Transforming variable transport delays into fixed ones: An application to a conveyor belt problem

Thomas Laurain, Zsófia Lendek, Jimmy Lauber, and Reinaldo M. Palhares

***Abstract*— This paper presents a systematic methodology to deal with variable transport delays by constructing a new discrete domain where the delay becomes fixed (a difference of samples). The Euler transformation is used to change from the continuous-time domain to this special discrete domain depending on the same variable as the transport delay. After this transformation, the model becomes nonlinear and the Takagi-Sugeno fuzzy representation is obtained to handle the nonlinearities. To highlight and illustrate the efficiency of the proposed methodology, a LMI based controller design is developed to control a conveyor belt with variable transport delay.**

Keywords: Variable transport delay; variable-dependent domain; discrete model-based control; Takagi-Sugeno representation; conveyor belt

I. INTRODUCTION

Time-varying delays represent a major problem from a control point of view. Delays are naturally present in various industrial and practical applications. There is a vast literature concerning constant known time-delays with different kind of strategies. In the frequency domain, Smith predictors have been designed. In the state space and in presence of (known or unknown) constant time-delays, the methods are based on Lyapunov-Krasovskii as well as Lyapunov-Razumikhin theory. Finding stability conditions for fixed time-delay systems has been studied in [1]–[3]. Delayed systems are often nonlinear so the Takagi-Sugeno representation has been considered [4], [5] and lead to more complex control laws [6]. This work is motivated by the stability conditions of systems with time-varying or unknown delays [7], [8].

This paper distinguishes between variable time and variable transport delays. A variable time delay is a delay that depends directly on the time. A variable transport delay is a delay appearing, e.g. when a certain distance has to be covered at a time-varying speed. Thus, it does not directly depend on the time, but on a variable that is time-varying. This work is focusing on this second category.

Variable transport delays are naturally present in many systems. [9] presents several sources of delay: a transfer between two industrial processes, an inextensible pipe that drives a flow, the position of a remote sensor, etc. Simulation, modeling and control in the presence of variable transport delays have been addressed in the application literature. Examples can be found in marine applications for cooling the engine [10], in irrigation problems [11], in heating systems [12], [13], in solar energy collecting systems [14] or in air-fuel ratio control where the lambda sensor is located in the exhaust manifold [15], [16].

The application considered in this paper is the conveyor belt. This system is used in industry for transporting materials from a point A to a point B. A transport delay is naturally present and depends on the speed of the engine driving the belt. Recent studies from the literature mainly focus on speed control for the belt conveyor [17]–[21].

This paper presents a transformation into a discrete-time domain where the sampling time is a function of the transport variables. In this new domain, the variable transport delay is expressed as a fixed one (related to different samples). Then, because the model becomes nonlinear, the Takagi-Sugeno fuzzy representation is employed and a state-feedback controller is designed.

The paper has the following structure: Section 2 details the notations and some preliminaries. Section 3 presents the main contribution, including the transformation to make the variable transport delay fixed and the controller design. Section 4 gives an application example of a conveyor belt which illustrates the interest in such a methodology. Finally, Section 5 concludes the paper.

II. PRELIMINARIES

A. Notations

Throughout this paper, the following notation is used: $x(t)$ represents the state in the time domain, $x(k)$ the state in the discrete-time domain and $x^{\theta}(k)$ the state in the domain given by the time-varying variable $\theta(t)$. I denotes the identity matrix and $(*)$ stands for the symmetric term of the left hand side.

B. Variable transport delay formulation

Let us consider the following linear state-space representation in the continuous-time domain with a delayed input:

$$\dot{x}(t) = Ax(t) + Bu\left(t - \delta(t)\right) \tag{1}$$

where A is the system matrix, B is the input matrix and δ is the variable transport delay, which depends on an external variable θ.

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Remark 1: The main contribution can be extended to the case where both input and state are delayed and to several types of delays, but for reason of simplicity, in this paper we focus on the delayed input with only one transport delay.

