced by symmetry in a matrix form, i.e., $\begin{pmatrix} A & B \\ (*) & C \end{pmatrix} = \begin{pmatrix} A & B \\ B^T & C \end{pmatrix}$, and the symmetrical part of the expression on the left-hand side in inline form, i.e., $X + (*) + Y = X + X^T + Y$. The subscripts '+' (as in A_{h+}, $\mathcal{P}_+$) and '$-$' stand for the scheduling vector being evaluated at the next sample and at the previous sample, i.e., at $\boldsymbol{z}(k+1)$ and at $\boldsymbol{z}(k-1)$, respectively. We will also make use of the following results:

Lemma 1. *[38] (Finsler's lemma) Consider a vector $\boldsymbol{x} \in \mathbb{R}^{n_x}$ and two matrices $Q = Q^T \in \mathbb{R}^{n_x \times n_x}$ and $\mathcal{R} \in \mathbb{R}^{m \times n_x}$ such that rank$(\mathcal{R}) < n_x$. The two following expressions are equivalent:*

1. $\boldsymbol{x}^T Q\boldsymbol{x} < 0$, $\boldsymbol{x} \in \{\boldsymbol{x} \in \mathbb{R}^{n_x}, \boldsymbol{x} \neq 0, \mathcal{R}\boldsymbol{x} = 0\}$
2. $\exists \mathcal{M} \in \mathbb{R}^{m \times n_x}$ *such that* $Q + \mathcal{M}\mathcal{R} + \mathcal{R}^T\mathcal{M}^T < 0$

Proposition 1. *(Congruence) Given a matrix $P = P^T$ and a full column rank matrix Q it holds that*

$$P > 0 \;\Rightarrow\; QPQ^T > 0$$

Proposition 2. *Let A and B be matrices of appropriate dimensions and ranks, with $B = B^T > 0$. Then*

$$(A - B)^T B^{-1}(A - B) \geq 0 \Longleftrightarrow A^T B^{-1} A \geq A + A^T - B$$

Proposition 3. [29] *(Schur complement) Consider a matrix $M = M^T = \begin{pmatrix} M_{11} & M_{12} \\ M_{12}^T & M_{22} \end{pmatrix}$, with M_{11} and M_{22} being square matrices. Then*

$$M < 0 \Leftrightarrow \begin{cases} M_{11} < 0 \\ M_{22} - M_{12}^T M_{11}^{-1} M_{12} < 0 \end{cases} \Leftrightarrow \begin{cases} M_{22} < 0 \\ M_{11} - M_{12} M_{22}^{-1} M_{12}^T < 0 \end{cases}$$

Proposition 4. *(S-procedure) Consider matrices $F_i = F_i^T \in \mathbb{R}^{n\times n}$, $\boldsymbol{x} \in \mathbb{R}^n$, such that $\boldsymbol{x}^T F_i \boldsymbol{x} \geq 0$, $i = 1, \ldots, p$, and the quadratic inequality condition*

$$\boldsymbol{x}^T F_0 \boldsymbol{x} > 0 \tag{8}$$

$\boldsymbol{x} \neq 0$. A sufficient condition for (8) *to hold is: there exist $\tau_i \geq 0$, $i = 1, \ldots, p$, such that*

$$F_0 - \sum_{i=1}^{p} \tau_i F_i > 0$$

Analysis and design for TS models often lead to double convex-sum negativity problems of the form

$$\boldsymbol{x}^T \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\boldsymbol{z}(k)) h_j(\boldsymbol{z}(k)) \Gamma_{ij} \boldsymbol{x} < 0 \tag{9}$$

where Γ_{ij}, $i, j = 1, 2, \ldots, r$ are matrices of appropriate dimensions and the functions $h_i(\boldsymbol{z}(k))$ satisfy the convex properties, i.e., $h_i(\boldsymbol{z}(k)) \in [0, 1]$ and $\sum_{i=1}^{r} h_i(\boldsymbol{z}(k)) = 1$, $\forall \boldsymbol{z}(k)$.

While in the literature an abundance of sufficient conditions to ensure the negativity of the double sum above exist, for simplicity, in this paper we will use the following lemma.

Lemma 2. *[39] The double convex sum* (9) *is negative, if*

$$\begin{aligned} &\Gamma_{ii} < 0 \\ &\Gamma_{ij} + \Gamma_{ji} < 0, \quad i, j = 1, 2, \ldots, r, \, i < j \end{aligned} \tag{10}$$

3. Local stabilization

Consider the descriptor TS model, repeated here for convenience:

$$E_v \boldsymbol{x}(k+1) = A_h \boldsymbol{x}(k) + B_h \boldsymbol{u}(k) \tag{11}$$

defined on a domain $\mathcal{D}$ including the origin.

Our goal is to determine a control law of the form

$$\boldsymbol{u}(k) = -\mathcal{F}\mathcal{H}^{-1}\boldsymbol{x}(k) \tag{12}$$

where $\mathcal{F}$ and $\mathcal{H}$ are (possibly fuzzy) controller gains such that the closed-loop system,

$$E_v \boldsymbol{x}(k+1) = (A_h - B_h \mathcal{F}\mathcal{H}^{-1})\boldsymbol{x}(k) \tag{13}$$

has a locally asymptotically stable equilibrium point in $\boldsymbol{x} = 0$ and determine a region of attraction $\mathcal{D}_S$. For this, let us first assume the following

Assumption 1. *There exists a domain $\mathcal{D}_R$ and a symmetric (possibly fuzzy) matrix expression $\mathcal{R} = \mathcal{R}^T$ so that*

$$\begin{pmatrix} \boldsymbol{x}(k) \\ \boldsymbol{x}(k+1) \end{pmatrix}^T \mathcal{R} \begin{pmatrix} \boldsymbol{x}(k) \\ \boldsymbol{x}(k+1) \end{pmatrix} \geq 0 \tag{14}$$

holds $\forall \boldsymbol{x}(k) \in \mathcal{D}_R$.

Remark: Note that Assumption 1 expresses the domain $\mathcal{D}_R$ as an implicit expression depending on the dynamics of the system, i.e., $\mathcal{D}_R$ is given by

$$\mathcal{D}_R = \left\{ \boldsymbol{x} \in \mathcal{D} \left| \begin{pmatrix} \boldsymbol{x} \\ E_v^{-1}(A_h - B_h \mathcal{F}\mathcal{H}^{-1})\boldsymbol{x} \end{pmatrix}^T \mathcal{R} \begin{pmatrix} \boldsymbol{x} \\ E_v^{-1}(A_h - B_h \mathcal{F}\mathcal{H}^{-1})\boldsymbol{x} \end{pmatrix} \geq 0 \right. \right\}$$

For a more compact and easier notation, in developing the conditions we will use the form (14).

Assumption 1 above can always be satisfied, e.g., by choosing a constant matrix $\mathcal{R} = R = R^T > 0$. The domain $\mathcal{D}_R$ depends on the system being analyzed, and in the worst case $\mathcal{D}_R = \{0\}$.

3.1. Local quadratic stabilization

We start with the following result.

Theorem 1. *Given a symmetric matrix expression $\mathcal{R} = \mathcal{R}^T$ such that Assumption 1 holds, the discrete-time nonlinear model* (13) *is locally asymptotically stable if there exist matrices $P = P^T > 0$, $\mathcal{F}$, $\mathcal{H}$, and scalar $\tau > 0$ so that*

$$\begin{pmatrix} -\mathcal{H} - \mathcal{H}^T + P & (*) \\ A_h\mathcal{H} - B_h\mathcal{F} & -E_v P - (*) + P \end{pmatrix} + \tau \begin{pmatrix} \mathcal{H}^T & 0 \\ 0 & P \end{pmatrix} \mathcal{R} \begin{pmatrix} \mathcal{H} & 0 \\ 0 & P \end{pmatrix} < 0 \tag{15}$$

Moreover, the region of attraction , i.e., the region from which all trajectories converge to zero, includes $\mathcal{D}_S$, where $\mathcal{D}_S$ is the largest Lyapunov level set included in $\mathcal{D}_R$.

