Particle Filters for Estimating Average Grain Diameter of Material Excavated by Hopper Dredger

Paweł Stano, Zsófia Lendek, Robert Babuška, Jelmer Braaksma and Cees de Keizer

***Abstract*— Hopper dredgers are massive ships that excavate sediments from the sea bottom while sailing. The excavated material is then transported and discharged at a specified location. The efficiency of this process is highly dependent on the detailed knowledge of the excavated soil. When the soil is composed mainly of sand, the parameter of the greatest importance is the average grain diameter. This, however cannot be directly measured by available sensors. Therefore, in this paper a particle filter is proposed to estimate the average grain diameter. The estimation is based on online measurements of the total height of the mixture in the hopper, total mass, the incoming mixture density and flow-rate and the height of a sand bed, together with estimates of the outgoing mixture density and flow-rate. The loading process is naturally decomposed into three phases and the filter is applied to the first two phases. In order to match different types of nonlinearities, a separate observer is proposed for each phase under consideration. This increases the modularity of the filter and makes tuning easier. The performance of the filter is evaluated in simulations and the results are encouraging.**

I. INTRODUCTION

The optimization of dredging operations is of vital importance for future improvement in efficiency, accuracy and from the viewpoint of labor saving. While modern hopper dredgers are equipped with advanced dynamic positioning and tracking systems, no on-board decision-support systems are yet available to optimize the dredging performance under given operating conditions (type of soil, dredging depth, water current, etc.). The manipulated variables must constantly be adjusted by two operators: the ship navigator and the dredge process operator. Consequently, the performance and efficiency of the entire process heavily depend on their insight and experience.

IHC Systems, a company specialized in the development and manufacturing of automation systems for dredgers, currently cooperates with the Delft Center for Systems and Control on the development of an adaptive decision-support system for hopper dredgers to advise the operators on the most suitable control strategy, given a specified performance goal. This can be, for instance, the minimization of the integral dredging costs per m^3 of sand or the maximization of the production per time unit. To this end, a control-oriented dynamic model of the hopper dredger has been developed and calibrated by using recorded process data. Based on this model, a suitable control strategy has been derived by using *Model-Predictive Control* (MPC) approach [1]. However, as only some of the state variables are measured by sensors, the use of on-line state estimation techniques is essential for an on-board application of this system.

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One of the variables that cannot be measured, but is required by the MPC controller, is the average grain diameter of the in situ material. This soil-dependent parameter is needed for the optimization of the separation process in the hopper. Therefore, the research presented in this paper addresses the estimation of this variable. For the estimation, the dynamical system describing the settling of sand in the hopper is used. This model is both nonlinear and non-Gaussian. Therefore, we use a Particle Filter (PF), which is a nonparametric method capable of estimating highly nonlinear and non-Gaussian systems.

The paper is organized as follows. Section II explains the dredging process and presents the dynamic sedimentation model. The estimation problem is stated in Section III. Section IV reviews the particle filtering methodology that is used for estimation. The results are presented in Section V and discussed in Section VI. Section VII concludes the paper.

II. PROBLEM STATEMENT

Trailing Suction Hopper Dredgers (TSHD) have been intensively studied in the recent years. In the literature models of the isolated components of the total system were developed [2], [3], [4], [5], together with the overall model taking into account the interactions between separate subsystems [1]. One of the most important parts of the system is the hopper model, which, among others, describes the sedimentation of the material excavated from the bottom. The sedimentation process has been extensively studied in the literature (see [5] and the reference therein). Existing models are in general very detailed descriptions of the physical phenomena in terms of partial differential equations and they contain a large number of uncertain parameters. For the aforementioned reasons they cannot be applied for control or optimization of the TSHD performance. Therefore, a simplified sedimentation model was proposed in [2] as a basis for the automation of the dredging process. This model contains uncertain parameters dependent only on the in situ soil properties. During the dredging, the ship is constantly sailing, hence the excavated soil can change. Therefore the parameters of the model have to adapt to new conditions. Current results [2], [6], [7] indicate that the sedimentation parameters can be approximated as functions of the average grain diameter of the excavated soil d_m. Furthermore, the knowledge of d_m can be used to estimate the uncertain parameters in other subsystems. Therefore, developing an accurate adaptive estimator of the average grain diameter is of crucial importance for the TSHD automation system.

