Switching Lyapunov functions for periodic TS systems

Zs. Lendek *,** J. Lauber *, T. M. Guerra *

* University of Valenciennes and Hainaut-Cambresis, LAMIH, Le Mont Houy, 59313 Valenciennes Cedex 9, France, (email: jimmy.lauber, thierry.guerra@univ-valenciennes.fr)
** Department of Automation, Technical University of Cluj-Napoca, Memorandumului 28, 400114 Cluj-Napoca, Romania, (email: zsofia.lendek@aut.utcluj.ro)

Abstract: This paper considers the stability analysis of periodic Takagi-Sugeno fuzzy models. For this we use a switching Lyapunov function defined at the time instants when the subsystems switch. Using the developed conditions we are able to prove the stability of periodic TS systems where the local models or even the subsystems are unstable. The application of the conditions is illustrated on numerical examples.

Keywords: Stability analysis, TS systems, periodic systems, non-quadratic Lyapunov function.

1. INTRODUCTION

Takagi-Sugeno (TS) fuzzy systems (Takagi and Sugeno, 1985) are nonlinear, convex combinations of local linear models, and have the property that they are able to exactly represent a large class of nonlinear systems (Lendek et al., 2010).

For the stability analysis and observer and controller design of TS systems the direct Lyapunov approach has been used. Stability conditions have been derived using quadratic Lyapunov functions (Tanaka et al., 1998; Tanaka and Wang, 2001; Sala et al., 2005), piecewise continuous Lyapunov functions (Johansson et al., 1999; Feng, 2004a), and more recently, to reduce the conservativeness of the conditions, nonquadratic Lyapunov functions (Guerra and Vermeiren, 2004; Kruszewski et al., 2008; Mozelli et al., 2009). The stability or design conditions are generally derived in the form of linear matrix inequalities (LMIs).

Non-quadratic Lyapunov functions have shown a real improvement of the design conditions in the discrete-time case (Guerra and Vermeiren, 2004; Ding et al., 2006; Dong and Yang, 2009a; Lee et al., 2011). It has been proven that the solutions obtained by non-quadratic Lyapunov functions include and extend the set of solutions obtained using the quadratic framework.

Non-quadratic Lyapunov functions have been extended to double-sum Lyapunov functions by (Ding et al., 2006) and later on to polynomial Lyapunov functions by (Sala and Ariño, 2007; Ding, 2010; Lee et al., 2010). A different type of improvement in the discrete case has been developed in (Kruszewski et al., 2008), conditions being obtained by replacing the classical one sample variation of the Lyapunov function by its variation over several samples (α-sample variation).

Switched TS systems are a class of nonlinear systems often described by continuous dynamics and discrete dynamics as well as their interactions. In the last decade, they have been investigated mainly in the continuous case where the stability is based on the use of a quadratic Lyapunov function (Tanaka et al., 2001; Lam et al., 2002, 2004; Ohtake et al., 2006) or a piecewise one (Feng, 2003, 2004b). Although results are available for discrete-time linear switching systems (Daafouz et al., 2002), for discrete-time TS models, few results exist (Doo et al., 2003; Dong and Yang, 2009b).

In this paper, we propose a switching non-quadratic Lyapunov function for the stability analysis of periodic TS fuzzy models. This Lyapunov function is useful for proving the stability of a periodic TS system having non-stable local models and even unstable subsystems. For simplicity, we present the results for a periodic system with two subsystems, although they can be easily generalized for a known number of subsystems.

The structure of the paper is as follows. Section 2 presents the notations used in this paper and the general form of the TS models. It also develops the proposed conditions for stability analysis of systems that switch at each sample time. The stability analysis of periodic systems is presented in Section 3. Section 4 illustrates the use of the conditions on a numerical example. Finally, Section 5 concludes the paper.