Proof. Consider the candidate Lyapunov function $V = \boldsymbol{x}^T(k)P^{-1}\boldsymbol{x}(k)$. The difference is

$$\begin{aligned} \Delta V &= \boldsymbol{x}^T(k+1)P^{-1}\boldsymbol{x}(k+1) - \boldsymbol{x}^T(k)P^{-1}\boldsymbol{x}(k) \\ &= \begin{pmatrix} \boldsymbol{x}(k) \\ \boldsymbol{x}(k+1) \end{pmatrix}^T \begin{pmatrix} -P^{-1} & 0 \\ 0 & P^{-1} \end{pmatrix} \begin{pmatrix} \boldsymbol{x}(k) \\ \boldsymbol{x}(k+1) \end{pmatrix} \end{aligned}$$

In the domain $\mathcal{D}_R$ Assumption 1 holds, thus, using Proposition 4, we have $\Delta V < 0$ if there exists $\tau > 0$ so that

$$\begin{gathered} \begin{pmatrix} \boldsymbol{x}(k) \\ \boldsymbol{x}(k+1) \end{pmatrix}^T \begin{pmatrix} -P^{-1} & 0 \\ 0 & P^{-1} \end{pmatrix} \begin{pmatrix} \boldsymbol{x}(k) \\ \boldsymbol{x}(k+1) \end{pmatrix} \\ + \tau \begin{pmatrix} \boldsymbol{x}(k) \\ \boldsymbol{x}(k+1) \end{pmatrix}^T \mathcal{R} \begin{pmatrix} \boldsymbol{x}(k) \\ \boldsymbol{x}(k+1) \end{pmatrix} \leq 0 \end{gathered}$$

Furthermore, the dynamics (11) can be written as

$$\begin{pmatrix} A_h - B_h\mathcal{F}\mathcal{H}^{-1} & -E_v \end{pmatrix} \begin{pmatrix} \boldsymbol{x}(k) \\ \boldsymbol{x}(k+1) \end{pmatrix} = 0$$

Using Lemma 1, we have $\Delta V < 0$ if there exist $\mathcal{M}$ so that

$$\mathcal{M} \begin{pmatrix} A_h - B_h\mathcal{F}\mathcal{H}^{-1} & -E_v \end{pmatrix} + (*) + \begin{pmatrix} -P^{-1} & 0 \\ 0 & P^{-1} \end{pmatrix} + \tau\mathcal{R} < 0$$

Choosing $\mathcal{M} = \begin{pmatrix} 0 \\ P^{-1} \end{pmatrix}$ leads to

$$\begin{pmatrix} -P^{-1} & (*) \\ P^{-1}A_h - P^{-1}B_h\mathcal{F}\mathcal{H}^{-1} & -P^{-1}E_v + (*) + P^{-1} \end{pmatrix} + \tau\mathcal{R} < 0$$

Congruence with $\begin{pmatrix} \mathcal{H}^T & 0 \\ 0 & P \end{pmatrix}$ and applying Proposition 2 gives directly conditions (15) and concludes the proof. Moreover, the region of attraction includes $\mathcal{D}_S$, where $\mathcal{D}_S$ is the largest Lyapunov level set included in $\mathcal{D}_R$. □

Before discussing the choice of the structure of $\mathcal{F}$ and $\mathcal{H}$, let us now consider the case when $\mathcal{R}$ is to be determined. Our aim is to effectively find the domain $\mathcal{D}_S$ where the system (13) is locally asymptotically stable. For this, similarly to the result in [32], we consider $\mathcal{R}$ as a decision variable, and, with a slight abuse of notation, $\tau\mathcal{R}$ will be denoted simply by $\mathcal{R}$. Thus, the following result can be formulated.

Theorem 2. *The discrete-time descriptor model* (13) *is locally asymptotically stable in the domain* $\mathcal{D}_S$ *if there exist matrices* $P = P^T > 0$, $\mathcal{F}$, $\mathcal{H}$, $i = 1, 2, \ldots, r$ *and* $W = W^T$ *so that*

$$\begin{pmatrix} -\mathcal{H} - \mathcal{H}^T + P & (*) \\ A_h\mathcal{H} - B_h\mathcal{F} & -E_vP - (*) + P \end{pmatrix} + W < 0 \tag{16}$$

where $\mathcal{D}_S$ *is the largest Lyapunov level set included in* $\mathcal{D}_R \bigcap \mathcal{D}$ *and* $\mathcal{D}_R$ *is given by*

$$\mathcal{D}_R = \left\{ \boldsymbol{x}(k) \in \mathcal{D} \,|\, \begin{pmatrix} \boldsymbol{x}(k) \\ \boldsymbol{x}(k+1) \end{pmatrix}^T \begin{pmatrix} \mathcal{H}^{-T} & 0 \\ 0 & P^{-1} \end{pmatrix} W \begin{pmatrix} \mathcal{H}^{-1} & 0 \\ 0 & P^{-1} \end{pmatrix} \begin{pmatrix} \boldsymbol{x}(k) \\ \boldsymbol{x}(k+1) \end{pmatrix} \geq 0 \right\}$$

Proof. The proof follows the same lines as that of Theorem 1 and denoting

$$\mathcal{R} = \begin{pmatrix} \mathcal{H}^{-T} & 0 \\ 0 & P^{-1} \end{pmatrix} W \begin{pmatrix} \mathcal{H}^{-1} & 0 \\ 0 & P^{-1} \end{pmatrix}$$

□

Remark: Classical results in the literature are usually based on the choice $\mathcal{M} = \begin{pmatrix} 0 \\ M \end{pmatrix}$, in Lemma 1. In this particular case, we chose $M = P^{-1}$.

As can be seen, the domain $\mathcal{D}_R$ and consequently the domain where local asymptotic stability of the closed-loop system will be established will depend on the controller gains and on the Lyapunov function. Moreover, since in the conditions (16), the matrix W "compensates" for the difference in the Lyapunov functions, several possibilities can be chosen, such as $W = \begin{pmatrix} W_1 & 0 \\ 0 & -I \end{pmatrix}$, which establishes a direct relation between $\boldsymbol{x}(k)$ and $\boldsymbol{x}(k+1)$; a full W, which will give a more complex relation between two consecutive samples and thus in principle a larger region, etc. In fact, as W is used to determine a region where the difference in the Lyapunov function is negative, it should have a structure to match this difference, and ultimately to match the nonlinearities in the system model.

Furthermore, since any trajectory that eventually gets in the domain $\mathcal{D}_S$ is a stable one, the region of attraction may be increased by looking at those trajectories that do not start in $\mathcal{D}_R$, but arrive in several steps to $\mathcal{D}_S$.

Let us now discuss the choice of the controller gains $\mathcal{F}$ and $\mathcal{H}$. Naturally, the simplest choice is $\mathcal{F} = F$, $\mathcal{H} = H$, F and H being matrices of appropriate dimensions. For such a choice, however, the controller is reduced to linear one, although (16) contains two sums (one in h and one in v, thus relaxations cannot be applied).

In order to be able to use relaxations such as Lemma 2, $\mathcal{F}$ and $\mathcal{H}$ should contain at least one sum in h, i.e., $\mathcal{F} = F_h$ and $\mathcal{H} = H_h$. This means that (16) contains two sums in h (for which the relaxations can be applied) and one in v. To increase the number of decision variables, and thus relax the conditions, a sum in v can still be included in $\mathcal{F}$ and $\mathcal{H}$, leading to $\mathcal{F} = F_{hv}$ and $\mathcal{H} = H_{hv}$, respectively. Note that this choice does not lead to an increase in the number of sums.

Finally, continuing the line of thought above, for the general case, one may choose $\mathcal{F} = F_{hh...hv...v}$ and $\mathcal{H} = H_{hh...hv...v}$. By increasing the number of sums – and, equivalently, the number of decision variables –, the controller becomes more general and the LMIs less conservative. However, at the same time, the computational complexity increases, eventually rendering the conditions computationally intractable in practice.

It should be noted that depending on the control problem considered other choices (e.g., using a constant H and $\mathcal{F} = F_{hv}$) may also be suitable. Moreover, choosing a constant $H = P$ recovers classic controllers from the literature.

3.2. Local non-quadratic stabilization

In what follows, we will consider local nonquadratic stabilization of the descriptor model (13). For this we will use the nonquadratic Lyapunov function $V = \boldsymbol{x}^T(k)\mathcal{P}^{-1}\boldsymbol{x}(k)$, together with the constraint $\begin{pmatrix}\boldsymbol{x}(k)\\ \boldsymbol{x}(k+1)\end{pmatrix}^T \mathcal{R} \begin{pmatrix}\boldsymbol{x}(k)\\ \boldsymbol{x}(k+1)\end{pmatrix}$ between consecutive samples. Then, following the lines of Theorem 2, we have the result:

Theorem 3. *The discrete-time nonlinear model* (13) *is locally asymptotically stable in the domain $\mathcal{D}_S$, if there exist $\mathcal{P}$, $\mathcal{F}$, $\mathcal{H}$, and $\mathcal{W}$, so that*

$$\begin{pmatrix} -\mathcal{H}^T - \mathcal{H} + \mathcal{P} & (*) \\ A_h\mathcal{H} - B_h\mathcal{F} & -E_v\mathcal{P}_+ + (*) + \mathcal{P}_+ \end{pmatrix} + \mathcal{W} < 0 \tag{17}$$

where $\mathcal{D}_R$ is given by

$$\mathcal{D}_R = \left\{ \boldsymbol{x}(k) \in \mathcal{D} \,|\, \begin{pmatrix}\boldsymbol{x}(k)\\ \boldsymbol{x}(k+1)\end{pmatrix}^T \begin{pmatrix}\mathcal{H}^{-T} & 0\\ 0 & \mathcal{P}_+^{-1}\end{pmatrix} \mathcal{W} \begin{pmatrix}\mathcal{H}^{-1} & 0\\ 0 & \mathcal{P}_+^{-1}\end{pmatrix} \begin{pmatrix}\boldsymbol{x}(k)\\ \boldsymbol{x}(k+1)\end{pmatrix} \geq 0 \right\}$$

and $\mathcal{D}_S$ is the largest Lyapunov level set included in $\mathcal{D}_R \bigcap \mathcal{D}$.

Proof. The proof follows the same lines as that of Theorem 2 and is therefore omitted. □

Once the sums (membership functions) to be used in $\mathcal{F}$, $\mathcal{H}$, $\mathcal{P}$, etc., are chosen, sufficient LMI conditions can easily be derived for the above conditions. However, in order to efficiently apply relaxations such as Lemma 2 to reduce the computational complexity, the sums used in the Lyapunov function and in the controller gains should be suitably chosen.

