

Modeling and parameter estimation for an activated sludge wastewater treatment process

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Abstract—Wastewater is treated in wastewater treatment plants (WWTP) through various physical, biological and chemical methods. A good control of WWTP processes could lead to better water quality and to an efficient use of energy. In order to control the treatment process all parameters need to be known. Some of them cannot be measured, so an estimate of the unknown parameters has to be used. This paper proposes the development of a linear model which will be used in future research for dynamic control design. To estimate the unknown parameters, a Luenberger observer and a Kalman filter are implemented.

Keywords - mathematical model, observer, parameter estimation, wastewater treatment plant, Kalman filter.

I. INTRODUCTION

Wastewater treatment processes are complex nonlinear systems with multiple inputs and outputs. The water treatment consists of several parts: preliminary treatment, primary treatment, secondary treatment, tertiary or advanced treatment and disinfection [1,2]. In the preliminary and primary treatment some solids, and especially the coarse solids are removed in order to protect the devices used in the process (especially pumping equipment) and to prevent blockages in pipes. The research described in this paper concerns the secondary treatment where the dissolved organic matter is removed by biochemical oxidation. A mixed population of bacteria, under specific conditions [3] decompose organic matter. In an aeration tank the organic matter is converted to simple elements like carbon dioxide and water. If the sewage is aerated, after a period (this can vary from three days to a few weeks) the new bacteria population form flocculent sludge. This sludge, named activated sludge, will be separated in the settling tanks and a part is recirculated to the aeration tank. This cyclic process is an efficient one compared to an anaerobic process. A model based on conservation mass and energy was developed in [4]. The purpose of the tertiary treatment is to improve water quality by removing unwanted elements such as detergents and suspended solids. For this, sand filters can be used. In the last part of the treatment, the disinfection, the microorganisms are destroyed.

There are many cases in which the model contains uncertain parameters that need to be known in order to use them to control the process [5]. The parameters estimated in this paper are: the biomass growth rate μ , which depends on the temperature, the substrate (S), the dissolved oxygen (DO)

and the decay rate b , which depends on the sludge type and the operating conditions. These parameters are usually determined by chemical methods, but this paper presents their estimation in terms of control engineering.

The structure of the paper is as follows: Section 2 presents the nonlinear mathematical model of the process which is linearized and discretized. Section 3 presents the two methods used for parameter estimation, the Luenberger observer and the Kalman filter. These methods are tested on both the linear and nonlinear process and the results are included. The conclusions are presented in section 4.

II. MATHEMATICAL MODELING OF THE ACTIVATED SLUDGE PROCESS

A. Description of the plant

The process is schematically presented in Figure 1. The physical system consists of two tanks, one for aeration and one for decantation. In the aeration tank, bacteria and other microorganisms feed on organic matter contained in the wastewater, thereby reducing the pollution level. The decanter is a gravity settling tank in which the sludge and clean water are separated.

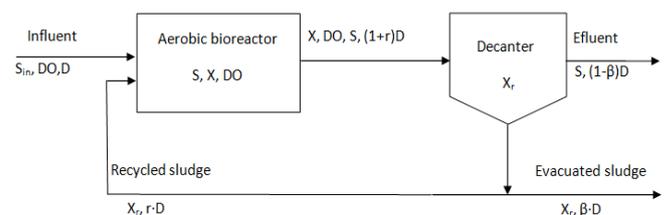


Figure 1. Biological wastewater treatment process schematics

Mathematical modeling is an important preliminary step to control the wastewater treatment plant. In what follows a simplified model of the aerobic biological process is presented. The most widely used model to describe the WWTP processes is the Activated Sludge Model Nr.1 (ASM1) [6]. With the purpose of developing a simpler model for the activated sludge process of the WWTP, the simplest possible case was taken into consideration [7,8]. This means that in the aeration bioreactor, the biological processes use only one type of microorganisms and one type of substrate. Only organic matter removal with wastewater

strength measured in terms of Chemical Oxygen Demand (COD-Cr) is considered. The model is based on the assumption that the aeration tank is continuously stirred and mixed. Part of the sludge is evacuated and part of it is recycled, meaning that it is returned to the aerobic bioreactor.