Dredgers are currently not equipped with sensors that can measure the height of the sand settled at the bottom of the hopper. However, design of such a sensor is an active research area and, under laboratory conditions, the height of the sand bed can be measured. The purpose of this research is to design a dynamic observer for estimating the average grain diameter of the sand excavated by the hopper dredger assuming that the height of the sand bed is measured.

A. LOADING PROCESS

The production process in a TSHD is naturally divided into three separate phases:

1) *The no-overflow phase*.
2) *The constant-volume phase*.
3) *The constant-tonnage phase*.

When the ship arrives at the dredging area, the loading begins. At first (no-overflow phase) all the excavated material is stored in the hopper. When the mixture level reaches a certain height, the second phase begins (constant-volume phase). During this stage the excess water (or a low density mixture) is being discharged overboard to keep the volume V_t of the stored material constant. As a result the density of the remaining mixture increases and therefore the total mass m_t of the material in the hopper also increases. The last loading phase begins after the maximum allowed mass in the hopper (determined by the maximum draught of the ship) has been reached. In order to prevent the ship from sinking a constant-tonnage controller is used. When necessary, the controller lowers the overflow height hence more mixture is disposed through the overflow pipe.

During this third phase the overflow losses increase up to the point when it is no longer economically efficient to continue dredging, at which point the loading stops.

B. DYNAMIC SEDIMENTATION MODEL

The sedimentation process in the hopper is described by a dynamic model with three state variables: the total mass in the hopper m_t, the total volume of the mixture in the hopper V_t and the mass of the sand bed m_s (see Figure 1). The dynamics are given by the following ordinary differential equations:

$$\dot{V}_t = Q_i - Q_o, \tag{1}$$
$$\dot{m}_t = Q_i\rho_i - Q_o\rho_o, \tag{2}$$
$$\dot{m}_s = F(d_m, m_s, h_t, h_s, Q_o, m_t, V_t). \tag{3}$$

Equations (1) and (2) represent the volume and mass balance, respectively. The two controllable input variables are the incoming flow rate Q_i and the density of the incoming mixture ρ_i. The overflow rate Q_o and the overflow density ρ_o are output variables which cannot be directly measured due to the lack of appropriate sensors in the overflow system and are in general estimated.

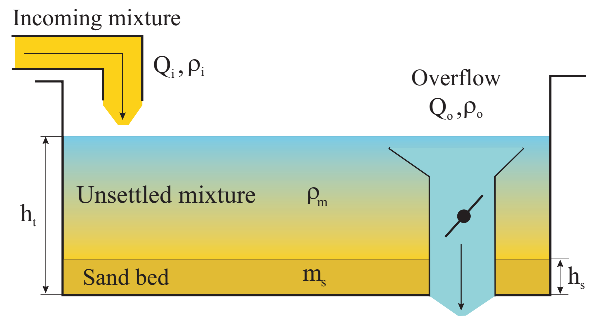

Fig. 1. Cross section of the hopper with variables that are used to describe the sedimentation process.

In the literature, a number of models of the overflow rate Q_o and the overflow density ρ_o have been proposed [2]. Unfortunately, those models contain too many uncertain parameters which lead to rather inaccurate approximations of the desired signals, when compared with the measured data. Therefore, a cascaded observer of the overflow rate Q_o and the overflow density ρ_o has been developed in [8] in order to obtain online estimates of both signals. Thus, in this paper, the two aforementioned variables are regarded as measured inputs of the system.

Equation (3) describes the sedimentation rate of the excavated material as a function of the average grain diameter d_m, sand bed mass m_s, the total height of the mixture in the hopper h_t, sand bed height h_s, the overflow rate Q_o, the total mass in the hopper m_t and the total volume of the mixture in the hopper V_t [2].

The nonlinear function F is factorized into two components

$$F = f_e(d_m, h_t, h_s, Q_o) f_s(d_m, m_s, m_t, V_t), \tag{4}$$

where each factor describes a different physical phenomenon. The scouring function f_e models the erosion of the already settled material as:

$$f_e(d_m, h_t, h_s, Q_o) = \max\left(1 - \frac{Q_o^2}{(k_e(d_m)(h_t - h_s))^2}, 0\right), \tag{5}$$

where the erosion pickup flux coefficient k_e is a soil dependent parameter expressed as a function of the mean grain diameter d_m of the in situ material.

