Improving observer design for discrete-time TS descriptor models under the quadratic framework

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Abstract: The paper presents a new quadratic Lyapunov function for observer design for discrete-time nonlinear descriptor systems. The main idea is to represent the original nonlinear model as a Takagi-Sugeno one and then use Lyapunov's direct method to design the observer. The well-known Finsler's Lemma is used to design a non-Parallel-Distributed-Compensator-like observer together with a quadratic Lyapunov function. This procedure yields design conditions in terms of linear matrix inequalities. The effectiveness of the proposed approaches is illustrated via numerical examples.

1. INTRODUCTION

Takagi-Sugeno models (Takagi and Sugeno, 1985) have become an interesting alternative for the analysis and controller/observer synthesis for nonlinear models; this is due to their convex structure that allows using the Lyapunov's direct method (Tanaka and Wang 2001). Moreover, when the sector nonlinearity is used, the resulting TS model is an exact representation of the original nonlinear one (Ohtake et al., 2001). A TS model is a blending of local linear models and nonlinear membership functions (MFs) (Lendek et al., 2010; Tanaka and Wang, 2001). The main design goal is to get conditions in terms of linear matrix inequalities (LMIs) (Boyd et al., 1994; Scherer and Weiland, 2005).

In (Wang et al., 1996) the so-called Parallel-Distributed-Compensator (PDC) together with a quadratic Lyapunov function (QLF) were introduced for control purposes. The observer design was treated in (Bergsten and Driankov, 2002; Palm and Driankov, 1999). The use of non-quadratic Lyapunov functions (NQLF) allows reducing the conservativeness of QLF; however, in the continuous-time case, researchers must face the difficulty of the time-derivatives of the MFs (Bernal and Guerra 2010; Blanco et al. 2001; Tanaka et al. 2003). However, in the discrete-time case the NQLF has yielded successful results (Ding et al. 2006; Guerra and Vermeiren 2004; Kruszewski et al. 2008; Lendek et al. 2014).

Recently, via some matrix manipulations (de Oliveira and Skelton 2001; Shaked 2001) some works have obtained advantages for the quadratic case altogether with a non-PDC controller/observer (Jaadari et al. 2012; Marquez et al. 2013; Marquez, et al. 2014).

Despite all the work mentioned above, there are few results referring to TS descriptor systems. This type of TS rewriting was first introduced in (Taniguchi et al., 1999) to represent nonlinear descriptor models which appear in mechanical systems (Luenberger, 1977). An exact TS representation of a model with several p nonlinear terms gives r number of rules ($r = 2^p$), thus it can easily reach computational intractability. Since its TS descriptor structure separates the non-constant terms in the two sides of the system (Estrada-Manzo et al., 2014b; Guelton et al., 2008; Taniguchi et al., 2000), keeping the descriptor form reduce the computational burden.

In most applications not all the states are available for control purposes; an observer is needed to estimate the missing states. In this paper, the main idea is to design an observer via a Lyapunov function similar as Case 1 in (Lendek et al. 2015). Using the well-known Finsler's Lemma together with QLFs, a non-PDC-like observer can be designed. Although it is well know that NQLFs are more relaxed than QLFs, we are interested in the study of QLFs because they give a co-negativity problem of 3 sums, which is less computationally complex than NQLF.

Summarizing, the aims of this work are: given a discrete-time TS descriptor model, 1) design non-PDC-like observer for such a TS model, 2) introduce a new structure on the Lyapunov function to perform the observer design conditions, 3) illustrate the advantages via numerical examples.

The rest of paper is organized as follows: Section 2 provides some useful notation and lemmas used along the paper; Section 3 introduces the discrete-time TS descriptor model and motives the study of them; Section 4 presents previous results in the literature and gives the main results on the observer design; Section 5 shows the performance of the proposed approaches via examples. Section 6 concludes the paper.