Four equations were used in order to determine the model:

- Balance equation of active sludge in aeration tank (1).
- Mass balance of the organic substrate in the aeration tank (2).
- Mass balance of dissolved oxygen concentration (3).
- Balance equation of the active sludge in the secondary settler (4).

$$\frac{dX(t)}{dt} = \mu(t) \cdot X(t) - D(t) \cdot (1+r(t)) \cdot X(t) + r(t) \cdot D(t) \cdot X_r(t) - b \cdot X(t) \quad (1)$$

$$\frac{dS(t)}{dt} = -\frac{\mu(t)}{Y} \cdot X(t) - D(t) \cdot (1+r(t)) \cdot S(t) + D(t) \cdot S_{in}(t) \quad (2)$$

$$\frac{dDO(t)}{dt} = -k_0 \cdot \frac{\mu(t)}{Y} \cdot X(t) - D(t) \cdot (1+r(t)) \cdot DO + D \cdot DO_i + \alpha \cdot W \cdot [DO_{max} - DO] \quad (3)$$

$$\frac{dX_r}{dt} = D(t) \cdot (1+r(t)) \cdot X(t) - D(t) \cdot (\beta+r(t)) \cdot X_r(t) \quad (4)$$

Where: $X(t)$ denotes the biomass, $S(t)$ is the substrate, $DO(t)$ is dissolved oxygen, DO_{max} is the maximum dissolved oxygen, $X_r(t)$ is the recycled biomass, $D(t)$ is the dilution rate, S_{in} the influent substrate, DO_i the dissolved oxygen concentrations, Y is the biomass production factor, α the oxygen transfer rate, W the aeration flow rate, k_0 a model constant, r is the ratio of the recycled flow and the influent flow, β the ratio of waste flow to influent flow, b the biomass decay rate and μ is the biomass growth rate.

The biomass growth rate μ is a complex function that depends on physical, chemical and biological factors. Many expressions have been suggested for the analytical modeling of this parameter [3,6]. The most popular is the Olson model that assumes that μ depends on the substrate concentrations (S), dissolved oxygen concentration (DO) and some kinetic parameters [9]:

$$\mu = \mu_{max} \cdot \frac{S}{k_S + S} \cdot \frac{DO}{k_{DO} + DO} \quad (5)$$

where μ_{max} represents the maximum specific growth rate.

B. Approximate linear model

In order to estimate the unknown variables using a Luenberger Observer and a Kalman filter, respectively, the first step is to develop a linear model [10].

Linearization is done by Taylor series expansion in the vicinity of an equilibrium point, neglecting terms that contain high order derivatives.

The values for the operating point in which linearization is performed are presented in Table 1.

TABLE I. NOMINAL VALUES OF THE VARIABLES

Variable	Steady-state value	Unit
X	565.5	[mg/l]
S	42.35	
DO	6.442	
X _r	1244	
DO _i	0.5	
S _{in}	765	
k	0.46	
k _s	181	
k _{DO}	0.2	
β	0.015	
r	0.8	
α	0.017	
Y	0.54	[m ³ /h]
W	340	
D	0.0638	[h ⁻¹]
μ _{max}	0.229	

The linear state-space model is the following:

$$\begin{cases} \dot{x}(t) = A \cdot x(t) + B \cdot u(t) \\ y(t) = C \cdot x(t) \end{cases} \quad (6)$$

With:

$x^T = [X \ S \ DO \ X_r]$ being the state vector,
 $y^T = [S \ DO]$ the output vector and
 $u^T = [W \ r \ \beta \ D \ S_{in} \ DO_{in}]$ the input vector.

As state variables were chosen the ones that have the dynamic behavior known (described in the first four equations).