The variable transport delay $\delta(t)$ is usually a function depending on a variable and its time-derivative, for instance in the case of the conveyor belt, dividing meters (the length) by the linear speed in meters per second leads to a delay in seconds. Let us introduce the variable $\theta(t)$ and its time-derivative $\dot{\theta}(t)$. Let us express the variable transport delay as:

$$\delta\left(\dot{\theta}(t)\right)=\frac{\theta_{fix}}{\dot{\theta}(t)} \tag{2}$$

where θ_{fix} represents a fixed value of the $\theta(t)$ variable, depending on the application. For example, it can be the length of the pipe, the length of the conveyor, the position of the sensor, etc. Let us assume that $\dot{\theta}(t)>0$ which makes sense in most of the applications where the start of the system is ensured by a dedicated module ($\dot{\theta}(t)=0$) and the system cannot work reversely ($\dot{\theta}(t)<0$). Then, equation (1) can be written as

$$\dot{x}(t)=Ax(t)+Bu\left(t-\delta\left(\dot{\theta}(t)\right)\right) \tag{3}$$

III. Main Contribution

A. Transformation to a new domain

Since a time-varying delay is not easy to handle, the main idea is to transform the system equation (3) into a domain in which the delay $\delta(\dot{\theta})$ becomes fixed. As presented in equation (2), the delay is a function of the variable $\dot{\theta}(t)$. Let us consider the θ domain. Then, $\dot{x}(t)$ can be expressed as:

$$\frac{dx(t)}{dt}=\frac{dx(t)}{d\theta(t)}\times\frac{d\theta(t)}{dt}=\frac{dx(t)}{d\theta(t)}\times\dot{\theta}(t) \tag{4}$$

where $\theta(t)$ stands for the variable of the new domain. Such a transformation is often used in engine control to go to crank-angle domain [22], [23]. The derivative of the state $x(t)$ with respect to $\theta(t)$ can be derived from equation (4):

$$\frac{dx(t)}{d\theta(t)}=\frac{1}{\dot{\theta}(t)}\frac{dx(t)}{dt} \tag{5}$$

In order to convert the continuous-time domain in the specific discrete θ domain, a discretization is realized using the Euler approximation:

$$\frac{dx(t)}{d\theta(t)}\approx\frac{x^{\theta}(k+1)-x^{\theta}(k)}{T_s^{\theta}} \tag{6}$$

where T_s^{θ} stands for a sampling value in the θ domain, that has to be chosen taking into account that T_s^{θ} should be small enough for not losing any information during the discretization. By combining (5) and (6), it is possible to deduce the recursive law between two samples in the θ domain:

$$x^{\theta}(k+1)\approx x^{\theta}(k)+\frac{T_s^{\theta}}{\dot{\theta}(k)}\frac{dx^{\theta}(k)}{dt} \tag{7}$$

B. Fixing the delay

Now that the θ domain is built and the transformation law (7) is defined, the continuous-time dynamical state-space representation (3) in the new domain becomes

$$x^{\theta}(k+1)=x^{\theta}(k)+\frac{T_s^{\theta}}{\dot{\theta}(k)}\left(Ax^{\theta}(k)+Bu^{\theta}\left(k-\Upsilon\left(\delta\left(\dot{\theta}\right)\right)\right)\right) \tag{8}$$

where $x^{\theta}(k)$ stands for the state expressed in the new θ domain at the sample k, $x^{\theta}(k+1)$ the state vector at the next sample according to the sampling period T_s^{θ}. u^{θ} is the control input in the new θ domain, and $\Upsilon(\bullet)$ the function that transforms the time-varying delay $\delta(\dot{\theta})$ into a delay expressed in the same unit as θ. Consequently, in order to respect the units, the function $\Upsilon(\bullet)$ may be defined as:

$$\Upsilon\left(\delta\left(\dot{\theta}\right)\right)=\dot{\theta}\times\delta\left(\dot{\theta}\right) \tag{9}$$

with $\dot{\theta}$ in $\left[\theta\ unit\times s^{-1}\right]$ and $\delta(\dot{\theta})$ in $[s]$. Then, the next step to make the delay fixed is to define the sampling value T_s^{θ} such as:

$$k-\Upsilon\left(\delta\left(\dot{\theta}\right)\right)=k-T_s^{\theta}\times\mu \tag{10}$$

where μ is a chosen integer representing the delay in number of samples. Then, combining (9) and (10) leads to the following expression for T_s^{θ}:

$$T_s^{\theta}=\frac{\dot{\theta}\times\delta\left(\dot{\theta}\right)}{\mu} \tag{11}$$