2. NOTATION AND TOOLS

Throughout the paper the following shorthand notation is used to represent convex sums of matrix expressions:

$$\Upsilon_h = \sum_{i=1}^{r} h_i\left(z(\kappa)\right)\Upsilon_i, \quad \Upsilon_v = \sum_{k=1}^{r_e} v_k\left(z(\kappa)\right)\Upsilon_k,$$

$$\Upsilon_{h^+} = \sum_{l=1}^{r} h_l\left(z(\kappa+1)\right)\Upsilon_l, \quad \Upsilon_h^{-1} = \left(\sum_{i=1}^{r} h_i\left(z(\kappa)\right)\Upsilon_i\right)^{-1},$$

where Υ_i, Υ_k, and Υ_l are matrices of appropriate dimensions. The subscript h and v denote the associated MF. In matrix expressions, an asterisk $(*)$ denotes the transpose of the symmetric element; for in-line expressions it represents the transpose of the terms on its left-hand side.

When double convex sums appear, the following relaxation lemma is employed to drop off the MFs.

Lemma 1 (Relaxation Lemma) (Tuan et al. 2001): Let Υ_{ij}^{k} $i,j \in \{1,2,\ldots,r\}$, $k \in \{1,2,\ldots,r_e\}$ be matrices of appropriate dimensions. Then

$$\Upsilon_{hh}^{v} = \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r_e} h_i\left(z(\kappa)\right)h_j\left(z(\kappa)\right)v_k\left(z(\kappa)\right)\Upsilon_{ij}^{k} < 0, \text{ holds if}$$

$$\Upsilon_{ii}^{k} < 0, \quad \forall i,k,$$
$$\frac{2}{r-1}\Upsilon_{ii}^{k} + \Upsilon_{ij}^{k} + \Upsilon_{ij}^{k} < 0, \quad i \neq j, \forall k. \tag{1}$$

Lemma 2 (Finsler's Lemma) (de Oliveira and Skelton 2001): Let $\mathcal{X} \in \mathbb{R}^n$, $\mathcal{Q} = \mathcal{Q}^T \in \mathbb{R}^{n\times n}$, and $\mathcal{R} \in \mathbb{R}^{m\times n}$ such that $rank(\mathcal{R}) < n$; the following expressions are equivalent:

a) $\mathcal{X}^T\mathcal{Q}\mathcal{X} < 0, \quad \forall \mathcal{X} \in \{\mathcal{X} \in \mathbb{R}^n : \mathcal{X} \neq 0, \mathcal{R}\mathcal{X} = 0\}$.

b) $\exists \mathcal{M} \in \mathbb{R}^{n\times m} : \mathcal{Q} + \mathcal{M}\mathcal{R} + \mathcal{R}^T\mathcal{M}^T < 0.$

Property 1: Let $\mathcal{Q} = \mathcal{Q}^T > 0$ and $\mathcal{X}$ be matrices of appropriated size. The following expression holds:

$$(\mathcal{X}-\mathcal{Q})^T \mathcal{Q}^{-1}(\mathcal{X}-\mathcal{Q}) \geq 0 \Leftrightarrow \mathcal{X}^T\mathcal{Q}^{-1}\mathcal{X} \geq \mathcal{X} + \mathcal{X}^T - \mathcal{Q}.$$

3. PROBLEM STATEMENT

Consider a nonlinear descriptor system in discrete-time:

$$\begin{aligned} E\left(x(\kappa)\right)x(\kappa+1) &= A\left(x(\kappa)\right)x(\kappa) + B\left(x(\kappa)\right)u(\kappa) \\ y(\kappa) &= C\left(x(\kappa)\right)x(\kappa), \end{aligned} \tag{2}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the control input vector, $y \in \mathbb{R}^o$ is the output vector, and κ is the current sample. Matrices $A\left(x(\kappa)\right)$, $B\left(x(\kappa)\right)$, and $C\left(x(\kappa)\right)$ are assumed to be bounded and smooth in a compact set of the state space Ω_x. This paper considers the descriptor matrix $E\left(x(\kappa)\right)$ as a non-singular matrix at least in a compact set Ω_x. In what follows, $x_{\kappa+}$ and x_κ will stand for $x(\kappa+1)$ and $x(\kappa)$ respectively. Arguments will be omitted when their meaning is clear.