The settling function f_s describes the process of settling of the sand particles suspended in the mixture above the sand bed. The sedimentation rate depends on the density of the mixture in the hopper. It has been experimentally shown [2] that above the sand bed, the mixture of water and sand that are being discharged into the hopper form a uniformly dense soup with a thin layer of water at the top. Thus, the density of the mixture can be approximated by the average density of the mixture ρ_m, given by [2]:

$$\rho_m = \frac{m_t - m_s}{V_t - m_s/\rho_s(d_m)} = \frac{\rho_s(d_m)(m_t - m_s)}{V_t\rho_s(d_m) - m_s}. \tag{6}$$

The sand bed density in the hopper ρ_s is another soil type dependent parameter and can also be derived as a function of the average grain diameter d_m. There are two more such parameters that play an important role in describing the sedimentation process: the undisturbed settling velocity of a single particle v_{s0} and the exponent β in the settling equation based on the particle Reynolds number [9]. Finally, the rate of sedimentation in a hopper of the form of a rectangular parallelepiped with a base area A $\left[\mathrm{m}^2\right]$ is given by:

$$f_s(d_m, m_s, m_t, V_t) = A\rho_s(d_m)v_{s0}(d_m)\frac{\rho_m - \rho_w}{\rho_s(d_m) - \rho_m}\left(\frac{\rho_q - \rho_m}{\rho_q - \rho_w}\right)^{\beta(d_m)}, \quad (7)$$

where ρ_w is the density of water (1024 kg/ m^3) and ρ_q is the density of quartz (approximately 2650 kg/ m^3).
Note that (3) can alternatively be written as

$$\dot{m}_s = A\rho_s(d_m)\frac{d}{dt}h_s, \quad (8)$$

from which it is straightforward to obtain the formula for the sand bed height h_s growth rate:

$$\frac{d}{dt}h_s = \frac{F(d_m, m_s, h_t, h_s, Q_o, m_t, V_t)}{A\rho_s(d_m)}. \quad (9)$$

Since the erosion of the sediment takes place only after the overflow begins (see (5)), there is a significant difference in sedimentation dynamics between the first and the second loading phase. The switching character of the process makes it difficult to design a single observer of the average grain diameter d_m for both phases. Instead, each loading stage is analyzed separately and an appropriate estimation technique is chosen with regard to the phase under consideration.

III. ESTIMATION PROBLEM

As explained in the previous section, due to the changes in sedimentation dynamics, different loading phases require different estimation methods. In this paper we propose filtering algorithms for the first two phases. The design of an observer for the constant-tonnage stage will be the subject of further research.

According to (1) and (2) the evolution of both total mass m_t and total volume V_t is determined by the incoming flow rate Q_i and the incoming density ρ_i (no-overflow period) together with the overflow flow rate Q_o and the overflow density ρ_o (constant-volume phase). The first two signals are available from the measurements and the last two are estimated [8]. Therefore, in both cases we can treat m_t and V_t as the inputs of the model. The outputs are: the total height of the mixture in the hopper h_t and the height of the sand bed h_s, both assumed to be corrupted by the zero-mean, time-invariant Gaussian noises e_t^o and e_s^o respectively. Since measurements are taken with a sampling period T_s, a discrete-time model is derived and used for estimation. The derivative of h_s at time step k is approximated by applying the Euler method:

$$\frac{d}{dt}h_{s,k} \approx \frac{h_{s,k} - h_{s,k-1}}{T_s} + e_{approx}, \quad (10)$$

where the last term is an approximation error (time invariant, zero-mean Gaussian) and T_s is the time to elapse between two measurements.

The dynamics of m_s are obtained by the discretization of (9) by using the Euler method and applying (10) afterwards:

$$\begin{aligned} m_{s,k+1} &= m_{s,k} + A\rho_s(d_{m,k})\left(h_{s,k} - h_{s,k-1}\right) \\ &+ T_sA\rho_s(d_{m,k})e_{approx} + e_m. \end{aligned} \quad (11)$$

Combining (9) and (10) gives:

$$h_{s,k+1} = h_{s,k} + T_se_{approx} + T_sG(d_{m,k}, h_{t,k}, h_{s,k}, \hat{Q}_{o,k}, \rho_m), \quad (12)$$

where G is the function defined by the right hand side of (9). During the first loading phase the variable Q_o is equal to zero, which simplifies the dynamics by setting (5) to one. After the overflow starts the erosion effect must be included in (12). Field data suggest that the mixture density ρ_m is equal to the overflow density ρ_o. Therefore, during the constant-volume phase an estimate of $\hat{\rho}_o$ is used in (12) instead of ρ_m. The state equations are augmented with a random-walk model for d_m:

$$d_{m,k+1} = d_{m,k} + e_d. \quad (13)$$

The e_m and e_d in (11) and (13) respectively are model uncertainties.