C. Controller design

Thanks to the proposed transformation, and choosing an appropriate T_s^{θ} according to equation (11), the time-varying delay $\delta(\dot{\theta})$ becomes fixed in the θ domain and is equal to μ samples:

$$x^{\theta}(k+1)=x^{\theta}(k)+\frac{T_s^{\theta}}{\dot{\theta}(k)}\left(Ax^{\theta}(k)+Bx^{\theta}(k-\mu)\right) \tag{12}$$

Two things are important to consider with such a transformation. First, from an application point of view, the choice of T_s^{θ} is often highly dependent on the sensors. Second, even if the considered continuous-time system (1) is a very simple linear model, by this transformation and the term $\frac{T_s^\theta}{\dot{\theta}(k)}$, it becomes nonlinear in the θ domain.

In the literature, several methods exist to deal with nonlinearities. The simplest one consists in linearizing the model at several operating points. Instead of using this methodology that does not guarantee the stability during transient phases, in what follows the Takagi-Sugeno (TS) fuzzy representation [24] is used.

TS models consist of a collection of linear subsystems linked together with some nonlinear functions h_i called 'membership functions'. These membership functions have to verify the convex sum property:

$$h_i\left(z(k)\right) > 0, \quad \sum_{i=1}^{r} h_i\left(z(k)\right) = 1 \tag{13}$$

where r is the number of local models.

The sector nonlinearity methodology for obtaining TS models has the advantage of providing an exact representation of the nonlinear system inside a domain of validity. The transformation to the fuzzy model in general is not detailed in this paper and the interested reader is referred to [25]. A TS model equivalent to (12) in the domain θ can be expressed as:

$$x^\theta(k+1) = \sum_{i=1}^{r} h_i\left(z(k)\right)\left(M_i x^\theta(k) + N_i u^\theta(k-\mu)\right) \tag{14}$$

where h_i are the membership functions and the matrices M_i and N_i stand for the linear subsystems depending on the bounds of the domain of validity. The notation is simplified along the rest of the paper, and M_z stands for $\sum_{i=1}^{r} h_i\left(z(k)\right)M_i$ and M_{z-1} for $\sum_{i=1}^{r} h_i\left(z(k-1)\right)M_i$. Thus, (14) becomes:

$$x^\theta(k+1) = M_z x^\theta(k) + N_z u^\theta(k-\mu) \tag{15}$$

Now, a fuzzy controller can be designed. For example, the simplest one consists of a state feedback including the membership functions, called Parallel Distributed Compensation (PDC) [26]:

$$u^\theta(k) = -F_z x^\theta(k) \tag{16}$$

where F_z is the fuzzy controller gain. Then, applying the control law (16) to the TS system (15), the closed-loop system is:

$$x^\theta(k+1) = M_z x^\theta(k) - N_z F_{z-\mu} x^\theta(k-\mu) \tag{17}$$

for which the following result can be established:

Theorem 1: Consider the Takagi-Sugeno model (15) and the control law (16). The closed-loop system (17) is asymptotically stable if there exist matrices L, Y, M_{2ij}, S_z such that:

$$\Gamma_{z,z-\mu} < 0 \tag{18}$$

with:

$$\Gamma_{z,z-\mu} = \begin{bmatrix} -L & (*) & (*) & (*) & (*) \\ L & -Y & (*) & (*) & (*) \\ M_z L & 0 & -W_{2z,z-\mu} - W_{2z,z-\mu}^T & (*) & (*) \\ 0 & 0 & W_{2z,z-\mu}^T & -L & (*) \\ M_z L & 0 & J_{z,z-\mu}^T & 0 & R_{z,z-\mu} \end{bmatrix} \tag{19}$$

where $R_{z,z-\mu} = -N_z S_{z-\mu} + (*) - Y$ with $(*)$ the symmetric term of the left hand side and $J_{z,z-\mu} = -S_{z-\mu}^T N_z^T - M_{2z,z-\mu}^T$.

Remark 2: Standard relaxations cannot be used because of $z(k-\mu)$ so the inequality (18) must be verified $\forall i, j$.