A similar line of choices of the sums used in the control gains and in the Lyapunov function as in the case of quadratic stabilization can be discussed also here. Since, due to the nonquadratic Lyapunov function, $\mathcal{P}_+$ appears in the conditions, the choice of delayed Lyapunov matrices, such as those in [40] may be warranted. When using fuzzy expression in both the gains and the Lyapunov function, the least number of sums that allows for the use of relaxations both in h and v is 5: $\mathcal{H} = H_h$, $\mathcal{F} = F_h$, $\mathcal{P} = P_{v-}$, where $v-$ denotes the membership functions v evaluated at the previous sampling instant. This choice leads to

$$\begin{pmatrix} -H_h - H_h^T + P_{v-} & (*) \\ A_h H_h - B_h F_h & -E_v P_v + (*) + P_v \end{pmatrix} + \mathcal{W} < 0 \tag{18}$$

The domain $\mathcal{D}_R$ is obtained based on $\mathcal{W}$, H_h and P_v. While even a constant matrix W may lead to a nonconvex set, in order to maximize the degrees of freedom in $\mathcal{W}$, one can choose $\mathcal{W} = W_{hhvvv-}$, i.e., include all the indices that appear on the left-hand side.

Then, the following sufficient LMI conditions may be formulated:

Corollary 1. The discrete-time descriptor model (13) is locally asymptotically stable if there exist matrices $P_k = P_k^T > 0$, H_i, F_i, $W_{ijklm} = W_{ijklm}^T$, $i, j = 1, 2, \ldots, r_e$, $k, l, m = 1, 2, \ldots, r$, so that (10) hold, with

$$\Gamma_{i,j,k,l,m} = \begin{pmatrix} -H_i - H_i^T + P_m & (*) \\ A_j H_i - B_j F_i & -E_k P_l + (*) + P_l \end{pmatrix} + W_{ijklm}$$

Moreover, the region of attraction includes $\mathcal{D}_S$, where $\mathcal{D}_S$ is the largest Lyapunov level set included in $\mathcal{D}_R$, where $\mathcal{D}_R$ is defined as

$$\mathcal{D}_R = \left\{ \boldsymbol{x}(k) \in \mathcal{D} \mid \begin{pmatrix} \boldsymbol{x}(k) \\ \boldsymbol{x}(k+1) \end{pmatrix}^T \begin{pmatrix} H_h^{-T} & 0 \\ 0 & P_v^{-1} \end{pmatrix} \mathcal{W} \begin{pmatrix} H_h^{-1} & 0 \\ 0 & P_v^{-1} \end{pmatrix} \begin{pmatrix} \boldsymbol{x}(k) \\ \boldsymbol{x}(k+1) \end{pmatrix} \geq 0 \right\}$$

4. Local set point tracking control

Consider now the discrete-time TS descriptor system (11)

$$\begin{aligned} E_v \boldsymbol{x}(k+1) &= A_h \boldsymbol{x}(k) + B_h \boldsymbol{u}(k) \\ \boldsymbol{y}(k) &= C\boldsymbol{x}(k) \end{aligned} \tag{19}$$

defined on a domain $\mathcal{D}$ including the origin, where $\boldsymbol{y}$ denotes the output.

Our goal is to determine a control law, such that $\boldsymbol{y}$ tracks a desired reference signal $\boldsymbol{y}_r$. For this, we will use the auxiliary state variable $\boldsymbol{x}^I(k)$ – corresponding to an integral term – with the dynamics given by $\boldsymbol{x}^I(k+1) = \boldsymbol{x}^I(k) - C\boldsymbol{x}(k) + \boldsymbol{y}_r$. The extended state dynamics are

$$\begin{aligned} E_v \boldsymbol{x}(k+1) &= A_h \boldsymbol{x}(k) + B_h \boldsymbol{u}(k) \\ \boldsymbol{x}^I(k+1) &= \boldsymbol{x}^I(k) - C\boldsymbol{x}(k) + \boldsymbol{y}_r \end{aligned}$$

Denoting $\boldsymbol{x}^e = \begin{pmatrix} \boldsymbol{x}(k) \\ \boldsymbol{x}^I(k) \end{pmatrix}$, $A^e = \begin{pmatrix} A_h & 0 \\ -C & I \end{pmatrix}$, $B_h^e = \begin{pmatrix} B_h \\ 0 \end{pmatrix}$, $E_v^e = \begin{pmatrix} E_v & 0 \\ 0 & I \end{pmatrix}$, $D = \begin{pmatrix} 0 \\ I \end{pmatrix}$ we have

$$E_v^e \boldsymbol{x}^e(k+1) = A_h^e \boldsymbol{x}^e(k) + B_h^e \boldsymbol{u}(k) + D\boldsymbol{y}_r$$

and we consider a control law $\boldsymbol{u}(k) = -\mathcal{F}\mathcal{H}^{-1}\boldsymbol{x}^e(k)$. The closed-loop system is

$$E_v^e \boldsymbol{x}^e(k+1) = (A_h^e - B_h^e \mathcal{F}\mathcal{H}^{-1})\boldsymbol{x}^e(k) + D\boldsymbol{y}_r \tag{20}$$

It is well-known [41] that if the closed-loop system (20) with $\boldsymbol{y}_r = 0$ is globally uniformly asymptotically stable, then it is also input-to-state stable (ISS) with respect to the exogenous input $\boldsymbol{y}_r$. In what follows, we determine conditions for the local tracking and determine a region where tracking is possible.

Similarly to the stabilization problem, consider the nonquadratic Lyapunov function $V = \boldsymbol{x}^{eT}(k)\mathcal{P}^{-1}\boldsymbol{x}^e(k)$, together with the constraint $\begin{pmatrix} \boldsymbol{x}^e(k) \\ \boldsymbol{x}^e(k+1) \end{pmatrix}^T \mathcal{R} \begin{pmatrix} \boldsymbol{x}^e(k) \\ \boldsymbol{x}^e(k+1) \end{pmatrix}$ between consecutive samples. Then, following the lines of Theorem 2, we have the result:

Theorem 4. *The discrete-time nonlinear model* (20) *is locally ISS with respect to the exogenous input $\boldsymbol{y}_r$ in the domain $\mathcal{D}_S$, if there exist $\mathcal{P}$, $\mathcal{F}$, $\mathcal{H}$, and $\mathcal{W}$, so that*

$$\begin{pmatrix} -\mathcal{H}^T - \mathcal{H} + \mathcal{P} & (*) \\ A_h^e\mathcal{H} - B_h^e\mathcal{F} & -E_v^e\mathcal{P}_+ + (*) + \mathcal{P}_+ \end{pmatrix} + \mathcal{W} < 0 \tag{21}$$

where $\mathcal{D}_R$ is given by

$$\mathcal{D}_R = \left\{ \boldsymbol{x}^e(k) \in \mathcal{D} \,|\, \begin{pmatrix} \boldsymbol{x}^e(k) \\ \boldsymbol{x}^e(k+1) \end{pmatrix}^T \begin{pmatrix} \mathcal{H}^{-T} & 0 \\ 0 & \mathcal{P}_+^{-1} \end{pmatrix} \mathcal{W} \begin{pmatrix} \mathcal{H}^{-1} & 0 \\ 0 & \mathcal{P}_+^{-1} \end{pmatrix} \begin{pmatrix} \boldsymbol{x}^e(k) \\ \boldsymbol{x}^e(k+1) \end{pmatrix} \geq 0 \right\}$$

and $\mathcal{D}_S$ is the largest Lyapunov level set included in $\mathcal{D}_R \bigcap \mathcal{D}$.

Proof. Consider the candidate Lyapunov function $V = (\boldsymbol{x}^e)^T(k)\mathcal{P}^{-1}\boldsymbol{x}^e(k)$. The difference in V is

$$\begin{aligned} \Delta V &= (\boldsymbol{x}^e)^T(k+1)\mathcal{P}_+^{-1}\boldsymbol{x}^e(k+1) - (\boldsymbol{x}^e)^T(k)\mathcal{P}^{-1}\boldsymbol{x}^e(k) \\ &= \begin{pmatrix} \boldsymbol{x}^e(k) \\ \boldsymbol{x}^e(k+1) \end{pmatrix}^T \begin{pmatrix} -\mathcal{P}^{-1} & 0 \\ 0 & \mathcal{P}_+^{-1} \end{pmatrix} \begin{pmatrix} \boldsymbol{x}^e(k) \\ \boldsymbol{x}^e(k+1) \end{pmatrix} \end{aligned}$$

The dynamics (20) can be written as

$$\begin{pmatrix} A_h^e - B_h^e\mathcal{F}\mathcal{H}^{-1} & -E_v^e & D \end{pmatrix} \begin{pmatrix} \boldsymbol{x}^e(k) \\ \boldsymbol{x}^e(k+1) \\ \boldsymbol{y}_r \end{pmatrix} = 0$$