The output vector consists of the variables intended to be controlled in future research and the input vector presents the variables that can be modified.

$$A = \begin{bmatrix} \mu - D(1+r) - b & X \cdot \mu_1 & X \cdot \mu_2 & r \cdot D \\ \frac{\mu}{Y} & -D(1+r) + \frac{1}{Y} \cdot X \cdot \mu_1 & \frac{1}{Y} \cdot X \cdot \mu_2 & 0 \\ \frac{k \cdot \mu}{Y} & \frac{k \cdot X}{Y} \cdot \mu_1 & \frac{k \cdot X}{Y} \cdot \mu_2 - D(1+r) - \alpha \cdot W & 0 \\ D(1+r) & 0 & 0 & -D(\beta+r) \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & D(X_r - X) & 0 & (1+r) \cdot X + r \cdot X_r & 0 & 0 \\ 0 & -D \cdot S & 0 & (1+r) \cdot S + S_{in} & D & 0 \\ \alpha \cdot [DO_{max} - DO] & D \cdot DO & 0 & (1+r) \cdot DO + DO_i & 0 & D \\ 0 & D(X - X_r) & -D \cdot X_r & (1+r) \cdot X - (\beta+r) \cdot X_r & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

In the matrices above μ_1 and μ_2 are the partial derivatives of μ with respect to the k_s and S , respectively evaluated in the nominal values and are given by:

$$\mu_1 = \mu_{max} \cdot \frac{k_s}{(k_s + S)^2} \cdot \frac{DO}{k_{DO} + DO} \quad (7)$$

$$\mu_2 = \mu_{max} \cdot \frac{S}{k_s + S} \cdot \frac{k_{DO}}{(k_{DO} + DO)^2} \quad (8)$$

The responses to a step input applied on the aeration flow rate (W) of both linear and nonlinear systems are presented in Figures 2 and 3. It can be observed that the outputs (S and DO) have the same shape in both cases and the linear model is close to the nonlinear one. The difference in the output is less than 0.01.

The linear model will be used to design the observers and the observers will also be tested on the nonlinear process.

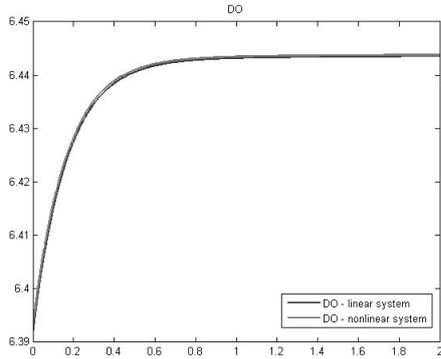


Figure 2. DO-output of linear and nonlinear model

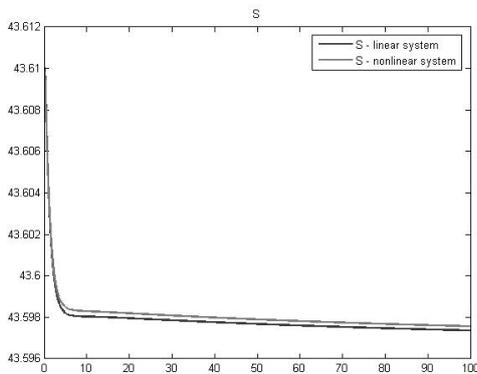


Figure 3. S-output of linear and nonlinear model

C. Discrete-time model

In what follows, a discrete time model of the process will be obtained.

The Euler method was used for discretization of the system, so the derivatives of the states are approximated as:

$$\dot{x}(t) \approx \frac{x_{k+1} - x_k}{T_s} \quad (9)$$

Where T_s is the sampling period which is chosen to be smaller than the smallest time constant of the process, in this case $T_s=0.1$ hours; the index k represents the current moment and $k + 1$ the next one.

The discrete-time state space model corresponding to the system (6) can be written as follows:

$$\begin{cases} x(k+1) = (A \cdot T_s + I_n) \cdot x(k) + B \cdot T_s \cdot u(k) \\ y(k) = C \cdot x(k) \end{cases} \quad (10)$$

III. PARAMETER ESTIMATION

Observers are used to estimate immeasurable state variables based on the measurements of the outputs and the control variables.

A. Observable model

To estimate the unknown parameters: μ_{max} , k_s , k_{DO} , Y , b , they were assumed to be constant and introduced in the state vector, which becomes:

$$[X \ S \ DO \ X_r \ \mu_{max} \ k_s \ k_{DO} \ Y \ b].$$

The corresponding dynamics are:

$$\dot{\mu}_{max} = 0, \quad \dot{k}_s = 0, \quad \dot{k}_{DO} = 0, \quad \dot{Y} = 0, \quad \dot{b} = 0.$$

Unfortunately, the resulting system was not observable. Observability of the linear system has been achieved by assuming μ a constant but unknown parameter and by also measuring X_r and Y , alongside S and DO .