Proof: Let us consider the Lyapunov function:

$$V(k) = x^\theta(k)^T P x^\theta(k) + \sum_{i=k-\mu}^{k-1} x^\theta(i)^T Q x^\theta(i) \tag{20}$$

Then, the difference $\Delta V = V(k+1) - V(k) < 0$ can be written as:

$$\begin{bmatrix} x^\theta(k) \\ x^\theta(k+1) \\ x^\theta(k-\mu) \end{bmatrix}^T \begin{bmatrix} -P+Q & 0 & 0 \\ 0 & P & 0 \\ 0 & 0 & -Q \end{bmatrix} \begin{bmatrix} x^\theta(k) \\ x^\theta(k+1) \\ x^\theta(k-\mu) \end{bmatrix} < 0 \tag{21}$$

The closed-loop system can also be written as:

$$\begin{bmatrix} M_z & -I & -N_z F_{z-\mu} \end{bmatrix} \begin{bmatrix} x^\theta(k) \\ x^\theta(k+1) \\ x^\theta(k-\mu) \end{bmatrix} = 0 \tag{22}$$

It is possible to use the Finsler lemma [27] to get:

$$\begin{bmatrix} 0 \\ W_{2z,z-\mu}^{-1} \\ Q \end{bmatrix} \begin{bmatrix} M_z & -I & -N_z F_{z-\mu} \end{bmatrix} + (*) + \begin{bmatrix} -P+Q & 0 & 0 \\ 0 & P & 0 \\ 0 & 0 & -Q \end{bmatrix} < 0 \tag{23}$$

where $(*)$ stands for the symmetric term of the left hand side. Then, a congruence transformation is applied with the matrix $diag\left(L, W_{2z,z-\mu}, Y\right)$ where $L = P^{-1}$ and $Y = Q^{-1}$, the change of variable $S_{z-\mu} = F_{z-\mu} Y$ and Schur complements on the term $L \cdot Q \cdot L^T$ in (1,1) and $W_{2z,z-\mu} \cdot P \cdot W_{2z,z-\mu}^T$ in (2,2) terms lead to the previously defined quantity (19).

Remark 3: Results of Theorem 1 only concern the stability of the discrete-time system (8), not the continuous-time system (3). To obtain stability results on the continuous-time system, the sampling value T_s^θ should be chosen with respect to Euler and Shannon theorems.

IV. Application to the Conveyor Belt System

The conveyor belt is an industrial application often used where a transport delay appears. The delay depends on the length of the belt and the speed of the motor. The proposed methodology is used to construct a new domain where the delay is fixed and to control the system.

A. Conveyor belt dynamical model

First, let us present the considered system and the dynamical equations. Fig. 1 has been adapted from [17]:

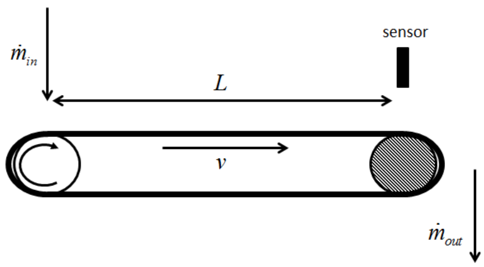

Figure 1. Conveyor belt modelling

where $\dot{m}_{in}$ and $\dot{m}_{out}$ are the mass flow entering and leaving the conveyor belt (g/s) respectively. v is the linear speed of the conveyor belt (m/s). The belt is $L = 10m$ long and $l = 1m$ wide. The sensor is measuring the height H of the matter on the belt (m). The control problem is to regulate the height by controlling the entering mass flow. The conveyor belt speed is considered as a disturbance. Based on the literature, such as the moisture model of [17], a first order model of the variation of the height is established, considering the mass flow input, the conveyor speed and the sensor dynamics:

$$\frac{dH(t)}{dt} = -\frac{1}{\tau}H(t) + \frac{1}{\tau\rho l}\frac{\dot{m}_{in}(t-\delta)}{v(t-\delta)} \tag{24}$$

In (24) τ is the time constant of the sensor (0.2s) and ρ is the volumetric mass of the matter (g/m^3). Let us consider for the simulation a conveyor transporting some quinoa. The volumetric mass of the quinoa is $\rho = 714.2857$. The variable transport delay δ depends on the speed of the conveyor belt:

$$\delta(v(t)) = \frac{l}{v(t)} \tag{25}$$

The state-space representation like (3) is realized with (24):

$$\dot{x}(t) = Ax(t) + Bu(t-\delta) \tag{26}$$

with $x(t) = H(t)$, $A = -\frac{1}{\tau}$, $B = \frac{1}{\tau\rho l}$ and $u(t) = \frac{\dot{m}_{in}(t)}{v(t)}$