Then, we have

$$
\begin{aligned}
\Delta V &= \begin{pmatrix} \boldsymbol{x}^e(k) \\ \boldsymbol{x}^e(k+1) \end{pmatrix}^T \begin{pmatrix} -\mathcal{P}^{-1} & 0 \\ 0 & \mathcal{P}_+^{-1} \end{pmatrix} \begin{pmatrix} \boldsymbol{x}^e(k) \\ \boldsymbol{x}^e(k+1) \end{pmatrix} \\
&+ \begin{pmatrix} \boldsymbol{x}^e(k) \\ \boldsymbol{x}^e(k+1) \\ \boldsymbol{y}_r \end{pmatrix}^T \left(\begin{pmatrix} 0 \\ \mathcal{P}_+^{-1} \\ 0 \end{pmatrix} \begin{pmatrix} A_h^e - B_h^e \mathcal{F}\mathcal{H}^{-1} & -E_v^e & D \end{pmatrix} + (*) \right) \begin{pmatrix} \boldsymbol{x}^e(k) \\ \boldsymbol{x}^e(k+1) \\ \boldsymbol{y}_r \end{pmatrix} \\
&= \begin{pmatrix} \boldsymbol{x}^e(k) \\ \boldsymbol{x}^e(k+1) \end{pmatrix}^T \begin{pmatrix} -\mathcal{P}^{-1} & (*) \\ \mathcal{P}_+^{-1}(A_h^e - B_h^e \mathcal{F}\mathcal{H}^{-1}) & -\mathcal{P}_+^{-1}E_v^e + (*) + \mathcal{P}_+^{-1} \end{pmatrix} \begin{pmatrix} \boldsymbol{x}^e(k) \\ \boldsymbol{x}^e(k+1) \end{pmatrix} \\
&+ 2\boldsymbol{y}_r \mathcal{P}_+^{-1} \left((A_h^e - B_h^e \mathcal{F}\mathcal{H}^{-1}) \boldsymbol{x}^e(k) + D\boldsymbol{y}_r \right) \\
&\leq \mathcal{Q}(\boldsymbol{x}^e) + 2\delta \|\boldsymbol{y}_r\| \|\boldsymbol{x}^e(k)\| + 2\alpha \|\boldsymbol{y}_r\|^2
\end{aligned}
$$

where

$$
\mathcal{Q}(\boldsymbol{x}^e) = \begin{pmatrix} \boldsymbol{x}^e(k) \\ \boldsymbol{x}^e(k+1) \end{pmatrix}^T \begin{pmatrix} -\mathcal{P}^{-1} & (*) \\ \mathcal{P}_+^{-1}(A_h^e - B_h^e \mathcal{F}\mathcal{H}^{-1}) & -\mathcal{P}_+^{-1}E_v^e + (*) + \mathcal{P}_+^{-1} \end{pmatrix} \begin{pmatrix} \boldsymbol{x}^e(k) \\ \boldsymbol{x}^e(k+1) \end{pmatrix}
$$

and $\delta \geq 0$ and $\alpha \geq 0$ are bounding constants, i.e., $\|\mathcal{P}_+^{-1}(A_h - B_h \mathcal{F}\mathcal{H}^{-1})\| \leq \delta$ and $\|\mathcal{P}_+^{-1} D\| \leq \alpha$.

Let us now consider $\mathcal{Q}(\boldsymbol{x}^e)$. Assuming that there exists $\mathcal{R}$ and a domain $\mathcal{D}_R$ such that

$$
\begin{pmatrix} \boldsymbol{x}^e(k) \\ \boldsymbol{x}^e(k+1) \end{pmatrix}^T \mathcal{R} \begin{pmatrix} \boldsymbol{x}^e(k) \\ \boldsymbol{x}^e(k+1) \end{pmatrix} \geq 0
$$

$\mathcal{Q}(\boldsymbol{x}^e) < 0$, if

$$
\begin{pmatrix} -\mathcal{P}^{-1} & (*) \\ \mathcal{P}_+^{-1}(A_h^e - B_h^e \mathcal{F}\mathcal{H}^{-1}) & -\mathcal{P}_+^{-1}E_v^e + (*) + \mathcal{P}_+^{-1} \end{pmatrix} + \mathcal{R} < 0
$$

Congruence with $\begin{pmatrix} \mathcal{H}^T & 0 \\ 0 & \mathcal{P}_+ \end{pmatrix}$ and applying Proposition 2 gives

$$
\begin{pmatrix} -\mathcal{H}^T - \mathcal{H} + \mathcal{P} & (*) \\ A_h^e \mathcal{H} - B_h^e \mathcal{F} & -E_v^e \mathcal{P}_+ + (*) + \mathcal{P}_+ \end{pmatrix} + \begin{pmatrix} \mathcal{H}^T & 0 \\ 0 & \mathcal{P}_+ \end{pmatrix} \mathcal{R} \begin{pmatrix} \mathcal{H} & 0 \\ 0 & \mathcal{P}_+ \end{pmatrix} < 0 \tag{22}
$$

or

$$
\begin{pmatrix} -\mathcal{H}^T - \mathcal{H} + \mathcal{P} & (*) \\ A_h^e \mathcal{H} - B_h^e \mathcal{F} & -E_v^e \mathcal{P}_+ + (*) + \mathcal{P}_+ \end{pmatrix} + \mathcal{W} < 0
$$

Furthermore in $\mathcal{D}_R$ where (22) holds, $\exists \lambda > 0$ so that $\mathcal{Q}(\boldsymbol{x}^e) < -\lambda \|\boldsymbol{x}^e(k)\|^2$. Consequently, in this domain,

$$
\Delta V \leq -\lambda \|\boldsymbol{x}^e(k)\|^2 + 2\delta \|\boldsymbol{y}_r\| \|\boldsymbol{x}^e(k)\| + 2\alpha \|\boldsymbol{y}_r\|^2
$$

and by the completion of squares, $2\delta \|\boldsymbol{y}_r\| \|\boldsymbol{x}^e(k)\| \leq \frac{1}{\theta} \|\boldsymbol{x}^e(k)\|^2 + \delta^2 \theta \|\boldsymbol{y}_r\|^2$, $\forall \theta > 0$, thus

$$
\Delta V \leq -\lambda \|\boldsymbol{x}^e(k)\|^2 + \frac{1}{\theta} \|\boldsymbol{x}^e(k)\|^2 + \delta^2 \theta \|\boldsymbol{y}_r\|^2 + 2\alpha \|\boldsymbol{y}_r\|^2
$$

Choosing $\theta > \frac{1}{\lambda}$ and denoting $c_1 = \lambda - \frac{1}{\theta} > 0$ and $c_2 = \delta^2\theta + 2\alpha$, we have

$$\Delta V \leq -c_1\|\boldsymbol{x}^e(k)\|^2 + c_2\|\boldsymbol{y}_r\|^2$$

Furthermore, consider $\tau \in (0,\, 1)$. Then,

$$\begin{aligned}\Delta V &\leq -(1-\tau)c_1\|\boldsymbol{x}^e(k)\|^2 - \tau c_1\|\boldsymbol{x}^e(k)\|^2 + c_2\|\boldsymbol{y}_r\|^2 \\ &\leq -(1-\tau)c_1\|\boldsymbol{x}^e(k)\|^2 \qquad \forall \|\boldsymbol{x}^e\|^2 \geq \frac{c_2}{\tau c_1}\|\boldsymbol{y}_r\|^2\end{aligned}$$

i.e., the closed-loop system (20) is ISS with respect to the exogenous input $\boldsymbol{y}_r$, with an ultimate bound given by $\frac{c_2}{\tau c_1}$. $\square$

While the bounding constants α and δ do not affect the feasibility of the developed LMI conditions, they do affect the bound above and how closely the systems output $\boldsymbol{y}$ will track the reference signal $\boldsymbol{y}_r$. Thus, in what follows, we develop LMI conditions for the minimization of this bound. Recall that $\|\mathcal{P}_+^{-1}(A_h^e - B_h^e\mathcal{F}\mathcal{H}^{-1})\| \leq \delta$ and $\|\mathcal{P}_+^{-1}D\| \leq \alpha$ and consider first $\|\mathcal{P}_+^{-1}D\| \leq \alpha$. This is satisfied if

$$\begin{aligned} D^T\mathcal{P}_+^{-1}\mathcal{P}_+^{-1}D &\leq \alpha^2 I \\ \alpha^2 I - D^T\mathcal{P}_+^{-1}\mathcal{P}_+^{-1}D &\geq 0\end{aligned}$$

By the Schur complement, we have

$$\begin{pmatrix} \alpha^2 I & D^T\mathcal{P}_+^{-1} \\ \mathcal{P}_+^{-1}D & I \end{pmatrix} \geq 0$$

Congruence with $\mathrm{diag}(I\,\mathcal{P}_+)$ gives

$$\begin{pmatrix} \alpha^2 I & D^T \\ D & \mathcal{P}_+\mathcal{P}_+ \end{pmatrix} \geq 0$$

and using Proposition 2 on $\mathcal{P}_+\mathcal{P}_+$ we obtain

$$\begin{pmatrix} \alpha^2 I & D^T \\ D & 2\mathcal{P}_+ - I \end{pmatrix} \geq 0 \tag{23}$$

Consider now $\|\mathcal{P}_+^{-1}(A_h^e - B_h^e\mathcal{F}\mathcal{H}^{-1})\| \leq \delta$. This is satisfied, if

$$(\mathcal{P}_+^{-1}(A_h^e - B_h^e\mathcal{F}\mathcal{H}^{-1}))^T\mathcal{P}_+^{-1}(A_h^e - B_h^e\mathcal{F}\mathcal{H}^{-1}) \leq \delta^2 I$$