The new model is a discrete one and it is presented by equations (11) and (12).

$$\begin{cases} x_{k+1} = A \cdot x_k + B \cdot u_k \\ y_k = C \cdot x_k \end{cases} \quad (11)$$

$$\begin{cases} x^T = [X \ S \ DO \ X_r \ \mu \ Y \ B] \\ y^T = [S \ DO \ X_r \ Y] \\ u^T = [S_m \ D] \end{cases} \quad (12)$$

The system is completely observable because the observability matrix has rank 7, same as the number of states.

This paper uses a Luenberger observer and a Kalman filter to determine the value of the parameters μ and b . References from the literature in the field [6] showed that these parameters are in the following intervals:

$$\begin{cases} b \in (0.002...0.066) \\ \mu \in (0.0038...0.4733) \end{cases} \quad (13)$$

B. Luenberger observer

1) Linear observer

Given the system (11), the mathematical model of the observer is:

$$\begin{cases} \hat{x}(k+1) = A \cdot \hat{x}(k) + B \cdot u(k) + L \cdot (y(k) - \hat{y}(k)) \\ \hat{y}(k) = C \cdot \hat{x}(k) \end{cases} \quad (14)$$

Where: \hat{x} denotes the estimated state; $\hat{y}(k)$ is the estimated output and L the observer gain matrix, which will be determined so that the estimation error converges to zero. The model (14) represents a prediction observer since the estimate $\hat{x}(k+1)$ is one sampling period ahead of the measurements $\hat{y}(k)$.

The estimation error is:

$$e(k) = x(k) - \hat{x}(k) \quad (15)$$

And its dynamics are:

$$\begin{aligned} e(k+1) &= A\hat{x}(k) + Bu(k) + L(Cx(k) - C\hat{x}(k)) - Ax(k) - Bu(k) \\ &= Ae(k) - LCe(k) = (A - LC)e(k) \end{aligned} \quad (16)$$

If the observer gain L is chosen such that the $A - L \cdot C$ is Hurwitz matrix (stable matrix), then the estimation error will converge to zero for any initial conditions.

Therefore the task is to place the poles of the error system. Usually the observer poles should be chosen to be about ten times faster than the system poles [11].

The close loop observer poles were chosen $[0.5 \ 0.2 \ 0.8 \ 0.4 \ -0.3 \ -0.2 \ -0.5]$ and the result L is:

$$L = \begin{bmatrix} -6.7382 & -2.4731 & -1.7111 & -0.088 & 0.0298 & 0 & 0.0262 \\ 11.2108 & 9.685 & 5.1244 & 0.1679 & -0.0827 & 0 & -0.0519 \\ 134.7528 & -11.1538 & -5.1188 & 2.3312 & 0.0921 & 0 & 0.3469 \\ 0 & 8.1383 & 3.7436 & 0 & 0 & 0.5 & 0 \end{bmatrix}$$

Figure 4 shows the estimated values of the parameters b , μ , and state X , respectively, when estimating the states of the linear system.

The results show that μ oscillates around 0.04, b value around 0.06 and X around 561 which are acceptable values for these parameters. The true values, used for simulation were:

$$\mu = 0.0421, \quad b = 0.04, \quad X = 563.5.$$

2) Nonlinear observer

Based on the linear observer structure, a nonlinear observer was built, having the following form:

$$\begin{cases} \hat{x}(k+1) = f(\hat{x}(k), u(k)) \cdot T_s + L(y(k) - \hat{y}(k)) \\ \hat{y}(k) = C \cdot \hat{x}(k) \end{cases} \quad (17)$$

where f is a vector that contains the nonlinear functions on the right-hand side of the equations (1), (2), (3) and (4), respectively and f_μ , f_y , f_b which are equal to zero and are added to the state vector to form an observable system.