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2. STABILITY OF CONTINUOUSLY SWITCHING SYSTEMS

2.1 Preliminaries

In this paper we consider stability analysis of discrete-time periodic TS systems. For the ease of notation, we only consider two subsystems of the form

\[ x(T + T_1) = \sum_{i=1}^{r_1} h_{i1}(z_1(T))A_{i1}x(T) \]
\[ = A_{11}x(T) \quad (1) \]

and

\[ x(T + T_2) = \sum_{i=1}^{r_2} h_{i2}(z_2(T))A_{i2}x(T) \]
\[ = A_{22}x(T) \quad (2) \]

where \( x \) denotes the state vector, \( T \) is the current time, \( T_1 \) and \( T_2 \) are the sampling periods of the subsystems (i.e., the two subsystems may have different sampling times), \( r_1 \) and \( r_2 \) are the number of rules, \( z_1 \) and \( z_2 \) are the scheduling vectors, \( h_{1i}, i = 1, 2, \ldots, r_1 \) and \( h_{2i}, i = 1, 2, \ldots, r_2 \) normalized membership functions, and \( A_{i1}, i = 1, 2, \ldots, r_1 \) and \( A_{2i}, i = 1, 2, \ldots, r_2 \) the local models.

We consider periodic systems, i.e., the two subsystems defined above are activated in a sequence \( 1, 1, \ldots, 1, 2, 2, \ldots, 2, 1, 1, \ldots, 1, \ldots \), etc., where \( p_1 \) and \( p_2 \) denote the periods of the subsystems.

In what follows, \( 0 \) and \( I \) denote the zero and identity matrices of appropriate dimensions, and \( \alpha (\cdot) \) denotes the term induced by symmetry. The subscript \( z + 1 \) (as in \( A_{1z+1} \)) stands for the scheduling vector evaluated at the next sampling time. Note that depending on the subsystem, the next sampling instant may be \( T + T_1 \) or \( T + T_2 \).

To derive stability conditions for periodic systems, we will make use of the following result:

**Lemma 1.** (Skelton et al., 1998) Consider a vector \( x \in \mathbb{R}^n \) and two matrices \( Q = Q^T \in \mathbb{R}^{n \times n} \) and \( R \in \mathbb{R}^{m \times n} \) such that \( \text{rank}(R) < n \). The two following expressions are equivalent:

1. \( x^TQx < 0, \ x \in \{x \in \mathbb{R}^n, x \neq 0, Rx = 0\} \)
2. \( \exists H \in \mathbb{R}^{m \times n} \) such that \( Q + HR + RH^T < 0 \)

2.2 Stability analysis

In this section, we consider the simplest case when the two subsystems defined in Section 2.1 are switched at every sampling time, i.e., \( p_1 = p_2 = 1 \). For such systems, the following results can be stated:

**Theorem 1.** The periodic TS system composed of the subsystems (1) and (2), with periods \( p_1 = p_2 = 1 \) is asymptotically stable, if there exist \( P_{11} = P_{11}^T > 0, M_{11}, \)
\( i = 1, 2, \ldots, r_1, P_{2i} = P_{2i}^T > 0, M_{2i}, i = 1, 2, \ldots, r_2, \) so that the following conditions are satisfied:

\[
\begin{align*}
- P_{11} & \in \mathbb{R}^{m \times m} , \\
M_{2i} - M_{1i} - M_{2i}^T + P_{2i+1} & > 0 \\
M_{1i} - M_{1i} - M_{1i}^T + P_{1i+1} & > 0 \\
- P_{1i} & > 0 \\
M_{1i} - M_{2i} - M_{2i}^T + P_{2i+1} & < 0 \\
M_{1i} - M_{1i} - M_{1i}^T + P_{1i+1} & < 0
\end{align*}
\]

**Proof:** Consider the switching Lyapunov function, similar to the one used by Daaouz et al. (2002),

\[
V(x(T), T) = \begin{cases} x(T)^T P_{11} x(T) & \text{active subsystem was } 1 \\
x(T)^T P_{22} x(T) & \text{otherwise}
\end{cases}
\]

Then the difference in the Lyapunov function is either

\[
V(x(T + T_2), T + T_2) - V(x(T), T) = \begin{pmatrix} x(T) \\ x(T + T_2) \end{pmatrix}^T \begin{pmatrix} -P_{11} & 0 \\ 0 & P_{22}+1 \end{pmatrix} \begin{pmatrix} x(T) \\ x(T + T_2) \end{pmatrix}
\]

if the switching is from the first subsystem to the second one (Case 1), or

\[
V(x(T + T_1), T + T_1) - V(x(T), T) = \begin{pmatrix} x(T) \\ x(T + T_1) \end{pmatrix}^T \begin{pmatrix} -P_{11} & 0 \\ 0 & P_{22}+1 \end{pmatrix} \begin{pmatrix} x(T) \\ x(T + T_1) \end{pmatrix}
\]

if the switching is from the second subsystem to the first one (Case 2).