Using the Schur complement and congruence with $\mathrm{diag}(\mathcal{H}^T\,\mathcal{P}_+)$ gives

$$\begin{pmatrix} \mathcal{H}^T\delta^2\mathcal{H} & (*) \\ A_h^e\mathcal{H} - B_h^e\mathcal{F} & \mathcal{P}_+\mathcal{P}_+ \end{pmatrix} \geq 0$$

and using Proposition 2 on both $\mathcal{H}^T\delta^2\mathcal{H}$ and $\mathcal{P}_+\mathcal{P}_+$ leads to

$$\begin{pmatrix} \mathcal{H}^T + \mathcal{H} - \frac{1}{\delta^2}I & (*) \\ A_h^e\mathcal{H} - B_h^e\mathcal{F} & 2\mathcal{P}_+ - I \end{pmatrix} \geq 0 \tag{24}$$

Furthermore, since the bound $\frac{c_2}{\tau c_1}$ depends on c_1, which in turn depends on λ, λ should be maximized. Then, $\mathcal{Q}(\boldsymbol{x}^e) \leq -\lambda\|\boldsymbol{x}^e(k)\|$, if

$$\begin{pmatrix} -\mathcal{P}^{-1}+\lambda & (*) \\ \mathcal{P}_+^{-1}(A_h^e - B_h^e\mathcal{F}\mathcal{H}^{-1}) & -\mathcal{P}_+^{-1}E_v^e + (*) + \mathcal{P}_+^{-1} \end{pmatrix} + \mathcal{R} < 0$$

Similarly to the proof of Theorem 4, congruence with $\begin{pmatrix} \mathcal{H}^T & 0 \\ 0 & \mathcal{P}_+ \end{pmatrix}$ and applying Proposition 2 gives

$$\begin{pmatrix} -\mathcal{H}^T - \mathcal{H} + \lambda\mathcal{H}^T\mathcal{H} + \mathcal{P} & (*) \\ A_h^e\mathcal{H} - B_h^e\mathcal{F} & -E_v^e\mathcal{P}_+ + (*) + \mathcal{P}_+ \end{pmatrix} + \mathcal{W} < 0$$

which, after applying the Schur complement results in

$$\begin{pmatrix} -\mathcal{H}^T - \mathcal{H} + \mathcal{P} & (*) & (*) \\ A_h^e\mathcal{H} - B_h^e\mathcal{F} & -E_v^e\mathcal{P}_+ + (*) + \mathcal{P}_+ & 0 \\ \mathcal{H}^T & 0 & -\frac{1}{\lambda}I \end{pmatrix} + \begin{pmatrix} \mathcal{W} & 0 \\ 0 & 0 \end{pmatrix} < 0 \tag{25}$$

This result can be summarized as follows.

Corollary 2. The discrete-time nonlinear model (20) is locally ISS with respect to the exogenous input $\boldsymbol{y}_r$ in the domain $\mathcal{D}_S$, if there exist $\mathcal{P}$, $\mathcal{F}$, $\mathcal{H}$, and $\mathcal{W}$, so that condition (21) holds. Furthermore, the ultimate bound on the states can be minimized by solving

minimize α, δ, maximize λ

subject to (23), (24), (25)

together with (21).

5. Examples

In this section we illustrate the proposed conditions first on a numerical example and then on a 2DOF robot arm.

5.1. Numerical example

Consider the discrete-time descriptor TS model:

$$\sum_{i=1}^{2} v_i(\boldsymbol{z}(k))E_i\boldsymbol{x}(k+1) = \sum_{i=1}^{4} h_i(\boldsymbol{z}(k))(A_i\boldsymbol{x}(k) + B\boldsymbol{u}(k)) \tag{26}$$

with $(x_1(k), x_2(k)) \in \mathbb{R}^2$, $\boldsymbol{z} = \boldsymbol{x}$,

$$\begin{aligned} v_1 &= \frac{1-\sin(x_1)}{2} & v_2 &= 1 - v_1 \\ h_1 &= \frac{1-\sin(x_1)}{2}\frac{1-\sin(x_2)}{2} & h_2 &= \frac{1-\sin(x_1)}{2}\frac{1+\sin(x_2)}{2} \\ h_3 &= \frac{1+\sin(x_1)}{2}\frac{1-\sin(x_2)}{2} & h_4 &= \frac{1+\sin(x_1)}{2}\frac{1+\sin(x_2)}{2} \end{aligned}$$

$$E_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad E_2 = \begin{pmatrix} 1 & 1 \\ 1 & 3 \end{pmatrix} \qquad B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$A_1 = \begin{pmatrix} -0.55 & -0.25 \\ -0.85 & 0.66 \end{pmatrix} \qquad A_2 = \begin{pmatrix} -0.85 & -0.12 \\ -1.20 & -0.06 \end{pmatrix}$$

$$A_3 = \begin{pmatrix} 0.48 & -0.15 \\ -0.6 & -0.43 \end{pmatrix} \qquad A_4 = \begin{pmatrix} -0.08 & -0.60 \\ 1.53 & -1.35 \end{pmatrix}$$

Our goal is to determine a controller that is able to track the reference signals presented in Figures 1(a) and 1(b). Using classical conditions for the global stabilization of this system, the LMIs are unfeasible. Thus, we consider a local approach. For the easier graphical representation, we consider a common quadratic Lyapunov function $V(\boldsymbol{x}^e) = \boldsymbol{x}^{eT}(k)P\boldsymbol{x}^e(k)$, $\boldsymbol{x}^e = \begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{x}^I \end{pmatrix}$ with $P = P^T > 0$ and the following choices: $\mathcal{H} = P$, $\mathcal{F} = F_{hv}$, $W = \begin{pmatrix} W_1 & 0 \\ 0 & -I \end{pmatrix}$. Note that with the choice of $\mathcal{H} = P$ the controller is reduced to a classic PDC controller. The following results have been obtained:

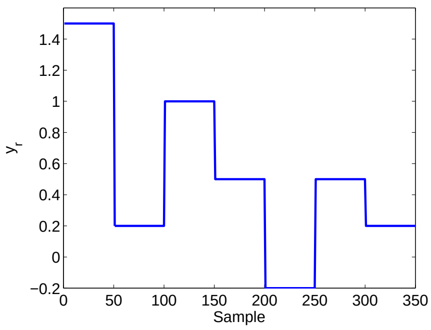
$$P = \begin{pmatrix} 1.36 & -0.21 & 1.55 \\ -0.21 & 2.34 & -1.63 \\ 1.55 & -1.63 & 5.95 \end{pmatrix} \qquad W_1 = \begin{pmatrix} 0.28 & 0 & 0 \\ 0 & 0.28 & 0 \\ 0 & 0 & 0.28 \end{pmatrix}$$

$$F_{1,1}H^{-1} = \begin{pmatrix} -1.06 & 0.68 & 0.25 \end{pmatrix} \qquad F_{1,2}H^{-1} = \begin{pmatrix} -0.48 & 1.03 & 0.46 \end{pmatrix}$$
$$F_{2,1}H^{-1} = \begin{pmatrix} -1.39 & -0.05 & 0.25 \end{pmatrix} \qquad F_{2,2}H^{-1} = \begin{pmatrix} -0.34 & 0.11 & 0.45 \end{pmatrix}$$
$$F_{3,1}H^{-1} = \begin{pmatrix} -0.89 & -0.42 & 0.25 \end{pmatrix} \qquad F_{3,2}H^{-1} = \begin{pmatrix} -1.79 & -0.21 & 0.45 \end{pmatrix}$$
$$F_{4,1}H^{-1} = \begin{pmatrix} 1.28 & -1.30 & 0.25 \end{pmatrix} \qquad F_{4,2}H^{-1} = \begin{pmatrix} 1.20 & -0.39 & 0.46 \end{pmatrix}$$

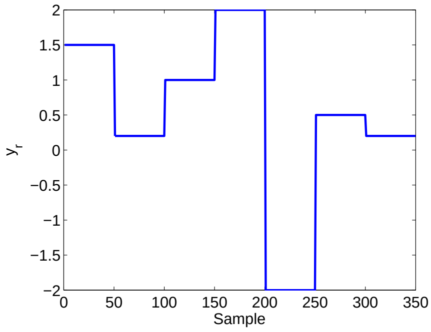
As can be seen in the tracking results illustrated in Figures 1(c) and 1(d), the tracking is indeed only possible locally: for instance, the reference $y_r = -2$ (see Figure 1(b)) cannot be tracked – at least with this choice of the control law and the Lyapunov functions –, although for $y_r = -0.2$, x_1 converges to the reference.