When using the sector nonlinearity approach (Ohtake et al. 2001), the p nonlinear terms in the right-hand side of (2) are captured via convex MFs $h_i\left(z(\kappa)\right) \geq 0$, $i \in \{1,2,\ldots,2^p\}$, $\sum_{i=1}^{r} h_i\left(z(\kappa)\right) = 1$. Proceeding similarly with the p_e nonlinear terms in the left-hand side, the MFs are $v_k\left(z(\kappa)\right) \geq 0$, $k \in \{1,2,\ldots,2^{p_e}\}$, $\sum_{k=1}^{r_e} v_k\left(z(\kappa)\right) = 1$. This method allows obtaining an exact TS descriptor model of the nonlinear descriptor one (more details are given in the pioneering work (Taniguchi et al. 2000)).

Using the above methodology, an exact representation of (2) in Ω_x is given by the following TS descriptor model:

$$\begin{aligned} \sum_{k=1}^{r_e} v_k\left(z(\kappa)\right)E_k x_{\kappa+} &= \sum_{i=1}^{r} h_i\left(z(\kappa)\right)\left(A_i x_\kappa + B_i u_\kappa\right) \\ y_\kappa &= \sum_{i=1}^{r} h_i\left(z(\kappa)\right)C_i x_\kappa, \end{aligned} \tag{3}$$

where matrices (A_i, B_i, C_i), $i \in \{1,2,\ldots,r\}$, represent the *i*-th linear right-hand side model (3) and E_k, $k \in \{1,2,\ldots,r_e\}$, represent the *k*-th linear left-hand side model of the TS descriptor model. In this work, the MFs depend on the premise variables grouped in the vector $z(\kappa)$ which are assumed to be known.

The following example illustrates why it is important to keep the nonlinear descriptor form instead of calculating the standard state space:

$$x_{\kappa+} = \bar{A}(x)x_\kappa + \bar{B}(x)u_\kappa, \tag{4}$$

with $\bar{A}(x) = E^{-1}(x)A(x)$ and $\bar{B}(x) = E^{-1}(x)B(x)$.

Example 1. Consider a nonlinear descriptor model:

$$E(x)x_{\kappa+} = A(x)x_\kappa + Bu_\kappa, \qquad y_\kappa = C(x)x_\kappa, \tag{5}$$

with $A(x) = \begin{bmatrix} -\cos(x_1) & -1 \\ 0.5 & -1.1 \end{bmatrix}$, $C(x) = \begin{bmatrix} \sin(x_1)/x_1 \\ 0 \end{bmatrix}^T$,

$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, and $E(x) = \begin{bmatrix} 2 & -1/\left(1+x_1^2\right) \\ 1/\left(1+x_1^2\right) & 1 \end{bmatrix}$. A TS representation in the form (3) gives $r_e = 2$ due to the nonlinear term $1/\left(1+x_1^2\right)$ and $r = 4$ due to the number of nonlinearities $\cos(x_1)$ and $\sin(x_1)/x_1$. Note that in (5) the matrix B is constant. Since $E(x)$ is non-singular, one can calculate its inverse and then construct a standard TS model:

$E^{-1}(x) = \dfrac{\left(1+x_2^2\right)^2}{3+4x_2^2+2x_2^4}\begin{bmatrix} 1 & 1/\left(1+x_2^2\right) \\ -1/\left(1+x_2^2\right) & 2 \end{bmatrix}$; from here, one can see that at least four different nonlinearities have to be considered, which results in $r = 16$. Moreover, the standard representation contains nonlinear terms in $\bar{B}(x)=E^{-1}(x)B$ which forces the use of sum relaxations for the controller design. Table I compares the observer design conditions for the standard TS and TS descriptor forms.

Approach	No. of sums	No. of LMI conditions	Feasible solution
Q Standard TS[1] + Lemma 1	3	$r^2+1=257$	No
NQ Standard TS[2] + Lemma 1	4	$r^3+r=4112$	No
Q TS Descriptor[3] + Lemma 1	3	$r_e\times r^2+1=33$	No
NQ TS Descriptor[3] + Lemma 1	4	$r_e\times r^3+r=132$	Yes

Table I. Computational Complexity for different results in Example 1. ◊

Via Table I, one can see that better results may be obtained with TS descriptor models. Therefore, the observer design is done for the TS descriptor models via the direct Lyapunov method. The next section recalls some previous results and gives new LMI conditions for observer design.