In the standard state-space approach, the outputs of the system are expressed as a (possibly nonlinear) function of states and noises. However, since it is too difficult to solve (9) in h_t, we propose to consider it as another input variable and include the noise e_t^o into the state equation (12). Then the augmented state, input and output vectors of the system are given by:

$$\mathbf{x} = \begin{pmatrix} m_s \\ h_s \\ d_m \end{pmatrix}, \quad \mathbf{u} = \begin{pmatrix} \hat{Q}_o \\ \hat{\rho}_o \\ h_t \end{pmatrix}, \quad y = h_s + e_s^o.$$

The final form of the state-space model in these variables is expressed as:

$$\begin{aligned} x_{1,k+1} &= x_{1,k} + A\rho_s(x_{3,k})\left(x_{2,k} - x_{2,k-1}\right) \\ &+ T_sA\rho_s(x_{3,k})e_{approx} + e_m, & (14) \\ x_{2,k+1} &= x_{2,k} + T_se_{approx} + T_sG(\mathbf{x}_k, \mathbf{u}_k, e_t^o), & (15) \\ x_{3,k+1} &= x_{3,k} + e_d, & (16) \\ y_k &= x_{2,k} + e_s^o. & (17) \end{aligned}$$

It is assumed that errors e_{approx} and e_m are independent and zero-mean Gaussians with variances σ^2_{approx} and σ^2_m respectively. Consequently, the random variable $T_sA\rho_s(x_{3,k})e_{approx} + e_m$ is also a zero-mean Gaussian with the variance $\sigma^2_{x_1}$ given by

$$\sigma^2_{x_1} = \left(T_sA\rho_s(x_{3,k})\sigma_{approx}\right)^2 + \sigma^2_m. \quad (18)$$

The random variable e_t^o influences only the erosion part f_e of the variable G. Therefore, since during the first phase f_e is constant, $G(\mathbf{x}_k, \mathbf{u}_k)$ in (15) becomes a deterministic function of state and observation. Thus, as long as there is no overflow ($Q_o = 0$), the variable $T_se_{approx} + T_sG(\mathbf{x}_k, \mathbf{u}_k)$

is normally distributed with mean μ_{x_2} and standard deviation σ_{x_2} given by:

$$\mu_{x_2} = T_s v_{s0}(x_{3,k}) \frac{\rho_{m,k} - \rho_w}{\rho_s(x_{3,k}) - \rho_{m,k}} \left(\frac{\rho_q - \rho_{m,k}}{\rho_q - \rho_w} \right)^{\beta(x_{3,k})}, \tag{19}$$

$$\sigma_{x_2} = T_s \sigma_{approx}. \tag{20}$$

The probabilistic model (15) for the constant-volume phase is more involved. The error $T_s e_{approx}$ and the random variable $T_s G(\mathbf{x}_k, \mathbf{u}_k, e_t^o)$ are independent, but the latter is not Gaussian. Thus, the probability density function (PDF) p_{x_2} of a sum of these variables is a convolution of their PDFs ($p_{x_2,e}$ and $p_{x_2,G}$ respectively):

$$p_{x_2}(y) = \int p_{x_2,e}(y - z) p_{x_2,G}(z) dz. \tag{21}$$

The PDF of the normally distributed variable $T_s e_{approx}$ is known. In order to derive the PDF of $T_s G(\mathbf{x}_k, \mathbf{u}_k, e_t^o)$ we use the following proposition:

Proposition 1: If X is a normally distributed random variable with mean μ and variance σ^2 and C_1, C_2 are certain positive constants then the probability density function of the variable $C_1 \max\left(0, 1 - \frac{C_2}{X^2}\right)$ is given by

$$p_{C_1 \max\left(0,1-\frac{C_2}{X^2}\right)}(x) = \sqrt{\frac{C_1 C_2}{2\pi\sigma^2 (C_1 - x)^3}} e^{-\frac{\frac{C_1 C_2}{C_1 - x} + \mu^2}{2\sigma^2}} \cosh\left(\frac{\mu\sqrt{C_1 C_2}}{\sigma^2 \sqrt{C_1 - x}}\right) \mathbf{1}_{(0,C_1]}(x) + \left(1 - \int_0^1 \sqrt{\frac{C_2}{2\pi\sigma^2 y^3}} e^{-\frac{\frac{C_2}{y} + \mu^2}{2\sigma^2}} \cosh\left(\frac{\mu\sqrt{C_2}}{\sigma^2\sqrt{y}}\right) dy\right) \delta_0(x), \tag{22}$$

where $\mathbf{1}_{(0,C_1]}$ is an indicator function and δ_0 is the Dirac delta.

Due to space constraints, the proof is skipped. At each time step k we define

$$X_k = k_e(d_{m,k}) \left(e_t^o + h_{t,k} - h_{s,k}\right), \tag{23}$$

$$C_{1,k} = T_s v_{s0}(d_{m,k}) \frac{\hat{\rho}_{o,k} - \rho_w}{\rho_s(d_{m,k}) - \hat{\rho}_{o,k}} \left(\frac{\rho_q - \hat{\rho}_{o,k}}{\rho_q - \rho_w} \right)^{\beta(d_{m,k})}, \tag{24}$$

$$C_{2,k} = \hat{Q}_{o,k}^2. \tag{25}$$

Such defined X_k is a normally distributed random variable with mean μ_k and variance σ_k^2 given by:

$$\mu_k = k_e(d_{m,k}) \left(h_{t,k} - h_{s,k}\right), \tag{26}$$

$$\sigma_k^2 = k_e(d_{m,k})^2 \sigma_t^2, \tag{27}$$

where σ_t^2 is a variance of the observation noise e_t^o. Furthermore, both $C_{1,k}$ and $C_{2,k}$ defined above are positive, and, therefore, the PDF of $T_s G(\mathbf{x}_k, \mathbf{u}_k, e_t^o)$ follows by applying Proposition 1.

From the description it can be seen that the system exhibits severe nonlinearities together with non-Gaussian probabilistic behavior. Due to this, standard parametric filters cannot be applied. Therefore we use a PF which is briefly described in the next section.

IV. PARTICLE FILTERS

The PF uses a probabilistic model which is based on equations (14) - (17) and specifies the probability density functions (PDF) for the state transition function and the measurement function, respectively:

$$p(x_k | x_{k-1}), \quad p(y_k | x_k).$$

The objective is to recursively construct the posterior PDF $p(x_k|y_k)$ of the state, given the measured output. The PF works in two stages:

1) The *prediction stage* uses the state-transition model (14) - (16) to predict the state PDF one step ahead. The PDF obtained is called the *prior*.
2) The *update stage* uses the latest measurement to correct the prior via the Bayes rule. The PDF obtained after the update is called the *posterior* PDF.

Particle filters represent the PDF by N random samples (particles) x^i with their associated weights w^i, normalized so that $\sum_{i=1}^N w^i = 1$. At time instant k, the prior PDF $p(x_{k-1}|y_{k-1})$ is represented by N samples x_{k-1}^i and the corresponding weights w_{k-1}^i. To approximate the posterior $p(x_k|y_k)$, new samples x_k^i and weights w_k^i are generated. Samples x_k^i are drawn from a (chosen) *importance density function* $q(x_k^i | x_{k-1}^i, y_k)$, and the weights are updated, using the current measurement y_k

$$\tilde{w}_k^i = w_{k-1}^i \frac{p(y_k | x_k^i)\, p(x_k^i | x_{k-1}^i)}{q(x_k^i | x_{k-1}^i, y_k)} \tag{28}$$

and normalized

$$w_k^i = \frac{\tilde{w}_k^i}{\sum_{j=1}^N \tilde{w}_k^j}.$$

The posterior PDF is represented by the set of weighted samples, conventionally denoted by:

$$p(x_k | y_k) \approx \sum_{i=1}^N w_k^i \delta(x_k - x_k^i).$$

Here, we choose the importance density $q(x_k | x_{k-1}, y_k)$ equal to the state-transition PDF $p(x_k | x_{k-1})$. The weight update equation (28) then becomes:

$$\tilde{w}_k^i = w_{k-1}^i p(y_k | x_k^i).$$

The PF algorithm is summarized in Algorithm 1.