Consider first Case 1. We have

\[
\Delta V = \begin{pmatrix} x(T) \\ x(T + T_2) \end{pmatrix}^T \begin{pmatrix} -P_{11} & 0 \\ 0 & P_{22}+1 \end{pmatrix} \begin{pmatrix} x(T) \\ x(T + T_2) \end{pmatrix}
\]

together with the system dynamics, which is

\[
(A_{22} - I) \begin{pmatrix} x(T) \\ x(T + T_2) \end{pmatrix} = 0
\]

since during the time \([T, T + T_2]\), the second subsystem is active. Using Lemma 1, the difference in the Lyapunov function is negative definite, if there exists \( H \) such that

\[
\begin{pmatrix} -P_{11} & 0 \\ 0 & P_{22}+1 \end{pmatrix} + H \begin{pmatrix} A_{22} - I \end{pmatrix} > 0
\]

Choosing \( H = \begin{pmatrix} 0 & M_{22} \end{pmatrix} \) leads directly to

\[
\begin{pmatrix} -P_{11} & 0 \\ M_{22} - M_{22} - M_{22}^T + P_{22}+1 \end{pmatrix} < 0
\]

For Case 2, we have

\[
\Delta V = \begin{pmatrix} x(T) \\ x(T + T_1) \end{pmatrix}^T \begin{pmatrix} -P_{22} & 0 \\ 0 & P_{11}+1 \end{pmatrix} \begin{pmatrix} x(T) \\ x(T + T_1) \end{pmatrix}
\]

and the dynamics

\[
(A_{11} - I) \begin{pmatrix} x(T) \\ x(T + T_1) \end{pmatrix} = 0
\]

which, by choosing \( H = \begin{pmatrix} 0 & M_{11} \end{pmatrix} \) leads to

\[
\begin{pmatrix} -P_{22} & 0 \\ M_{11} - M_{11} - M_{11}^T + P_{11}+1 \end{pmatrix} < 0
\]

\(\square\)

3. STABILITY ANALYSIS OF PERIODIC SYSTEMS

In the previous section, we considered the special case when the subsystems are switching at each sample time. Consider now the case when the subsystems switch after \( p_1 \), respectively \( p_2 \) samples. Then, the following result can be stated.
Theorem 2. The periodic TS system composed of the subsystems (1) and (2), with periods \( p_1 \geq 1 \) and \( p_2 \geq 1 \) is asymptotically stable, if there exist \( P_{1i} = P_{1i}^T > 0 \), \( M_{1i} \), \( i = 1, 2, \ldots, r_1 \), \( P_{2i} = P_{2i}^T > 0 \), \( M_{2i} \), \( i = 1, 2, \ldots, r_2 \), so that the following conditions are satisfied:

\[
\begin{bmatrix}
-P_{1i} & (\ast) \\
M_{2i} - M_{2i} - M_{2i}^T & \cdots & (\ast) \\
\vdots & \vdots & \vdots \\
0 & 0 & M_{2i+p_2-1} A_{2i+p_2-1} - \Omega_{2i+p_2}
\end{bmatrix} < 0
\]

(4)

where \( \Omega_{1i+p_1} = -M_{1i+p_1-1} - M_{1i+p_1-1}^T + P_{1i+p_1} \), and \( \Omega_{2i+p_2} = -M_{2i+p_2-1} - M_{2i+p_2-1}^T + P_{2i+p_2} \), and the subscript \( \alpha + \beta \) denotes the scheduling vector being evaluated at time \( T + \alpha T_1 \) or \( T + \alpha T_2 \), depending on which subsystem is active.

Proof: Consider again the switching Lyapunov function

\[ V(x(T), T) = \begin{cases} x(T)^T P_{1i} x(T) & \text{active subsystem was 1} \\ x(T)^T P_{2i} x(T) & \text{otherwise} \end{cases} \]

defined only in the time instants when the system dynamics switches from one subsystem to another.

