Local quadratic and nonquadratic stabilization of discrete-time TS fuzzy systems

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TS fuzzy models

- Nonlinear, convex combinations of local linear models
- Well-established methods

Stabilization

- Continuous time: global, local
- Discrete-time: global, (few) local

This paper:

Local stabilization for discrete-time TS systems

Outline

Outline

Motivating example

- System: $x_1(k) \in [-a,\ a]$

$$x_1(k+1) = x_1^2(k)$$
$$x_2(k+1) = 3x_1(k) + 10x_2(k) + u(k)$$

- Exact TS model:

$$\mathbf{x}(k+1) = h_1(x_1(k))A_1\mathbf{x} + h_2(x_1(k))A_2\mathbf{x}$$
$$A_1 = \begin{pmatrix} -a & 0 \\ 3 & 10 \end{pmatrix} \qquad A_2 = \begin{pmatrix} a & 0 \\ 3 & 10 \end{pmatrix} \qquad B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$
$$h_1(x_1) = \frac{a - x_1(k)}{2a} \qquad h_2(x_1(k)) = 1 - h_1(x_1(k))$$

Motivating example – cont'd

Stabilization:

- Nonlinear system:
  - $a \in (-1, 1)$: can be stabilized to zero
  - $|a| > 1$: unstable
- TS model: depends on the modeling region
  - $a \in (-1, 1)$: can be stabilized to zero
  - $|a| > 1$: no solution

Add contraint: $x_1{}^2(k+1) < 0.9 x_1{}^2(k)$

Notations

- System: $\mathbf{x}(k+1) = \sum_{i=1}^{r} h_i(z(k))A_i\mathbf{x}(k)$

$$\sum_{i=1}^{r} h_i(z(k))A_i\mathbf{x}(k) \Leftrightarrow A_z\mathbf{x}(k)$$

$$\sum_{i=1}^{r} h_i(z(k+1))A_i\mathbf{x}(k) \Leftrightarrow A_{z+1}\mathbf{x}(k)$$

- In results: $(*)$ - term induced by symmetry

Assumptions

Assumption:

$$\begin{pmatrix} \mathbf{x}(k) \\ \mathbf{x}(k+1) \end{pmatrix}^T R \begin{pmatrix} \mathbf{x}(k) \\ \mathbf{x}(k+1) \end{pmatrix} \geq 0$$

- Can always be satisfied
- S-procedure

What is R?

This paper: stabilization

Outline

Quadratic stabilization

- System and control law:

$$\mathbf{x}(k+1) = A_z\mathbf{x}(k) + B_z\mathbf{u}(k)$$
$$\mathbf{u}(k) = -F_zP^{-1}\mathbf{x}(k)$$

- Lyapunov function

$$V(\mathbf{x}(k)) = \mathbf{x}^T(k)P^{-1}\mathbf{x}(k)$$

- Add constraint:

$$\begin{pmatrix} \mathbf{x}(k) \\ \mathbf{x}(k+1) \end{pmatrix}^T R \begin{pmatrix} \mathbf{x}(k) \\ \mathbf{x}(k+1) \end{pmatrix} \geq 0$$

Quadratic stabilization - conditions

- S-procedure
- Finsler's lemma
- Nice condition:

$$\begin{pmatrix} -P & (*) \\ A_z P - B_z F_z & -P \end{pmatrix} + W < 0$$

- $R = \begin{pmatrix} P^{-1} & 0 \\ 0 & P^{-1} \end{pmatrix} W \begin{pmatrix} P^{-1} & 0 \\ 0 & P^{-1} \end{pmatrix}$
- Region: largest Lyapunov level set included in $\mathcal{D}_R$

Non-quadratic stabilization

- System and control law:

$$\mathbf{x}(k+1) = A_z\mathbf{x}(k) + B_z\mathbf{u}(k)$$
$$\mathbf{u}(k) = -F_z H_z^{-1}\mathbf{x}(k)$$

- Lyapunov functions

$$V(\mathbf{x}(k)) = \mathbf{x}^T(k) H_z^{-T} P_z H_z^{-1}\mathbf{x}(k)$$
$$V(\mathbf{x}(k)) = \mathbf{x}^T(k) P_z^{-1}\mathbf{x}(k)$$

- Add constraint:

$$\begin{pmatrix}\mathbf{x}(k)\\ \mathbf{x}(k+1)\end{pmatrix}^T R \begin{pmatrix}\mathbf{x}(k)\\ \mathbf{x}(k+1)\end{pmatrix} \geq 0$$