The domain in this case is determined by

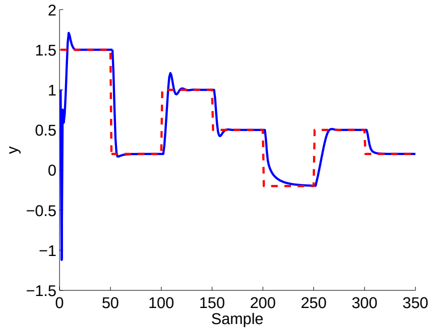
$$\begin{aligned}
\mathcal{D}_R &= \left\{ \boldsymbol{x}^e(k) \in \mathcal{D} \,|\, \begin{pmatrix} \boldsymbol{x}^e(k) \\ \boldsymbol{x}^e(k+1) \end{pmatrix}^T \begin{pmatrix} P^{-1} & 0 \\ 0 & P^{-1} \end{pmatrix} \begin{pmatrix} W_1 & 0 \\ 0 & -I \end{pmatrix} \begin{pmatrix} P^{-1} & 0 \\ 0 & P^{-1} \end{pmatrix} \begin{pmatrix} \boldsymbol{x}^e(k) \\ \boldsymbol{x}^e(k+1) \end{pmatrix} \geq 0 \right\} \\
&= \left\{ \boldsymbol{x}^e(k) \in \mathcal{D} \,|\, \boldsymbol{x}^e(k)^T \begin{pmatrix} 0.35 & -0.07 & -0.12 \\ -0.07 & 0.1 & 0.05 \\ -0.12 & 0.05 & 0.06 \end{pmatrix} \boldsymbol{x}^e(k) + \right. \\
&\quad \left. + \, \boldsymbol{x}^e(k+1)^T \begin{pmatrix} -1.26 & 0.25 & 0.45 \\ 0.25 & -0.34 & -0.19 \\ 0.45 & -0.19 & -0.22 \end{pmatrix} \boldsymbol{x}^e(k+1) \geq 0 \right\} \\
&= \left\{ \boldsymbol{x}^e \in \mathcal{D} \,|\, \boldsymbol{x}^{eT} \begin{pmatrix} 0.35 & -0.07 & -0.12 \\ -0.07 & 0.1 & 0.05 \\ -0.12 & 0.05 & 0.06 \end{pmatrix} \boldsymbol{x}^e + \right. \\
&\quad \left. + \, \boldsymbol{x}^{eT}(A_h^e - B_h^e \mathcal{F}\mathcal{H}^{-1})^T (E_v^{eT})^{-1} \begin{pmatrix} -1.26 & 0.25 & 0.45 \\ 0.25 & -0.34 & -0.19 \\ 0.45 & -0.19 & -0.22 \end{pmatrix} (*) \boldsymbol{x}^e \geq 0 \right\}
\end{aligned}$$



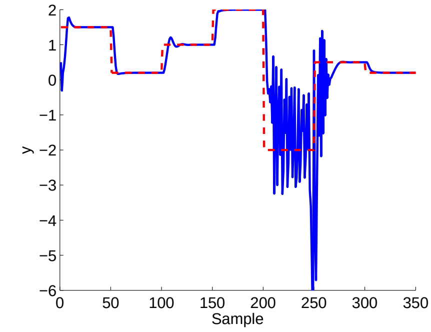
(a) Reference signal for Example 5.1



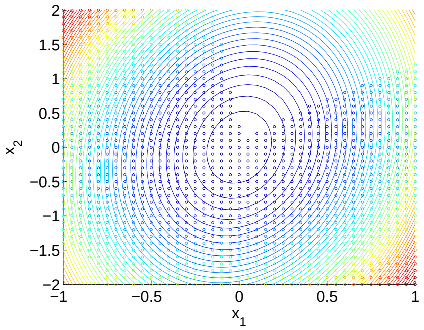
(b) Reference signal for Example 5.1



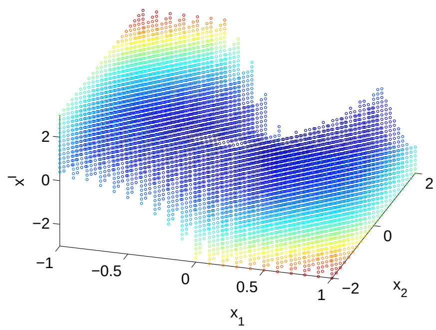
(c) Results for the reference in Figure 1(a)



(d) Results for the reference in Figure 1(b)



(e) Lyapunov level sets corresponding to $x^I = 0$ and points that are in $\mathcal{D}_R$.



(f) The domain $\mathcal{D}_R$

Figure 1: Results for Example 5.1.

Since the Lyapunov function depends on x_1, x_2 and x^I, its projection on the $x_1 - x_2$ plane, i.e., the level sets for $x^I = 0$, and those points (denoted by 'o') which are in $\mathcal{D}_R$ are presented in Figure 1(e). The points that are in $\mathcal{D}_R$ for $x_1 \in [-1, 1]$, x_2, $x^I \in [-2, 2]$ are graphically represented in Figure 1(f).

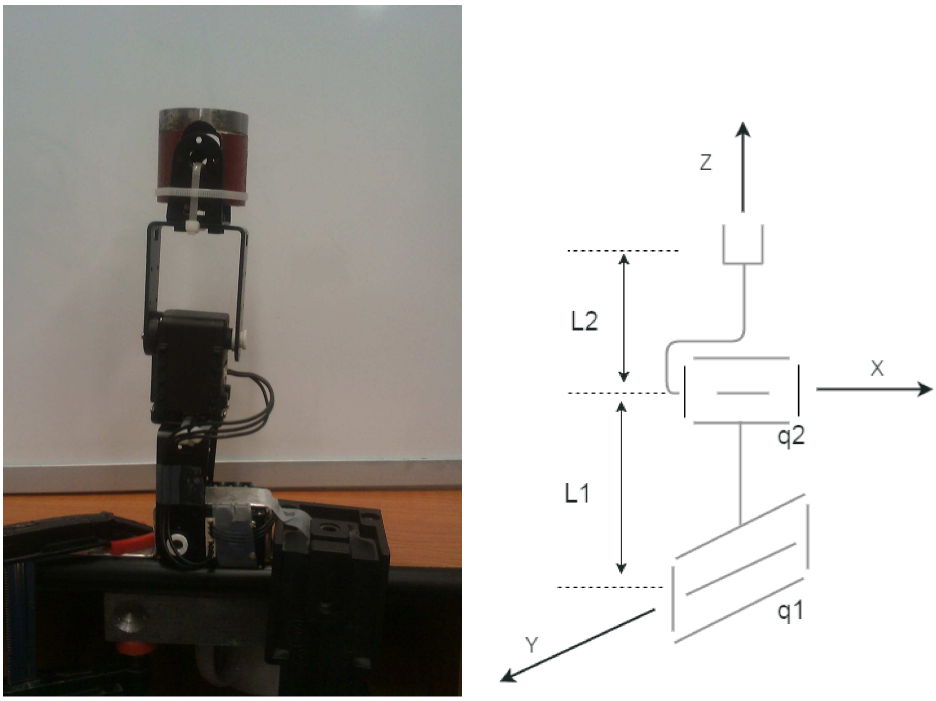
Two issues need to be noted here:

- Although not the case for asymptotic stabilization, in case of ISS, the obtained domain also depends on the auxiliary integral variable $\boldsymbol{x}^I$. Since this variable directly depends on the exogenous input $\boldsymbol{y}_r$, it naturally follows that stability will only be ensured for some reference signals.
- The domain of attraction is actually determined by an implicit equation of the state (stabilization) and of $\boldsymbol{y}_r$ (in case of ISS). Depending on the nonlinearities in the membership functions, the choice of the sums in the controller gains, and the (possibly delayed) Lyapunov matrix, analytically solving this equation may not be possible.

Remark: For comparison purposes we also computed a linear state-feedback control law $\boldsymbol{u} = -F\boldsymbol{x}$ for the linearized model, and obtained the controller gain $F = \begin{pmatrix}-0.29 & -0.35 & -0.05\end{pmatrix}$. However, this control cannot even stabilize the original nonlinear system from the initial point $\boldsymbol{x} = \begin{pmatrix}0.1 & 0.1\end{pmatrix}^T$, not to mention tracking a reference. This clearly illustrates the advantages of designing a local nonlinear controller.

5.2. Tracking for a robot arm

In what follows, we consider a tracking problem for the 2DOF robot arm shown in Figure 2(a). The schematic representation of this arm is shown in Figure 2(b).



(a) A 2DOF robot arm

(b) Schematic representation of a 2DOF robot arm

Figure 2: A 2DOF robot arm

Using the Euler-Lagrange modeling approach, the dynamic model of this arm can be obtained as

$$M(\boldsymbol{q})\ddot{\boldsymbol{q}} = -D(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + I\tau \tag{27}$$

where $\boldsymbol{q} = \begin{pmatrix} q_1 & q_2 \end{pmatrix}^T$ are the angles of the two joints, $\dot{\boldsymbol{q}} = \begin{pmatrix} \dot{q}_1 & \dot{q}_2 \end{pmatrix}^T$ are the angular velocities, $\tau = \begin{pmatrix} \tau_1 & \tau_2 \end{pmatrix}^T$ are the torques. M represents the mass matrix and D contains the Coriolis, centrifugal and friction forces. The parameters of the system have been experimentally identified and are presented in Table 1.