A common problem of PF is the particle degeneracy: after several iterations, all but one particle will have negligible weights. Therefore, particles must be resampled. A standard measure of the degeneracy is the effective sample size, computed by:

$$N_{\text{eff}} = \frac{1}{\sum_{i=1}^N (w_k^i)^2}$$

If N_{eff} drops below a specified threshold $N_{\text{T}} \in [1, N]$, particles are resampled by using Algorithm 2.

Algorithm 1 Particle filter

Require: $p(x_k|x_{k-1})$, $p(y_k|x_k)$, $p(x_0)$, N, N_T
Initialize:
for $i = 1, 2, \ldots, N$ **do**
Draw a new particle: $x_1^i \sim p(x_0)$
Assign weight: $w_1^i = \frac{1}{N}$
end for
At every time step $k = 2, 3, \ldots$
for $i = 1, 2, \ldots, N$ **do**
Draw particle from importance distribution:
$x_k^i \sim p(x_k^i|x_{k-1}^i)$
Use measured y_k to update the weight:
$\tilde{w}_k^i = w_{k-1}^i p(y_k|x_k^i)$
end for
Normalize weights: $w_k^i = \frac{\tilde{w}_k^i}{\sum_{j=1}^N \tilde{w}_k^j}$
if $\frac{1}{\sum_{i=1}^N (w_k^i)^2} < N_\mathrm{T}$ **then**
Resample using Algorithm 2.
end if

Algorithm 2 Resampling

Require: $\{(x^i, w^i)\}_{i=1}^N$
Ensure: $\{(x_\mathrm{new}^i, w_\mathrm{new}^i)\}_{i=1}^N$
for $i = 1, 2, \ldots, N$ **do**
Compute cumulative sum of weights: $w_c^i = \sum_{j=1}^i w_k^j$
end for
Draw u_1 from $\mathcal{U}(0, \frac{1}{N})$
for $i = 1, 2, \ldots, N$ **do**
Find x^{+i}, the first sample for which $w_c^i \geq u_i$.
Replace particle i: $x_\mathrm{new}^i = x^{+i}$, $w_\mathrm{new}^i = \frac{1}{N}$
$u_{i+1} = u_i + \frac{1}{N}$
end for

The state estimate is in general computed as the weighted mean of the particles:

$$\hat{x}_k = \sum_{i=1}^N w_k^i x_k^i \,.$$

For more details on particle filters, refer to [10], [11], [12].

V. ESTIMATION RESULTS

The PF presented in Section IV is applied to simulated data, obtained with the dynamic sedimentation model described in Section II-B. For simplicity reasons the sampling time T_s is set to one. For illustrative purposes a separate data set is generated for both loading phases. In each case, the value of the average grain diameter d_m is changed twice during the phase. In the real process this corresponds to approaching a dredging area with a different type of the in situ material. To analyze the tracking properties of the filter, each change in d_m is set to be steep (during the real dredging operation those transitions are generally smoother).

The hopper used for the simulations is of a rectangular parallelepiped form with the base area $A = 600[\mathrm{m}^2]$. Furthermore, it is assumed that the excavated soil is sand with the mean grain diameter d_m varying between 0.35[mm] and 0.7[mm]. In such a regime, for the given hopper width, the soil dependent parameters k_e, ρ_s, v_{s0} and β can be approximated by the following functions of d_m [2], [6], [7]:

$$k_e(d_m) = 28.06\sqrt{d_m} - 6.35 \tag{29}$$

$$\rho_s(d_m) = 34.81\sqrt{d_m} + 1926 \tag{30}$$

$$v_{s0}(d_m) = \frac{8.925}{d_m}\left(\sqrt{1 + 95\frac{\rho_q - \rho_w}{\rho_w}d_m^3} - 1\right) \tag{31}$$

$$\beta(d_m) = \frac{4.7 + 0.41\left(-2.289 + 41.53d_m + 118.6d_m^2\right)^{0.75}}{1 + 0.175\left(-2.289 + 41.53d_m + 118.6d_m^2\right)^{0.75}} \tag{32}$$

The results were obtained with $N = 1000$ particles and the threshold N_T for an effective sample size was experimentally set to 500 (i.e., 50% of the number of particles N). The standard deviations of the variables defined in Section II-B are as follows: $\sigma_m = 1000$, $\sigma_d = 0.1$, $\sigma_t^o = 0.1$, $\sigma_s^o = 0.05$ and $\sigma_{approx} = 0.001$.
In the prediction stage of the PF designed for the constant-volume phase, it is required, among others, to sample from the random variable defined in (15). However, drawing from the PDF given by (21) - (27) is in general not straightforward. Therefore, for each particle $x_{2,k-1}^i$ an approximation of a true random sample is generated by Algorithm 3.