Non-quadratic stabilization

- Lyapunov function

$$V(\mathbf{x}(k)) = \mathbf{x}^T(k) H_z^{-T} P_z H_z^{-1} \mathbf{x}(k)$$

- S-procedure
- Finsler's lemma
- Conditions:

$$\begin{pmatrix} -P_z & (*) \\ A_z H_z - B_z F_z & -H_{z+} - H_{z+}^T + P_{z+} \end{pmatrix} + R < 0$$

- Region $\begin{pmatrix} \mathbf{x}(k) \\ \mathbf{x}(k+1) \end{pmatrix}^T \begin{pmatrix} H_z^{-T} & 0 \\ 0 & H_{z+}^{-T} \end{pmatrix} R \begin{pmatrix} H_z^{-1} & 0 \\ 0 & H_{z+}^{-1} \end{pmatrix} \begin{pmatrix} \mathbf{x}(k) \\ \mathbf{x}(k+1) \end{pmatrix} > 0$
- Region: largest Lyapunov level set included in $\mathcal{D}_R$

Outline

What does it mean?

Convergence:

only in the largest Lyapunov level set included in $\mathcal{D}_R \bigcap \mathcal{D}$.

- The domain $\mathcal{D}_R$ has to be verified
- Back to the example: with $a = 2$

$$\mathbf{x}(k+1) = h_1(x_1(k))A_1\mathbf{x} + h_2(x_1(k))A_2\mathbf{x}$$

$$A_1 = \begin{pmatrix} -a & 0 \\ 3 & 10 \end{pmatrix} \qquad A_2 = \begin{pmatrix} a & 0 \\ 3 & 10 \end{pmatrix} \qquad B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$h_1(x_1) = \frac{a - x_1(k)}{2a} \qquad h_2(x_1(k)) = 1 - h_1(x_1(k))$$

- Try different Rs

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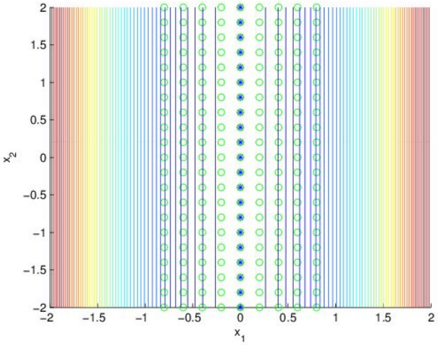
Results - quadratic stabilization

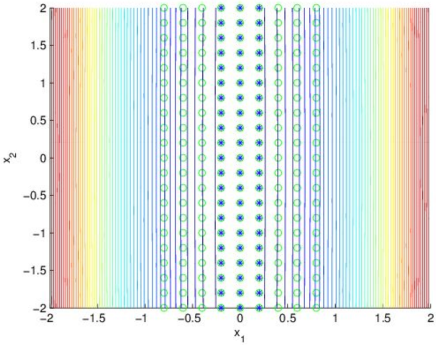
R full



Only 0 .....

Results - quadratic stabilization

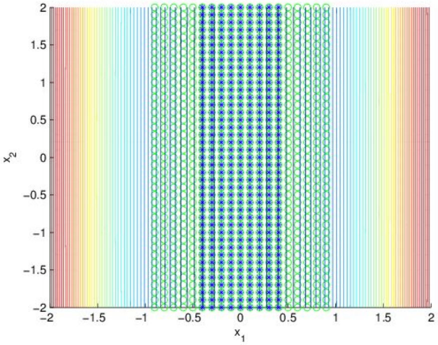
R block-diagonal



Increased...

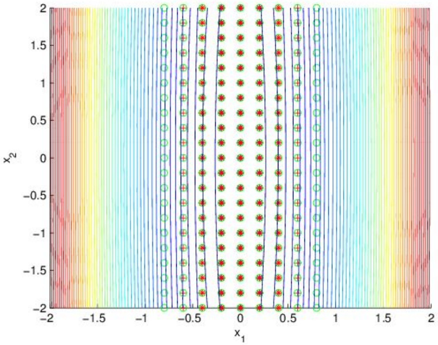
Results - non-quadratic stabilization

R diagonal



Interesting result - quadratic stabilization

Verify for 2 steps



Almost full...

Outline

Conclusions

- Local stabilization
- Estimation of the domain of attraction

- Obvious extensions: α-sample variation, etc.
- Past samples
- Structure of R

Thank you!

Questions?

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