4. RESULTS

An observer for the descriptor model (3) is given by:

$$\begin{aligned} E_v\hat{x}_{\kappa^+} &= A_h\hat{x}_\kappa + B_h u_\kappa + \mathcal{K}_{(\cdot)}\left(y_\kappa-\hat{y}_\kappa\right) \\ \hat{y}_\kappa &= C_h\hat{x}_\kappa, \end{aligned} \tag{6}$$

where the observer gain $\mathcal{K}_{(\cdot)}$ may change according to the approach under study.

Defining the estimation error $e_\kappa = x_\kappa-\hat{x}_\kappa$, its dynamics are as follows:

$$E_v e_{\kappa^+} = \left(A_h-\mathcal{K}_{(\cdot)}C_h\right)e_\kappa, \tag{7}$$

which can be expressed as the following equality constraint

$$\begin{bmatrix} A_h-\mathcal{K}_{(\cdot)}C_h & -E_v \end{bmatrix}\begin{bmatrix} e_\kappa \\ e_{\kappa^+}\end{bmatrix}=0. \tag{8}$$

For design purposes consider the following Lyapunov function candidate:

$$V\left(e_\kappa\right)=e_\kappa^T\mathcal{P}e_\kappa>0,\quad \mathcal{P}=\mathcal{P}^T>0, \tag{9}$$

where $\mathcal{P}$ may be constant (quadratic approach) or depend on MFs (non-quadratic approach). The variation of the Lyapunov function (9) writes

$$\Delta V\left(e_k\right)=e_{k+}^T\mathcal{P}_+e_{k+}-e_k^T\mathcal{P}e_k<0. \tag{10}$$

In order to use Finsler's Lemma the inequality (10) is expressed as follows:

$$\Delta V\left(e_\kappa\right)=\begin{bmatrix} e_\kappa \\ e_{\kappa^+}\end{bmatrix}^T\begin{bmatrix} -\mathcal{P} & 0 \\ 0 & \mathcal{P}_+\end{bmatrix}\begin{bmatrix} e_\kappa \\ e_{\kappa^+}\end{bmatrix}<0. \tag{11}$$

Thus, using Finsler's Lemma the inequality constraint (11) together with the equality one (8) gives

$$\begin{bmatrix} -\mathcal{P} & 0 \\ 0 & \mathcal{P}_+\end{bmatrix}+\mathcal{M}_{(\cdot)}\begin{bmatrix} A_h-\mathcal{K}_{(\cdot)}C_h & -E_v\end{bmatrix}+(*)<0, \tag{12}$$

where $\mathcal{M}_{(\cdot)}\in\mathbb{R}^{2n\times n}$ will be defined later on.

4.1 Previous results

The quadratic approach in (Estrada-Manzo et al. 2014a) is summarized in the following Lemma :

Lemma 3: (Estrada-Manzo et al. 2014a) The estimation error e_κ is asymptotically stable if there exist matrices $P=P^T>0$ and F_{hv}, such that the following inequality holds

$$\Upsilon_{hh}^{v}=\begin{bmatrix} -P & (*) \\ PA_h-F_{hv}C_h & -PE_v-E_v^TP+P\end{bmatrix}<0. \tag{13}$$

The observer gains are recovered with $L_{hv}=P^{-1}F_{hv}$. The final observer structure is

$$\begin{aligned} E_v\hat{x}_{\kappa^+} &= A_h\hat{x}_\kappa + B_h u_\kappa + L_{hv}\left(y_\kappa-\hat{y}_\kappa\right) \\ \hat{y}_\kappa &= C_h\hat{x}_\kappa. \end{aligned} \tag{14}$$

Proof: see Theorem 1 in (Estrada-Manzo et al. 2014b). □

The non-quadratic case presented in (Estrada-Manzo et al. 2014a) is summarized in the following Lemma.

Lemma 4: (Estrada-Manzo et al. 2014a) The estimation error e_κ is asymptotically stable if there exist matrices $P_h=P_h^T>0$, H_h, L_{hv}, such that the next inequality holds

$$\Upsilon_{hhh^+}^{h}=\begin{bmatrix} -P_h & (*) \\ H_hA_h-L_{hv}C_h & -H_hE_v-E_v^TH_h^T+P_{h^+}\end{bmatrix}<0. \tag{15}$$

The final observer structure is

$$\begin{aligned} E_v\hat{x}_{\kappa^+} &= A_h\hat{x}_\kappa + B_h u_\kappa + H_h^{-1}L_{hv}\left(y_\kappa-\hat{y}_\kappa\right) \\ \hat{y}_\kappa &= C_h\hat{x}_\kappa. \end{aligned} \tag{16}$$