Then the difference in the Lyapunov function is either

\[
V(x(T+p_2 T_2), T+p_2 T_2) - V(x(T), T) = \begin{bmatrix} x(T) \\ x(T+p_2 T_2) \end{bmatrix}^T \begin{pmatrix} -P_{1i} & 0 \\ 0 & P_{2i+p_2} \end{pmatrix} \begin{bmatrix} x(T) \\ x(T+p_2 T_2) \end{bmatrix}
\]

if the switching is from the first subsystem to the second one (Case 1), or

\[
V(x(T+p_1 T_1), T+p_1 T_1) - V(x(T), T) = \begin{bmatrix} x(T) \\ x(T+p_1 T_1) \end{bmatrix}^T \begin{pmatrix} -P_{2i} & 0 \\ 0 & P_{1i+p_1} \end{pmatrix} \begin{bmatrix} x(T) \\ x(T+p_1 T_1) \end{bmatrix}
\]

if the switching is from the second subsystem to the first one (Case 2).

Consider first Case 1. We have

\[
\Delta V = \begin{bmatrix} x(T) \\ x(T+p_2 T_2) \end{bmatrix}^T \begin{pmatrix} -P_{1i} & 0 \\ 0 & P_{2i+p_2} \end{pmatrix} \begin{bmatrix} x(T) \\ x(T+p_2 T_2) \end{bmatrix}
\]

together with the system dynamics, which is

\[
\begin{bmatrix} A_{2i} & -I & 0 & \cdots & 0 \\ 0 & A_{2i+1} & -I & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & -I \\ \end{bmatrix} \begin{bmatrix} x(T) \\ x(T+p_2 T_2) \end{bmatrix} = 0
\]

since during the time \( [T, T+p_2 T_2] \), the second subsystem is active. Using Lemma 1, the difference in the Lyapunov function is negative definite, if there exists \( H \) such that

\[
\begin{bmatrix} -P_{1i} & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & P_{2i+p_2} \end{bmatrix} + H \begin{bmatrix} A_{2i} & -I & 0 & \cdots & 0 \\ 0 & A_{2i+1} & -I & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & -I \\ \end{bmatrix} \prec 0
\]

Choosing

\[
H = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ M_{2i} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & M_{2i+p_2-1} \end{pmatrix}
\]

leads directly to

\[
\begin{bmatrix} -P_{1i} & (\ast) \\
M_{2i} - M_{2i} - M_{2i}^T & \cdots & (\ast) \\
\vdots & \vdots & \vdots \\
0 & 0 & M_{2i+p_2-1} A_{2i+p_2-1} - \Omega_{2i+p_2}
\end{bmatrix} < 0
\]

For Case 2, we have the dynamics

\[
\begin{pmatrix} A_{1i} & -I & 0 & \cdots & 0 \\ 0 & A_{1i+1} & -I & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & -I \\ \end{pmatrix} \begin{bmatrix} x(T) \\ x(T+p_1 T_1) \end{bmatrix} = 0
\]

and by choosing

\[
H = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ M_{1i} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & M_{1i+p_1-1} \end{pmatrix}
\]

we obtain

\[
\begin{bmatrix} -P_{2i} & (\ast) \\
M_{1i} - M_{1i} - M_{1i}^T & \cdots & (\ast) \\
\vdots & \vdots & \vdots \\
0 & 0 & M_{1i+p_1-1} A_{1i+p_1-1} - \Omega_{1i+p_1}
\end{bmatrix} < 0
\]

\[ \Box \]

Remarks:

1. The conditions of Theorems 1 and 2 can easily be transformed into LMIs, and the relaxations of (Wang et al., 1996) or (Tuan et al., 2001) can be used.

2. When developing the conditions we exploited the fact that the subsystems switch in finite time. If the system can remain in one mode (i.e., one of the subsystems can be continuously active), the developed conditions are not sufficient to guarantee stability of the whole system. However, similar conditions can be derived.

3. Also due to the fact that the subsystems switch in finite time, it is not necessary that the local models of the TS subsystems or even the subsystems themselves to be stable, as it will be illustrated in the next section. Indeed, neither the conditions of Theorem 1, nor those of Theorem 2 require the subsystems to be stable.

4. Although the conditions of Theorems 1 and 2 concern only two subsystems, they can be easily extended to a fixed number of subsystems, leading to the conditions: the periodic system with \( n \) subsystems, each having period \( p_i \), is asymptotically stable, if there exist \( P_{ij} = P_{ij}^T > 0 \), \( M_{ij} \), \( i = 1, 2, \ldots, n \), \( j = 1, 2, \ldots, r_i \), so that
Consider the switching fuzzy system with two subsystems.