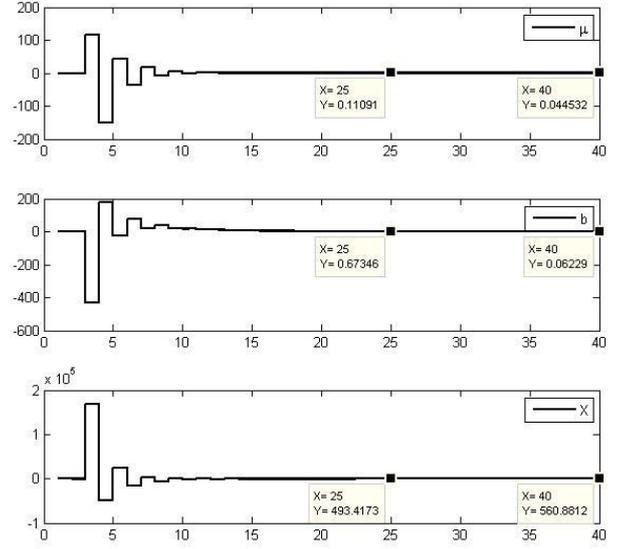


Figure 4. Estimated values of X , b and μ

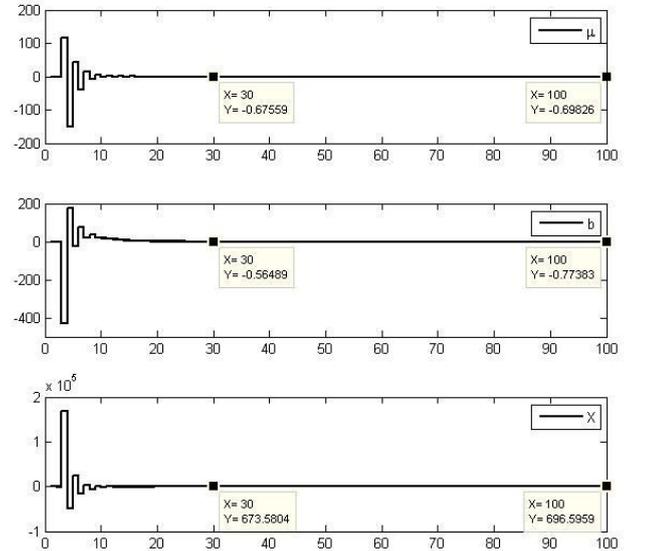


Figure 5. Estimated parameter of X , b and μ

Figure 5 shows the estimated values of the parameters b , μ , and state X , respectively, when using the nonlinear discrete-time process.

According to the nonlinear observer results the estimated μ and b values are negatives. The estimated X takes values in the vicinity of 696.

There are some difference between the linear observer result and the nonlinear observer, the linear system is only an approximation of the nonlinear one.

C. The Kalman filter

1) Overview

The Kalman filter is a standard method for unconstrained state estimations and it follows a two-step procedure [12]. The first step is the time update which uses the system model to predict the current state of the system based on the last estimate. The second step is the measurement update. The prediction from the previous step is updated/corrected by using the measured data.

Algorithm steps:

1. Time update (prediction step)

- Prediction of the state:

$$\hat{x}_{k/k-1} = A\hat{x}_{k-1/k-1} + Bu_{k-1} \quad (18)$$

- Prediction of the error covariance:

$$P_{k/k-1} = AP_{k-1}A^T + Q \quad (19)$$

2. Measurement update (correction step)

- Computation of the Kalman gain:

$$K_k = P_{k/k-1}C^T(CP_{k/k-1}C^T + R)^{-1} \quad (20)$$

- Update of the estimate with measurement:

$$\hat{x}_{k/k} = \hat{x}_{k/k-1} + K_k(y_k - C\hat{x}_{k/k-1}) \quad (21)$$

- Update of the error covariance:

$$P_k = (I - K_kC)P_{k/k-1}(I - K_kC)^T + K_kRK_k^T \quad (22)$$

The matrices Q and R represent the covariance matrices of the model and measurement noise. $Q = 0.01 \cdot I_7$ where I_7 is the identity matrix of size $[7 \times 7]$ and $R = 0.01 \cdot I_4$ where I_4 is the identity matrix of size $[4 \times 4]$.

2) Kalman filter used for the linear process

Figure 6 shows the estimated values for parameters X , b and μ , using the linear model.

It can be seen that the Kalman filter works properly and the parameters reach values in the interval (13).