B. New domain transformation

According to the expression of the variable transport delay δ given in (25), the delay depends on the speed of the conveyor belt $v(t)$, so $\dot{\theta}(t) = v(t)$. Then, θ stands for the distance travelled by the conveyor belt. The transformation is applied to go from the continuous-time domain to the particular discrete-time domain:

$$x^\theta(k+1) = x^\theta(k) + \frac{T_s^\theta}{v(k)}\frac{dx(k)}{dt} \tag{27}$$

In order to get the sampling value T_s^θ in meters, let us consider the discretized delay equal to $\mu = 10$. Then, by using (11), we have:

$$T_s^\theta = \frac{\dot{\theta}\times\delta(\dot{\theta})}{\mu} = \frac{v(k)\times\frac{l}{v(k)}}{10} = \frac{l}{10} = 1\,m \tag{28}$$

Consequently, in this new meter-domain computed every meter, the variable transport delay becomes fixed (equal to 10 samples). Now, for control design purposes, let us convert the state-space system (26) to the new domain:

$$x^\theta(k+1) = Ax^\theta(k) + Bu(k-10) \tag{29}$$

$A = 1 - NL_1 \times \frac{1}{\tau}$ and $B = NL_1 \times \frac{1}{\tau\rho l}$ where $NL_1 = \frac{T_s^\theta}{v(k)}$. The bounds are chosen as $v(k) \in [3;5]$ so that the assumption $\dot{\theta}(t) > 0$ is respected. Equation (29) can be written as:

$$x^\theta(k+1) = \sum_{i=1}^{2} h_i(k)\left(M_i x^\theta(k) + N_i u(k-10)\right) \tag{30}$$

By solving the LMI conditions presented in Theorem 1, we obtain the following controller gains for the control law:

$$u^\theta(k) = -F_z x^\theta(k) \tag{31}$$

$F_1 = -45.3246$, $F_2 = -5.61$. In order to follow a step reference, a term is added to the control law as follow:

$$u^\theta(k) = -F_z x^\theta(k) + \frac{y_{ref}}{C_z \cdot (I - A_z + B_z \cdot F_z)^{-1} \cdot B_z} \tag{32}$$

which is a direct extension of what is done in the linear context [28] for the Takagi-Sugeno representation. More details can be found in [29].

C. Simulation results

This controller is implemented in simulation to control the conveyor belt system, i.e. to regulate the height of mass leaving the conveyor (for packaging purposes for example). The conveyor speed is simulated for the values depicted in Fig. 2.

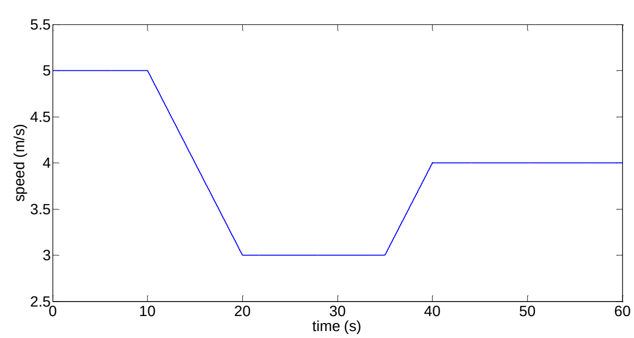

Figure 2. Speed of the conveyor (m/s)

The variable transport delay is increasing corresponding to the speed as presented in Fig. 3. However, in the θ domain, the delay does not change.

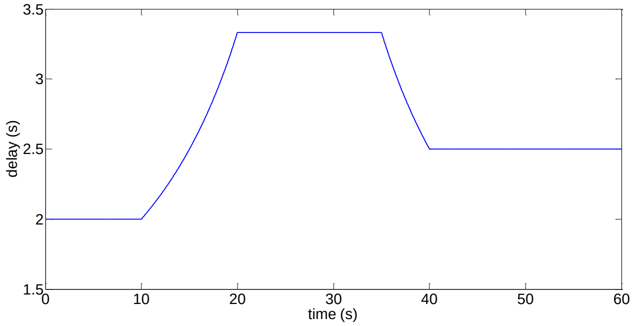

Figure 3. Variable transport delay (s)