In this section we illustrate the use of the developed approach, we consider the switching fuzzy system with two subsystems as follows:

\[
\begin{pmatrix}
-P_{i2} & 0 & \ldots & 0 \\
\Gamma_{i0} & -M_{i+12} - M_{i+12}^T & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & \Gamma_{i,p_i+1} \Omega_{i+12+p_i+1}
\end{pmatrix} < 0
\]

with \( \Gamma_{ij} = M_{i+12+j-1}A_{i+12+j-1}, i = 1, 2, \ldots, n, \ j = 0, 1, \ldots, p_i+1 \), \( \Omega_{i+12+p_i+1} = -M_{i+12+p_i+1} - M_{i+12+p_i+1}^T + P_{i+p_i+1} \) and the \( n+1 \)th subsystem by definition being the first one.

4. EXAMPLES

In this section we illustrate the use of the developed conditions on numerical examples.

Consider the switching fuzzy system with two subsystems as follows:

\[
x(T + T_1) = \sum_{i=1}^{2} h_{i1}(z_1(T)) A_{i1} x(T)
\]

with

\[
A_{11} = \begin{pmatrix} -0.16 & -0.1 \\ 0.4 & 0.7 \end{pmatrix}, \quad A_{12} = \begin{pmatrix} -1.1 & -0.15 \\ -0.67 & 0.24 \end{pmatrix}
\]

with \( T_1 = 1, h_{11} \) randomly generated in \([0, 1], h_{12} = 1 - h_{11}\) and

\[
x(T + T_2) = \sum_{i=1}^{2} h_{i2}(z_1(T)) A_{i2} x(T)
\]

with

\[
A_{21} = \begin{pmatrix} 0.5 & 0.6 \\ 0.5 & 0.67 \end{pmatrix}, \quad A_{22} = \begin{pmatrix} 0.4 & -0.4 \\ 0.16 & 0.36 \end{pmatrix}
\]

with \( T_2 = 3, h_{21} = \cos(x_1)^2, h_{22} = 1 - h_{21} \).

Note that both \( A_{12} \) and \( A_{21} \) are unstable, their eigenvalues being \((-1.1712, 0.3112)\) and \((-0.0307, 1.1393)\), respectively. The stability of the first subsystem depends on the exact values of the membership functions, while the second subsystem is locally stable, but not locally asymptotically stable (see Figure 1). Moreover, since the local models are unstable, this means that existing methods from the literature cannot be applied to prove the stability of this switching system.

However, by switching between the two subsystems at every time step, the whole system is asymptotically stable, as illustrated in Figure 2.

Note that for this switching system it is not possible to find either a quadratic or a nonquadratic Lyapunov function, as the corresponding LMI become unfeasible.

According to the proposed approach, we consider the switching Lyapunov function

\[
V(x(T), T) = \begin{cases} x(T)^T P_{12} x(T) & \text{active subsystem was 1} \\ x(T)^T P_{22} x(T) & \text{otherwise} \end{cases}
\]

with \( P_{12} = h_{11}(T) P_{11} + h_{12}(T) P_{12}, P_{22} = h_{21}(T) P_{21} + h_{22}(T) P_{22} \).

The LMI corresponding to the conditions of Theorem 1, when using the relaxation of Wang et al. (1996) are

\[
\begin{align*}
\Gamma_{ik} &< 0 \\
\Gamma_{ijk} + \Gamma_{ijk} &< 0
\end{align*}
\]

where

\[
\Gamma_{ijk} = \begin{pmatrix} -P_{i1} & \ast \\ -M_{2i} A_{2j} - M_{2i}^T & P_{2k} \end{pmatrix}
\]

or

\[
\Gamma_{ijk} = \begin{pmatrix} -P_{i2} & \ast \\ -M_{1i} A_{1j} - M_{1i}^T & P_{1k} \end{pmatrix}
\]

By solving the conditions (6), we obtain

\[
\begin{align*}
P_{11} &= \begin{pmatrix} 4.4614 & 2.3067 \\ 2.3067 & 7.1567 \end{pmatrix} \\
P_{22} &= \begin{pmatrix} 5.0813 & -0.6803 \\ -0.6803 & 6.0733 \end{pmatrix} \\
P_{21} &= \begin{pmatrix} 6.6673 & -0.7481 \\ 1.2718 & 6.9281 \end{pmatrix} \\
P_{22} &= \begin{pmatrix} -1.8734 & 6.5859 \\ -1.8734 & 6.9846 \end{pmatrix} \\
P_{22} &= \begin{pmatrix} 13.3010 & -2.7835 \\ -2.7835 & 4.3840 \end{pmatrix} \\
P_{22} &= \begin{pmatrix} -8.0200 & -3.7586 \\ -6.0206 & 6.1986 \end{pmatrix} \\
\end{align*}
\]

and thereby prove the stability of the switching system.