Table 1: Margin settings

$L_1[m]$	0.095	length first-second joint
$L_2[m]$	0.1	length second joint end-effector
$M_1[kg]$	0.0.095	mass first joint
$M_2[kg]$	0.37	mass second joint
$g[m/s^2]$	9.81	gravitational acceleration
$I_{1x}[kgm^2]$	$2.27 \cdot 10^{-2}$	moment of inertia
$I_{1y}[kgm^2]$	$8.33 \cdot 10^{-5}$	moment of inertia
$I_{1z}[kgm^2]$	$1.81 \cdot 10^{-5}$	moment of inertia
$I_{2x}[kgm^2]$	$8.33 \cdot 10^{-5}$	moment of inertia
$I_{2y}[kgm^2]$	$2.27 \cdot 10^{-2}$	moment of inertia
$I_{2z}[kgm^2]$	$7.07 \cdot 10^{-5}$	moment of inertia
$b_1[-]$	0.24	friction coefficient, first joint
$b_2[-]$	0.16	friction coefficient, second joint

The model (27) is naturally in descriptor form. With the notation $\boldsymbol{x} = \begin{pmatrix} q_1 & q_2 & \dot{q}_1 & \dot{q}_2 \end{pmatrix}^T$, we have

$$E_c(\boldsymbol{x}) = \begin{pmatrix} I & 0 \\ 0 & M(\boldsymbol{x}) \end{pmatrix} \qquad A_c(\boldsymbol{x}) = \begin{pmatrix} 0 & I \\ 0 & -D(\boldsymbol{x}) \end{pmatrix} \qquad B_c = \begin{pmatrix} 0 \\ I \end{pmatrix}$$

$$\begin{aligned} M(\boldsymbol{x}) &= \begin{pmatrix} I_{1x} + I_{2z} + \cos(x_2)^2(I_{2x} - I_{2z}) + M_2(L_1 + \frac{L_2\cos(x_2)}{2})^2 & 0 \\ 0 & \frac{M_2L_2^2}{4} + I_{2y} \end{pmatrix} \\ D(\boldsymbol{x}) &= \begin{pmatrix} -x_4(\sin(2x_2)(\frac{M_2L_2^2}{4} + I_{2x} - I_{2z}) + L_1L_2M_2\sin(x_2)) + b_1 & 0 \\ x_3(\frac{\sin(2x_2)}{2}(I_{2x} - I_{2z} + \frac{L_2}{4}) + \frac{L_2M_2L_1\sin(x_2)}{2}) & b_2 \end{pmatrix} \end{aligned} \tag{28}$$

The state variables are bounded as $q_1, q_2 \in \left(\frac{-3\pi}{4}, \quad \frac{3\pi}{4}\right)$ and $\dot{q}_1, \dot{q}_2 \in \left(-3, \quad 3\right)$. Using a simple forward Euler discretization with sampling time $T_s = 0.045$ and the sector nonlinearity approach, an equivalent TS descriptor model is:

$$\sum_{i=1}^{2} v_i(\boldsymbol{z}(k))E_i\boldsymbol{x}(k+1) = \sum_{i=1}^{4} h_i(\boldsymbol{z}(k))(A_i\boldsymbol{x}(k) + B\boldsymbol{u}(k))$$

with the matrices

$$E_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0.0242 & 0 \\ 0 & 0 & 0 & 0.0236 \end{pmatrix} \qquad E_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0.0307 & 0 \\ 0 & 0 & 0 & 0.0236 \end{pmatrix}$$

$$A_1 = \begin{pmatrix} 1 & 0 & 0.045 & 0 \\ 0 & 1 & 0 & 0.045 \\ 0 & 0 & 0.0132 & 0 \\ 0 & 0 & -0.003 & 0.0164 \end{pmatrix} \qquad A_2 = \begin{pmatrix} 1 & 0 & 0.045 & 0 \\ 0 & 1 & 0 & 0.045 \\ 0 & 0 & 0.0132 & 0 \\ 0 & 0 & 0.003 & 0.0164 \end{pmatrix}$$

$$A_3 = \begin{pmatrix} 1 & 0 & 0.045 & 0 \\ 0 & 1 & 0 & 0.045 \\ 0 & 0 & 0.02 & 0 \\ 0 & 0 & -0.003 & 0.0164 \end{pmatrix} \qquad A_4 = \begin{pmatrix} 1 & 0 & 0.045 & 0 \\ 0 & 1 & 0 & 0.045 \\ 0 & 0 & 0.02 & 0 \\ 0 & 0 & 0.003 & 0.0164 \end{pmatrix}$$

$$B = 0.045 B_c \qquad C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Similarly to the numerical example in the previous section, our goal is to determine a controller such that the angles x_1 and x_2 track a given reference signal. Using classical conditions for the global stabilization of this system, the LMIs for the descriptor form are unfeasible. Thus, we consider a local approach. We consider a common quadratic Lyapunov function $V(\boldsymbol{x}^e) = \boldsymbol{x}^{eT}(k)P\boldsymbol{x}^e(k)$, with $P = P^T > 0$ and the following choices: $\mathcal{H} = P$, $\mathcal{F} = F_{hv}$, $W = \begin{pmatrix} W_1 & 0 \\ 0 & -I \end{pmatrix}$. The obtained Lyapunov matrix, W and some of the gains are:

$$P = \begin{pmatrix} 0.13 & 0 & -0.05 & 0 & 0.68 & 0 \\ 0 & 0.13 & 0 & -0.05 & 0 & 0.68 \\ -0.05 & -0.00 & 1.27 & 0 & -0.5 & 0 \\ 0 & -0.05 & 0 & 1.26 & 0 & -0.50 \\ 0.68 & 0 & -0.5 & 0 & 9.92 & 0 \\ 0 & 0.68 & 0 & -0.50 & 0 & 9.92 \end{pmatrix}$$

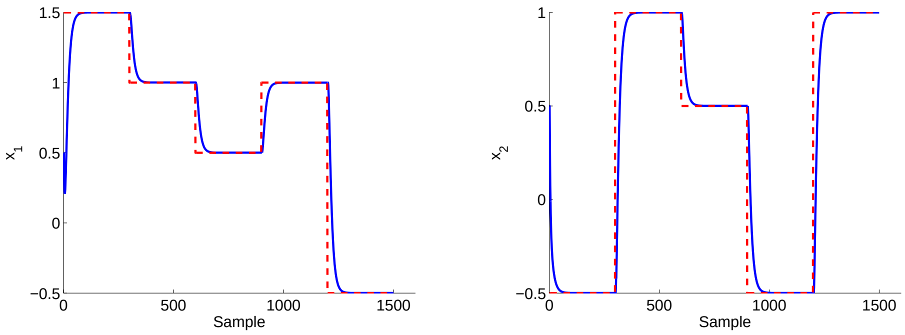
$$W_1 = \begin{pmatrix} -0.10 & 0 & -0.1 & 0 & 0.04 & 0 \\ 0 & -0.1 & 0 & -0.1 & 0 & 0.4 \\ -0.1 & 0 & 1.23 & 0 & -0.18 & 0 \\ 0 & -0.1 & 0 & 1.22 & 0 & -0.18 \\ 0.04 & 0 & -0.18 & 0 & 1.95 & 0 \\ 0 & 0.04 & 0 & -0.18 & 0 & 1.95 \end{pmatrix}$$

$$F_{1,1}H^{-1} = \begin{pmatrix} 4.18 & 0 & 0.47 & 0 & -0.22 & 0 \\ 0 & 4.2 & -0.006 & 0.54 & 0 & -0.22 \end{pmatrix}$$

$$F_{4,2}H^{-1} = \begin{pmatrix} 4.11 & 0 & 0.62 & 0 & -0.21 & 0 \\ 0 & 4.2 & 0.006 & 0.54 & 0 & -0.22 \end{pmatrix}$$

The tracking results are presented in Figure 3.

Note that the domain in which the tracking controller is locally asymptotically stable



(a) Tracking results for x_1 (b) Tracking results for x_2

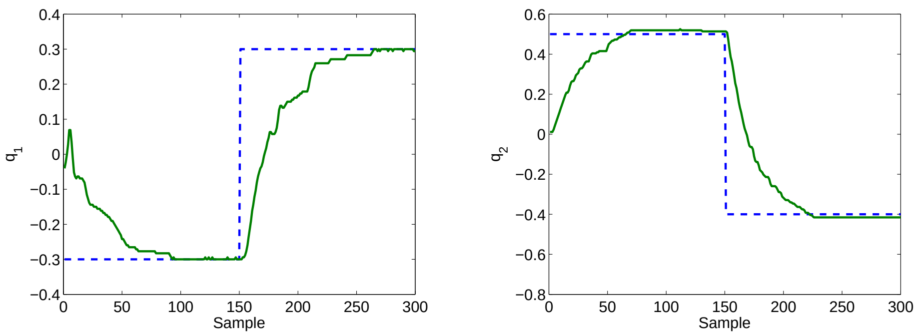
Figure 3: Tracking results for a robot arm.

is determined by

$$\mathcal{D}_R = \left\{ \boldsymbol{x}(k) \in \mathcal{D} \,|\, \begin{pmatrix} \boldsymbol{x}^e(k) \\ \boldsymbol{x}^e(k+1) \end{pmatrix}^T \begin{pmatrix} P^{-1}W_1P^{-1} & 0 \\ 0 & P^{-2} \end{pmatrix} \begin{pmatrix} \boldsymbol{x}^e(k) \\ \boldsymbol{x}^e(k+1) \end{pmatrix} \geq 0 \right\}$$

thus it will also depend on the reference signal to be tracked.

Finally, the experimental results obtained for the 2DOF robot arm are shown in Figure 4. Since the angular velocities for this robot arm are actually not measured, a linear observer has been used to estimate them. The observer has been computed for the linearized model by placing the poles of the estimation error dynamics at 0.6, 0.65, 0.7, and 0.75. The observer gain is $L = \begin{pmatrix} 0.3 & 0 \\ 0 & 0.34 \\ 0.83 & 0 \\ 0 & 0.23 \end{pmatrix}$. The controller uses the measured angles and the estimated angular velocities.



(a) Tracking results for x_1 (b) Tracking results for x_2

Figure 4: Experimental results for the robot arm.