In addition to this change of speed, a change of reference is imposed, for example due to a change in the type of product. The reference height changes from 1 centimeter to 2 centimeters. Moreover, a sensor noise is added at the output of the system to simulate real conditions. For comparison purposes, a PI controller is designed with gains whose achieve the better result that can be obtained empirically to realize a trade-off between oscillations and performance. A PI controller with a Smith predictor-based structure (since the delay is time-varying) is also designed since it can better handle delayed systems. Fig. 4 depicts the height of the traveled mass along the conveyor belt with the proposed controller, the PI and the PI + Smith one.

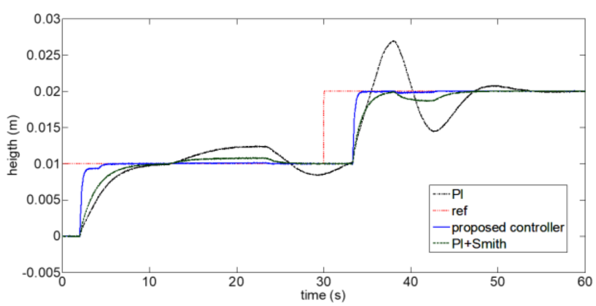

Figure 4. Height of transported quinoa (m) controlled by three different controllers: PI (black dotted), PI+Smith (green discontinuous) and proposed controller (blue)

As one can see, the PI and PI+Smith controllers do not have a good performance. Increasing the gain and the time constant of the PI controllers lead to more oscillations. The PI controller with the Smith predictor structure has better performance than the standard PI, however, it suffers from changes of speed, i.e. changes of delay, as one can see in Fig. 5 which presents a zoom of Fig. 4 from 0 to 30 seconds. Fig. 6 presents the command generated by the TS discrete controller triggered every meter of the conveyor belt.

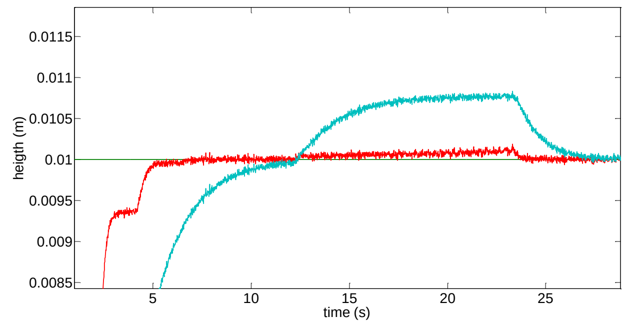

Figure 5. Zoom of the behavior of the PI+Smith (blue) and the proposed controller (red) around the reference $H_{ref} = 0.01$

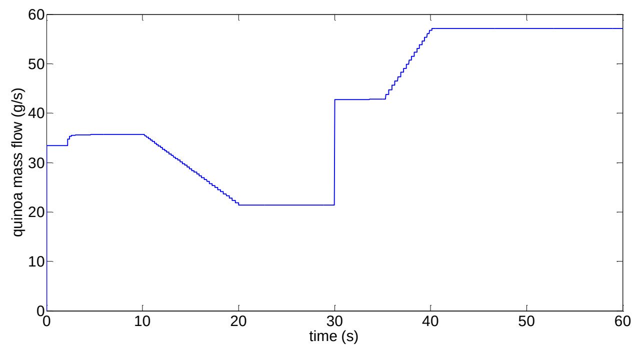

Figure 6. Entering quinoa mass flow (g/s)

V. CONCLUSION

This paper presented a new and original way to deal with variable transport delays. It detailed the systematic construction of a new discrete domain where the delay becomes fixed. Going to this domain leads to adding nonlinearities to the problem. The nonlinear aspect is considered using the Takagi-Sugeno representation. A Takagi-Sugeno state feedback controller is designed. As an application, the conveyor belt problem illustrates that our methodology provides good performances compared to a PI controller with a Smith predictor structure. Future work will focus on including the delayed input into the state so less conservative results can be obtained, using a non-quadratic delayed Lyapunov function and finding the largest sampling value T_s^{θ} for which the system in the new θ domain is still controllable. Furthermore, a transformation will convert the results about the stability of the system from the θ domain to the original continuous-time domain.

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