To illustrate the conditions of Theorem 2, consider the periodic fuzzy system with two subsystems as follows:

\[
x(T + T_1) = \sum_{i=1}^{2} h_{i1}(z_1(T)) A_{i1} x(T)
\]

with

\[
A_{11} = \begin{pmatrix} -0.44 & -0.26 \\ -0.65 & 0.62 \end{pmatrix}, \quad A_{12} = \begin{pmatrix} 1.1 & -0.2 \\ 0.53 & -0.27 \end{pmatrix}
\]

\[\text{For solving LMI, the feasp function of Matlab has been used, with the default options.}\]
with $T_1 = 1$, $h_{11}$ randomly generated in $[0, 1]$, $h_{12} = 1 - h_{11}$ and

$$x(T + T_2) = \sum_{i=1}^{2} h_{2i}(z_1(T)) A_{2i} x(T)$$

with

$$A_{21} = \begin{pmatrix} 0.02 & 0.6 \\ -0.22 & -0.44 \end{pmatrix}, \quad A_{22} = \begin{pmatrix} 0.32 & -0.15 \\ -1 & 0.8 \end{pmatrix}$$

with $T_2 = 3$, $h_{21} = \cos(x_1)^2$, $h_{22} = 1 - h_{21}$.

The local models $A_{12}$ and $A_{22}$ are unstable, their eigenvalues being $(1.01177 -0.1877)$ and $(0.1044 1.0156)$, respectively. Again, this means that existing results from the literature cannot be applied. By switching between the two subsystems with a period $p_1 = 2$ for the first subsystem and $p_2 = 3$ for the second subsystem, the resulting switching system is asymptotically stable, as illustrated in Figure 3.

For this switching system it is not possible to find either a quadratic or a nonquadratic Lyapunov function, as the corresponding LMIs become unfeasible.

The conditions of Theorem 2 concerning the first subsystem are (the LMIs are obtained similarly for the second subsystem)

$$
\begin{pmatrix}
-P_{22} & (*) \\
(M_{12} A_{12} - M_{12} - M_{12}^T) & (*) \\
0 & (M_{12} A_{12} - M_{12} - M_{12}^T + P_{22})
\end{pmatrix} < 0
$$

that is,

$$
\sum_{i,j,k,l,m=1}^{2} \begin{pmatrix}
-P_{2i} & (*) \\
(M_{1j} A_{1j} - M_{1j} - M_{1j}^T) & (*) \\
0 & (M_{1j} A_{1j} - M_{1j} - M_{1j}^T + P_{2m})
\end{pmatrix} < 0
$$

for which relaxations such as those in (Wang et al., 1996) or (Tuan et al., 2001) can be used. In this paper, we used the relaxations of (Wang et al., 1996), and obtained

$$
P_{11} = P_{12} = \begin{pmatrix} 0.1984 & -0.0815 \\
-0.0815 & 0.2034 \end{pmatrix}
$$

$$
M_{11} = M_{12} = \begin{pmatrix} 0.1973 & -0.0711 \\
-0.0537 & 0.1662 \end{pmatrix}
$$

$$
P_{21} = P_{22} = \begin{pmatrix} 0.2395 & 0.0035 \\
0.0035 & 0.0867 \end{pmatrix}
$$

$$
M_{21} = M_{22} = \begin{pmatrix} 0.2215 & 0.0139 \\
-0.0153 & 0.1214 \end{pmatrix}
$$

Consequently, the periodic TS model is asymptotically stable.

5. CONCLUSIONS

In this paper we have developed conditions for the stability of periodic TS systems. For this, we used a switching Lyapunov function, defined in the points where the subsystems themselves switch. The conditions can guarantee the stability of periodic systems even if the local models or even the subsystems are unstable.

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