In fact, μ oscillates around 0.04, b around 0.04 and X around 563.4 which are acceptable values for these parameters. The true values, used for simulation were:

$$\mu = 0.0421, \quad b = 0.04, \quad X = 563.5.$$

3) Kalman filter used for the nonlinear process

Figure 7 presents the estimates obtained by the Kalman filter for the nonlinear process. It can be seen that the parameters need more time to converge than in the linear case. μ oscillates around 0.054, b around 0.058 and X around 562.8 which are acceptable values for these parameters.

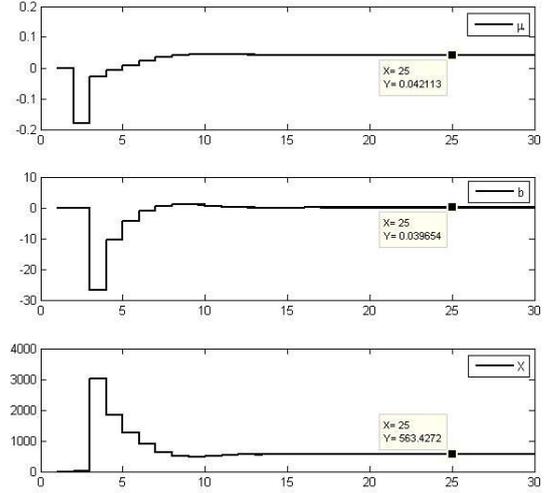


Figure 6. Estimated parameters – linear model

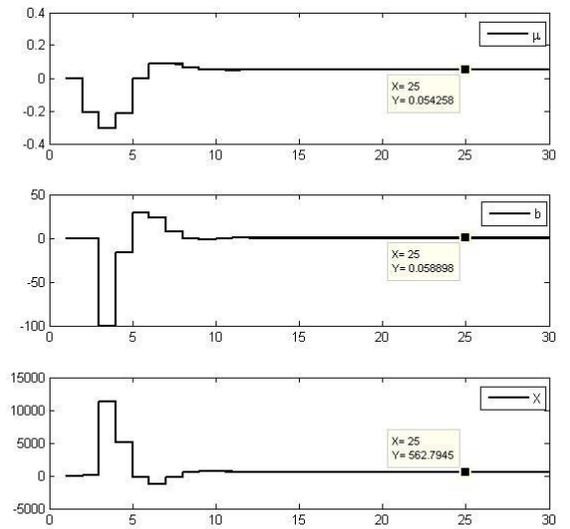


Figure 7. Estimated parameters – nonlinear model

D. The Luenberger Observer versus the Kalman filter

In this section a brief comparison between the two estimation methods is presented.

1) Linear model

Figure 8 presents the estimated states when using the Kalman filter and the Luenberger observer on the linear discret-time system. It can be seen that in both cases the parameters converge to values acceptable for the process described.

Using the Kalman filter the parameters converge faster to a stationary value.

IV. CONCLUSION

In this paper a Luenberger observer and a Kalman filter have been used to estimate the unknown parameters of a wastewater treatment process.

The desired parameters to be determined are μ_{max} , k_S , k_{DO} , Y , b which were introduced in the initial state vector. However, in this case the system is not observable. The observability condition was accomplished only if μ_{max} , k_S , k_{DO} were represented as μ and the output vector was updated with X_r and Y (the output vector was initially represented by S and DO).

The methods have been tested on the nonlinear process, resulting that the Kalman filter is more suitable for both the linear and nonlinear process. This result was expected, because the Luenberger Observer has the same gain during all the simulation time (it does not change) while the Kalman filter modifies its gain at every iteration.