Proof: Recall (12). Choosing the observer gain as $\mathcal{K}_{(\cdot)}=H_h^{-1}L_{hv}$, the Lyapunov matrix as $\mathcal{P}=P_h$, and $\mathcal{M}_{(\cdot)}=\begin{bmatrix}0 & H_h^T\end{bmatrix}^T$ yields (15), thus the proof is ended. □

Remark 1: The inequality conditions in Lemma 3 and 4 are easily transformed into LMIs once the MFs are removed. To this end, Lemma 1 could be applied, and many other relaxation lemmas are available (Kim and Lee 2000; Sala and Ariño 2007; Wang et al. 1996).

[1] (Lendek et al. 2010)
[2] (Guerra et al. 2012)
[3] (Estrada-Manzo et al. 2014a)

4.2 New quadratic Lyapunov function

In this subsection a new Lyapunov function is presented. Recall (10) and select $\mathcal{P} = G^T P^{-1} G$, therefore a new Lyapunov function yields

$$V(e_\kappa) = e_\kappa^T G^T P^{-1} G e_\kappa > 0, \tag{17}$$

where $P = P^T > 0$, thus it is non-singular. The following result can be stated:

Theorem 1: The estimation error e_κ is asymptotically stable if there exist matrices $P = P^T > 0$, G, and F_{hv}, such that the following inequality holds

$$\Upsilon_{hh}^v = \begin{bmatrix} -G - G^T + P & (*) & (*) \\ PA_h - F_{hv} C_h & -PE_v - E_v^T P & (*) \\ 0 & G & -P \end{bmatrix} < 0. \tag{18}$$

The observer gains are recovered with $L_{hv} = P^{-1} F_{hv}$. The final observer structure is

$$\begin{aligned} E_v \hat{x}_{\kappa^+} &= A_h \hat{x}_\kappa + B_h u_\kappa + L_{hv} (y_\kappa - \hat{y}_\kappa) \\ \hat{y}_\kappa &= C_h \hat{x}_\kappa . \end{aligned} \tag{19}$$

Proof: Recall (12). Choosing the observer gain as $\mathcal{K}_{(\cdot)} = L_{hv}$, the Lyapunov matrix as $\mathcal{P} = G^T P^{-1} G$, and $\mathcal{M}_{(\cdot)} = \begin{bmatrix} 0 & P \end{bmatrix}^T$ yields

$$\begin{bmatrix} -G^T P^{-1} G & (*) \\ PA_h - F_{hv} C_h & -PE_v - E_v^T P + G^T P^{-1} G \end{bmatrix} < 0, \tag{20}$$

with the change of variable $F_{hv} = PL_{hv}$. Finally, using Property 1 on the position (1,1) and the Schur complement on the position (2,2) renders (18); thus concluding the proof. □

Since Finsler's Lemma allows "separating" — in a sense — the observer gain and the Lyapunov matrix (Marquez et al. 2013), a way to take advantage of the classical non-PDC-like observer is obtained by choosing $\mathcal{K}_{(\cdot)} = H_h^{-1} L_{hv}$; this result is summarized in the following:

Theorem 2: The estimation error e_κ is asymptotically stable if there exist matrices $P = P^T > 0$, G, H_h, and L_{hv} such that the following inequality holds

$$\Upsilon_{hh}^v = \begin{bmatrix} -G - G^T + P & (*) & (*) \\ H_h A_h - L_{hv} C_h & -H_h E_v - E_v^T H_h^T & (*) \\ 0 & G & -P \end{bmatrix} < 0. \tag{21}$$

The final observer structure is

$$\begin{aligned} E_v \hat{x}_{\kappa^+} &= A_h \hat{x}_\kappa + B_h u_\kappa + H_h^{-1} L_{hv} (y_\kappa - \hat{y}_\kappa) \\ \hat{y}_\kappa &= C_h \hat{x}_\kappa . \end{aligned} \tag{22}$$