Algorithm 3 Approximate sampling

Require: $x_{2,k-1}$, μ_{k-1}, σ_{k-1}, $C_{1,k-1}$, $C_{2,k-1}$, T_s, σ_{approx}
for $i = 1, 2, \ldots, N$ **do**
Draw two independent samples:
$x_1^i \sim \mathcal{N}\left(x_{2,k-1}^i, T_s\sigma_{approx}\right)$
$x_2^i \sim \mathcal{N}\left(\mu_{k-1}^i, \sigma_{k-1}^i\right)$
Perform nonlinear transformation:
$\tilde{x}_2^i = T_s C_{1,k-1}^i \max\left(0, 1 - C_{2,k-1}^i/x_2^i\right)$
Assign approximated sample:
$x_{2,k}^i = x_1^i + \tilde{x}_2^i$
end for

Comparison of the estimated and simulated signals for the no-overflow phase is presented in Figure 2, while Figure 3 shows the aforementioned signals for the constant-volume phase.

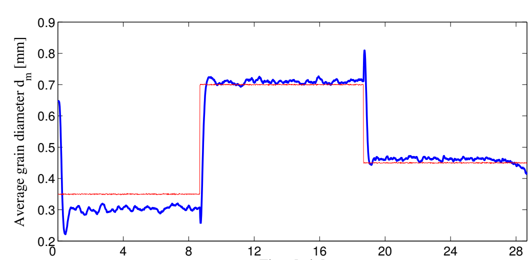

Fig. 2. Simulation results for the no-overflow phase (thin line: simulated variable, thick line: variable estimated by the particle filter).

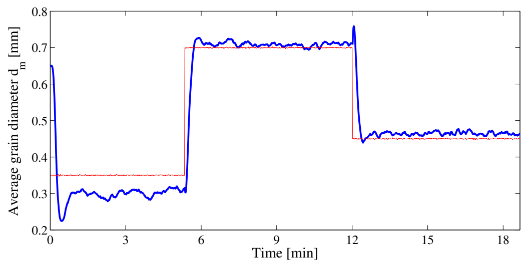

Fig. 3. Simulation results for the constant-volume phase (thin line: simulated variable, thick line: variable estimated by the particle filter).

VI. PERFORMANCE EVALUATION

From Figures 2 and 3 it can be observed that the PF works very well and that its performance in the two loading phases is comparable. In each case the convergence times are approximately $40[\text{s}]$, $20[\text{s}]$ and $20[\text{s}]$ respectively, which in each case is acceptable for the TSHD's controller developed in [1]. Moreover, as has been stated before, in real dredging operations the changes in average grain diameter are smoother and therefore the filter converges much faster. For both phases the estimate is slightly biased. This is due to the difference between the models used to generate data and to estimate the d_m. However, the absolute estimation errors are smaller than the standard deviation σ_d of the estimated variable d_m which make the errors acceptable. In fact for higher values of d_m the bias is negligible. This is confirmed by analyzing the *residuals* i.e. the difference between estimated and simulated values after the filter reaches the steady state. In case of tracking the d_m equal to $0.35[\text{mm}]$ the signal is underestimated. The average grain diameter residual has a mean of $-0.049[\text{mm}]$ and $-0.047[\text{mm}]$ and a standard deviation of $0.0095[\text{mm}]$ and $0.0077[\text{mm}]$ for the no-overflow and constant-volume phase respectively. In each loading phases the average bias is approximately 14% which is tolerable during the actual dredging. When the true signal is set to $0.7[\text{mm}]$ and $0.45[\text{mm}]$ the observer overestimates the reference signal. In the former case the residual has a mean of $0.01[\text{mm}]$ and $0.009[\text{mm}]$ and standard deviation of $0.0064[\text{mm}]$ and $0.0066[\text{mm}]$ for the no-overflow and constant-volume phase respectively. In the latter case the residual has a mean of $0.014[\text{mm}]$ and $0.009[\text{mm}]$ and standard deviation of $0.0057[\text{mm}]$ and $0.009[\text{mm}]$ for the no-overflow and constant-volume phase respectively. In these cases, the average bias is less than 3% for both loading phases, which is an excellent result from a practical point of view.