REFERENCES

- [1] Michael R. Templeton, David Butler, "Introduction to Wastewater Treatment", bookboon.com, 2011
- [2] Claudia Maria Simionescu, "Epurarea biologică a apelor uzate", Matrix Rom, 2009
- [3] Diana Robescu, Attila Verestoy, Szabolcs Lanyi, Dan Robescu, "Modelarea și simularea proceselor de epurare", Editura Tehnică, 2004
- [4] James W. Haefner, "Modeling Biological Systems", Springer, 2005
- [5] Carl-Frederik Lindberg, "Control and estimation strategies applied to the activated sludge process", Uppsala University, 1997
- [6] Henze, M., Gujer, W., Mino, T. & Van Loosdrecht, M. 2000. Activated sludge models ASM1, ASM2, ASM2d and ASM3. *IWA Publishing, London, Uk.*
- [7] Muntean I., Both R., Crisan R., Nascu I.: RGA analysis and decentralized control for a wastewater treatment plant, Proceedings of the 2015 IEEE International Conference on Industrial Technology (ICIT), 17-19 March, Seville, Spain, Pages: 453 – 458
- [8] Cristescu, S. M., Nașcu, I., Nașcu, I. 2015. Sensitivity Analyses of an Activated Sludge Model for a Wastewater Treatment Plant. 17th International Conference on System Theory, Control and Computing (ICSTCC), 14-19 Oct. 2015, Cheile Gradistei, pp: 595 - 600, ISBN: 978-1-4799-8481-7.
- [9] Gustaf Olsson, Bob Newell, "Wastewater Treatment Systems", IWA Publishing, 1999
- [10] Katalin Hangos, Ian Cameron, "Process Modeling and Model Analysis", Academic Press, 2001
- [11] Katsuhiko Ogata, "Modern Control Engineering", Prentice Hall, 2002
- [12] Zsófia Lendek, "Distributed Fuzzy and Stochastic Observers for Nonlinear Systems", Phd Thesis, Delft, The Netherlands, 2009.

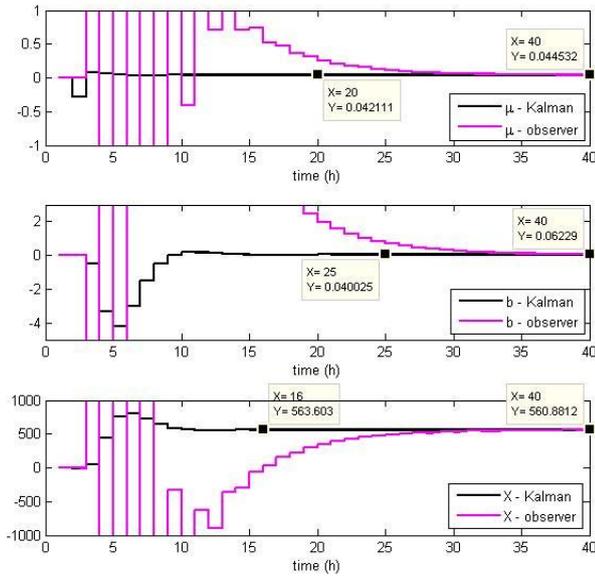


Figure 8. Luenberger Observer vs Kalman filter – linear process

2) Nonlinear model

As can be seen in Figure 9, the Kalman filter gives better results in estimating the states. The linear observer is not suitable for the nonlinear process as it does not give results in the interval (13) for parameters μ and b . The statement about the speed of convergence remains the same as the one for the linear system.

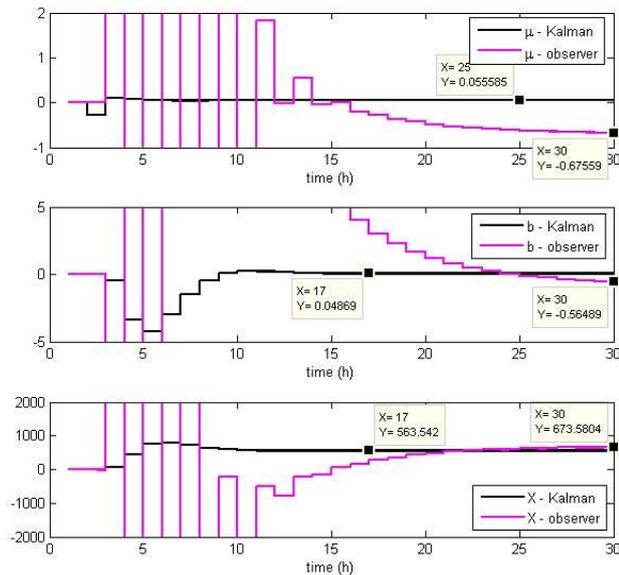


Figure 9. Luenberger Observer vs Kalman filter – nonlinear process