Proof: Recall (12). Choosing the observer gain as $\mathcal{K}_{(\cdot)} = H_h^{-1} L_{hv}$, the Lyapunov matrix as $\mathcal{P} = G^T P^{-1} G$, and $\mathcal{M}_{(\cdot)} = \begin{bmatrix} 0 & H_h^T \end{bmatrix}^T$ yields

$$\begin{bmatrix} -G^T P^{-1} G & (*) \\ H_h A_h - L_{hv} C_h & -H_h E_v - E_v^T H_h^T + G^T P^{-1} G \end{bmatrix} < 0. \tag{23}$$

Using Property 1 on the position (1,1) and the Schur complement on the position (2,2) gives (21); thus concluding the proof. □

Remark 2: Note that the PDC-like observer structure in Theorem 1 is the same as in Lemma 3, while the non-PDC-like observer in Theorem 2 is the same as in Lemma 4; but the design procedure is done via different Lyapunov functions. The new Lyapunov function may be less conservative since it naturally introduces more slack variables; it could be seen as the dual of the one presented in (Lendek et al. 2015) for control purposes.

The complexity in terms of number of decision variable and LMI conditions is summarized in Table II.

Approach	No. of decision variables	No. of LMI conditions
Lemma 3	$0.5n(n+1) + (r \times r_e) \times (n \times o)$	$r_e \times r^2 + 1$
Lemma 4	$0.5r \times n(n+1) + (r \times r_e) \times (n \times o) + r \times n^2$	$r_e \times r^3 + r$
Theorem 1	$0.5n(n+1) + n^2 + (r \times r_e) \times (n \times o)$	$r_e \times r^2 + 1$
Theorem 2	$0.5n(n+1) + n^2 + r \times n^2 + (r \times r_e) \times (n \times o)$	$r_e \times r^2 + 1$

Table II. Computational complexity of the various approaches.

5. EXAMPLES

The following example is adopted from (Estrada-Manzo et al. 2014a).

Example 2. (Estrada-Manzo et al. 2014a) Consider a discrete-time TS descriptor model as in (3) with $r = r_e = 2$,

$$E_1 = \begin{bmatrix} 0.9 & 0.1 + a \\ -0.4 - b & 1.1 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0.9 & 0.1 - a \\ -0.4 + b & 1.1 \end{bmatrix},$$
$$A_1 = \begin{bmatrix} -1 & 1 + a \\ -1.5 & 0.5 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -1 & 1 - a \\ -1.5 & 0.5 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} 0 \\ 1 - b \end{bmatrix}^T, \quad C_2 = \begin{bmatrix} 0 \\ 1 + b \end{bmatrix}^T.$$

The parameters are defined as $a \in [-1, 1]$ and $b \in [-1, 1]$.

Fig. 1 shows the feasible regions for the proposed approaches. Results obtained via Lemma 3 (□), via Lemma 4 (×), and via Theorem 1 (+).

Example 2 illustrates the improvements made by the new Lyapunov function. In Fig. 1, one can see that the new Lyapunov function together with a PDC-like observer overcomes the quadratic approach in (Estrada-Manzo et al. 2014a).

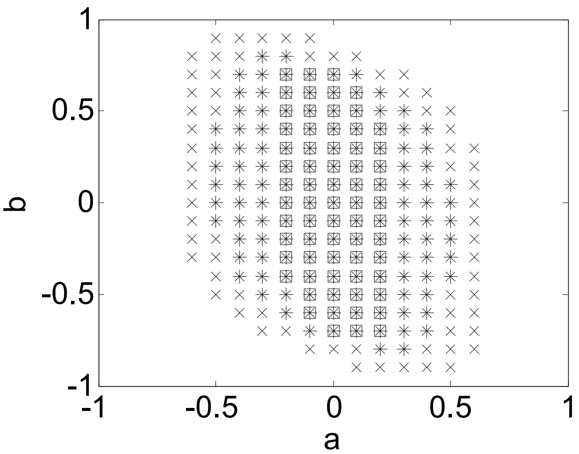

Fig. 1. Feasibility sets for Example 2.