On board of the real TSHD the measurements are taken every $5[\text{s}]$, and hence this is the time in which the MPC controller [1] has to compute the optimal action. The proposed PF computes the estimate in less than one second[1], thus the filter can be integrated into the aforementioned controller on board of the commercial dredges.

[1]The algorithm was executed in Matlab 7 on a PC with an Intel Core 2 Duo E6550 2.33 GHz CPU with 3 GB RAM.

When the sand bed height h_s approaches the total height of the mixture h_t, the performance of the filter decreases, which can be observed in Figure 3 (last minutes of the phase). A possible explanation is that Algorithm 3 fails to approximate the true PDF (22) for the low values of the denominator in (5) . Also, an estimator of d_m for the third loading phase has not been designed yet. These problems will be addressed in our future research.

VII. CONCLUSIONS

A particle filter has been applied to the estimation of the average grain diameter of the material excavated by a hopper dredger. The soil-type dependent variable is estimated on the basis of the measured total hopper volume, hopper mass, incoming mixture density and flow-rate and the height of the sand bed in the hopper. The performance has been evaluated in simulations of two loading phases. During the constant-volume phase the estimate of the overflow losses is also used by the observer. The results are encouraging, therefore the next step in our research will be the testing of the filter on measured data. Furthermore, theoretical research has to be done for reducing the estimation bias. Eventually, after the filter is fully developed it will be integrated in an automatic control system for the future use on board of the hopper dredger.

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REFERENCES

[1] Braaksma, J., Klaassens, J. B., Babuška R., de Keizer, C., "Model Predictive Control for optimizing the overall dredging performance of a trailing suction hopper dredger", *Proceedings of the Eighteenth World Dredging Congress (WODCON XVIII)*, 2007, pp. 1263-1274.
[2] Braaksma, J., Klaassens, J. B., Babuška R., de Keizer, C., "A computationally efficient model for predicting overflow mixture density in a hopper dredger", *Terra et Aqua*, vol. 106, 2007, pp. 16-25.
[3] Matoušek, V.,"Pressure drops and flow patterns in sand-mixture pipes", *Experimental Thermal and Fluid Science*, vol. 26, 2002, pp. 693-702.
[4] Miedema, S. A., "The cutting of densely compacted sand under water", *Terra et Aqua*, vol. 28, 1984, pp. 4-10.
[5] van Rhee, C., "Modelling the sedimentation process in a trailing suction hopper dredger", *Terra et Aqua*, vol. 86, 2002, pp. 18-27.
[6] Vlasblom, W. J. and Miedema, S. A., "A theory for determining sedimentation and overflow losses in hoppers", *Proc. of the 14th World Dredging Congress*, Amsterdam 1995, pp. 183-202.
[7] Rowe, P. N., "A convenient empirical equation for estimation of the Richardson-Zaki exponent", *Chemical Engineering Science* vol. 43(11), 1987, pp. 2795-2796.
[8] Lendek, Zs., Babuška R., Braaksma, J., de Keizer C. "Decentralized estimation of overflow losses in a hopper dredger", *Control Engineering Practice*, vol. 16, 2008, pp. 392-406.
[9] Richardson J., Zaki W., "Sedimentation and fluidisation: Part I", *Transactions of the Institution of Chemical Engineers*, vol. 32, 1954, pp. 35-53.
[10] Doucet A., de Freitas N., Gordon N., *Sequential Monte Carlo Methods in Practice*. Springer Verlag, 2001.
[11] Arulampalam S., Maskell N., Gordon N., Clapp T. "A tutorial on particle filters for on-line nonlinear/non-gaussian bayesian tracking", *IEEE Transactions of signal Processing*, vol. 50, no 2, pp. 174-188, 2002.
[12] Ristic B., Arulampalam S., Gordon N. *Beyond the Kalman Filter. Particle Filters for Tracking Applications*. Artech House, Boston, 2004.