6. Conclusions

In this paper, we have developed conditions for the local asymptotic stability and input-to-state-stability of discrete-time descriptor TS models. The conditions have been formulated as linear matrix inequality conditions that are easy to solve. An estimate of the domain of attraction has also been determined based on the Lyapunov function used and on the system matrices. The application of the proposed conditions has been illustrated on a numerical example and experimentally on a 2DOF robot arm.

In our future work, we will extend the conditions for general delayed Lyapunov functions and controller gains.

Acknowledgements

This work was supported by a grant of the Romanian National Authority for Scientific Research and Innovation, CNCS – UEFISCDI, project number PN-II-RU-TE-2014-4-0942, contract number 88/01.10.2015.

References

[1] F. Lewis, D. Dawson, C. Adballah, Robot Manipulator Control: Theory and Practice, Marcel Dekker, Inc., New York, USA, 2004.

[2] S. M. Megahed, Principles of robot modelling and simulation, Wiley, New York, 1993.

[3] M. Spong, S. Hutchinson, M. Vidyasagar, Robot Modeling and Control, John Wiley & Sons, 2005.

[4] R. P. M. Chan, K. A. Stol, C. R. Halkyard, Review of modelling and control of two-wheeled robots, Annual Reviews in Control 37 (1) (2013) 89–103.

[5] K. Guelton, S. Delprat, T.-M. Guerra, An alternative to inverse dynamics joint torques estimation in human stance based on a Takagi-Sugeno unknown-inputs observer in the descriptor form, Control Engineering Practice 16 (12) (2008) 1414–1426.

[6] P. Nikdel, M.Hosseinpour, M. A. Badamchizadeh, M. A. Akbari, Improved Takagi-Sugeno fuzzy model-based control of flexible joint robot via hybrid-Taguchi genetic algorithm, Engineering Applications of Artificial Intelligence 33 (2014) 12–20.

[7] D. Luenberger, Dynamic equations in descriptor form, IEEE Transactions on Automatic Control 22 (3) (1977) 312–321.

[8] Y.-J. Chen, W.-J. Wang, C.-L. Chang, Guaranteed cost control for an overhead crane with practical constraints: Fuzzy descriptor system approach, Engineering Applications of Artificial Intelligence 22 (2009) 639–645.

[9] T. Takagi, M. Sugeno, Fuzzy identification of systems and its applications to modeling and control, IEEE Transactions on Systems, Man, and Cybernetics 15 (1) (1985) 116–132.

[10] H. Ohtake, K. Tanaka, H. Wang, Fuzzy modeling via sector nonlinearity concept, in: Proceedings of the Joint 9th IFSA World Congress and 20th NAFIPS International Conference, vol. 1, Vancouver, Canada, 127–132, 2001.

[11] T. Taniguchi, K. Tanaka, H. Ohtake, H. Wang, Model construction, rule reduction, and robust compensation for generalized form of Takagi-Sugeno fuzzy systems, IEEE Transactions on Fuzzy Systems 9 (4) (2001) 525–538.

[12] K. Tanaka, H. O. Wang, Fuzzy Control System Design and Analysis: A Linear Matrix Inequality Approach, John Wiley & Sons, New York, NY, USA, 2001.

[13] T. Taniguchi, K. Tanaka, K. Yamafuji, H. Wang, Nonlinear model following control via Takagi-Sugeno fuzzy model, in: Proceedings of the 1999 American Control Conference, vol. 3, San Diego, CA, USA, 1837–1841, 1999.

[14] V. Estrada-Manzo, Zs. Lendek, T. M. Guerra, P. Pudlo, Controller design for discrete-time descriptor models: a systematic LMI approach, IEEE Transactions on Fuzzy Systems 23 (5) (2015) 1608–1621.

[15] H. Zhang, G. Feng, Stability Analysis and H_∞ Controller Design of Discrete-Time Fuzzy Large-Scale Systems Based on Piecewise Lyapunov Functions, IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics 38 (5) (2008) 1390–1401.

[16] M. Chadli, M. Darouach, Novel bounded real lemma for discrete-time descriptor systems: Application to control design, Automatica 48 (2) (2012) 449–453.

[17] T. M. Guerra, M. Bernal, A. Kruszewski, M. Afroun, A way to improve results for the stabilization of continuous-time fuzzy descriptor models, in: Proceedings of the 46th IEEE Conference on Decision and Control, 5960–5964, 2007.

[18] T. Taniguchi, K. Tanaka, H. O. Wang, Fuzzy descriptor systems and nonlinear model following control, IEEE Transactions of Fuzzy Systems 8 (4) (2000) 442–452.

[19] K. Tanaka, T. Ikeda, H. Wang, Fuzzy regulators and fuzzy observers: relaxed stability conditions and LMI-based designs, IEEE Transactions on Fuzzy Systems 6 (2) (1998) 250–265.

[20] A. Sala, T. M. Guerra, R. Babuška, Perspectives of fuzzy systems and control, Fuzzy Sets and Systems 156 (3) (2005) 432–444.

[21] M. Johansson, A. Rantzer, K. Arzen, Piecewise quadratic stability of fuzzy systems, IEEE Transactions on Fuzzy Systems 7 (6) (1999) 713–722.

[22] G. Feng, Stability analysis of discrete-time fuzzy dynamic systems based on piecewise Lyapunov functions, IEEE Transactions on Fuzzy Systems 12 (1) (2004) 22–28.

[23] T. M. Guerra, L. Vermeiren, LMI-based relaxed nonquadratic stabilization conditions for nonlinear systems in the Takagi-Sugeno's form, Automatica 40 (5) (2004) 823–829.

[24] A. Kruszewski, R. Wang, T. M. Guerra, Nonquadratic Stabilization Conditions for a Class of Uncertain Nonlinear Discrete Time TS Fuzzy Models: A New Approach, IEEE Transactions on Automatic Control 53 (2) (2008) 606–611.

[25] L. A. Mozelli, R. M. Palhares, F. O. Souza, E. M. A. M. Mendes, Reducing conservativeness in recent stability conditions of TS fuzzy systems, Automatica 45 (6) (2009) 1580–1583.

[26] B. Ding, H. Sun, P. Yang, Further studies on LMI-based relaxed stabilization conditions for nonlinear systems in Takagi-Sugeno's form, Automatica 42 (3) (2006) 503–508, ISSN 0005–1098.

[27] J. Dong, G. Yang, Dynamic output feedback H_∞ control synthesis for discrete-time T-S fuzzy systems via switching fuzzy controllers, Fuzzy Sets and Systems 160 (19) (2009) 482–499.

[28] D. H. Lee, J. B. Park, Y. H. Joo, Approaches to extended non-quadratic stability and stabilization conditions for discrete-time Takagi-Sugeno fuzzy systems, Automatica 47 (3) (2011) 534–538, ISSN 0005-1098.

[29] S. Boyd, L. El Ghaoui, E. Féron, V. Balakrishnan, Linear Matrix Inequalities in System and Control Theory, Studies in Applied Mathematics, Society for Industrial and Applied Mathematics, Philadelphia, PA, USA, 1994.

[30] C. Scherer, S. Weiland, Linear Matrix Inequalities in Control, Delft University, The Netherlands., 2005.

[31] Zs. Lendek, J. Lauber, Local stability of discrete-time TS fuzzy systems, in: 4th IFAC International Conference on Intelligent Control and Automation Sciences, Reims, France, 7–12, 2016.

[32] Zs. Lendek, J. Lauber, Local quadratic and nonquadratic stabilization of discrete-time TS fuzzy systems, in: Proceedings of the 2016 IEEE World Congress on Computational Intelligence, IEEE International Conference on Fuzzy Systems, Vancouver, Canada, 2182–2187, 2016.

[33] J. da Silva, S. Tarbouriech, Local stabilization of discrete-time linear systems with saturating controls: an LMI-based approach, IEEE Transactions on Automatic Control 46 (1) (2001) 119–124.

[34] J. da Silva, S. Tarbouriech, Anti-windup design with guaranteed regions of stability for discrete-time linear systems., Systems & Control Letters 55 (3) (2006) 184–192.

[35] T. Hu, Z. Lin, B. M. Chen, Analysis and design for discrete-time linear systems subject to actuator saturation, System & Control Letters 45 (2001) 97–112.

[36] Y. Cao, Z. Lin, Stability analysis of discrete-time systems with actuator saturation by a saturation-dependent Lyapunov function., Automatica 39 (7) (2003) 1235–1241.

[37] K. Kiriakidis, Nonlinear modeling by interpolation between linear dynamics and its application in control, Journal of Dynamic Systems, Measurement, and Control 129 (6) (2007) 813–824.

[38] R. E. Skelton, T. Iwasaki, K. Grigoriadis, A Unified Approach to Linear Control Design, Taylor & Francis, 1998.

[39] H. Wang, K. Tanaka, M. Griffin, An approach to fuzzy control of nonlinear systems: stability and design issues, IEEE Transactions on Fuzzy Systems 4 (1) (1996) 14–23.

[40] T. M. Guerra, H. Kerkeni, J. Lauber, L. Vermeiren, An efficient Lyapunov function for discrete TS models: observer design, IEEE Transactions of Fuzzy Systems 20 (1) (2012) 187 – 192, doi: 10.1109/TFUZZ.2011.2165545.

[41] Zs. Lendek, T. M. Guerra, R. Babuška, B. De Schutter, Stability analysis and nonlinear observer design using Takagi-Sugeno fuzzy models, vol. 262 of *Studies in Fuzziness and Soft Computing*, Springer Germany, ISBN ISBN 978-3-642-16775-1, 2010.