Using results in Theorem 2, it is possible to obtain the same feasibility set as in Lemma 4 for this specific Example 2. Both conditions allow designing the same observer structure (16). Table II shows that conditions in Theorem 2 are less demanding than Lemma 4. ◊

Example 3. Recall the nonlinear descriptor in Example 1. Considering the compact set $\Omega_x = \mathbb{R}^2$ and using the sector nonlinearity approach a TS descriptor model as in (3) , we obtain: on the left-hand side:

$$r_e = 2, \quad E_1 = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}, \quad \text{and} \quad E_2 = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix};$$

on the right-hand side:

$$r = 4, \quad A_1 = A_2 = \begin{bmatrix} 1 & -1 \\ 0.5 & -1.1 \end{bmatrix}, \quad A_3 = A_4 = \begin{bmatrix} -1 & -1 \\ 0.5 & -1.1 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C_1 = C_3 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}^T, \quad \text{and} \quad C_2 = C_4 = \begin{bmatrix} -0.2167 \\ 0 \end{bmatrix}^T.$$

The MFs are: $v_1 = 1/\left(1+x_1^2\right)$, $v_2 = 1 - v_1$, $h_1 = \omega_0^1\omega_0^2$, $h_2 = \omega_0^1\omega_1^2$, $h_3 = \omega_1^1\omega_0^2$, $h_4 = \omega_1^1\omega_1^2$; with $\omega_0^1 = \left(\cos\left(x_1\right)-1\right)/2$, $\omega_0^2 = \left(\sin\left(x_1\right)/x_1 + 0.2167\right)/1.2167$, $\omega_1^1 = 1-\omega_0^1$, and $\omega_1^2 = 1-\omega_0^2$. The state variable x_1 is available. Conditions in Lemma 3 and Theorem 1 are not feasible. Feasible results are obtained via Theorem 2 (non-PDC-like observer and the new quadratic Lyapunov function); for brevity, only some of the resulting gains are given:

$$P = \begin{bmatrix} 1.23 & -0.48 \\ -0.48 & 1.22 \end{bmatrix}, G = \begin{bmatrix} 1.18 & -0.49 \\ -0.52 & 1.20 \end{bmatrix}, L_{11} = \begin{bmatrix} 0.18 \\ -0.04 \end{bmatrix},$$

$$L_{12} = \begin{bmatrix} 0.28 \\ -0.11 \end{bmatrix}, L_{22} = \begin{bmatrix} 0.05 \\ 0.01 \end{bmatrix}, L_{32} = \begin{bmatrix} -0.52 \\ 0.26 \end{bmatrix}, L_{41} = \begin{bmatrix} -0.04 \\ 0.02 \end{bmatrix},$$

$$H_1 = \begin{bmatrix} 0.69 & -0.19 \\ -0.32 & 1.06 \end{bmatrix}, \quad H_3 = \begin{bmatrix} 0.68 & -0.02 \\ -0.32 & 0.99 \end{bmatrix}.$$

For this example, conditions in Lemma 4 are also feasible. However, the number of LMIs in Theorem 2 is 33 while for Lemma 4 is 132 (see Table II).

The TS descriptor exactly represents the nonlinear model in the compact set $\Omega_x = \mathbb{R}^2$, thus the designed observer allows the asymptotic converge of the estimation error for any initial condition of the original nonlinear model (see Fig. 2).

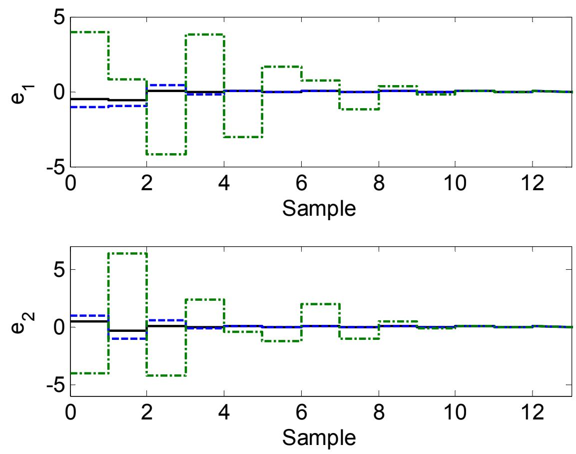

Fig. 2. Evolution of the estimation error for several initial conditions for Example 3. ◊

6. CONCLUSIONS

In this paper, LMI conditions are established for the observer design of TS descriptor models. Using Finsler's Lemma together with a quadratic Lyapunov function it is possible to design a non-PDC-like observer; thus relaxing previous results. Numerical examples illustrate the advantages of the presented approaches.

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