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Handling non-smooth effects in control of real robotic systems

Young Teams Project
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Objectives of the 2015-2016 phase

Assistive robotics [1], or, specifically, the use of assistive robotic arms is motivated by the societal need of increasing the independence of elderly and disabled people. Higher living standard may be obtained by employing robotic assistance. Aerial vehicles, in particular vertical take-off and landing ones such as quadrotors [2, 3, 4], have received a growing interest of the robotics research community due to the numerous applications that can be addressed with such systems, like surveillance, inspection, or mapping [5, 6]. However, for model-based control, these systems require methods and algorithms that are able to 1) reliably estimate variables of interest while compensating for sensor limitations and disturbances; and 2) achieve the desired control objective in spite of actuator limitations and significant changes in the model due to external, possibly discontinuous destabilizing effects. In this context, the objectives delineated in the implementation plan were the following:

- Analysis and design: model development, theoretical analysis and synthesis. This involved on the one hand the development of the dynamic model – based on first principles – for robot arms and aerial vehicles, and the other hand the analysis of these models and the design of controllers and observers for them. Current Lyapunov-based analysis and design methods are quite general, but this generality comes with increased computational costs and reduced specificity. Methods adapted to specific classes of systems have been developed that are computationally more efficient, and we focused in particular on problems that, due to physical constraints, accept a local solution.
- Applications: experiment design, estimation and control experiments. In addition to benchmarking the techniques sitting at the core of the project, another aim here is to show that developed techniques can address important unsolved problems in nonlinear control, thereby increasing their visibility and acceptance in the field.

In addition, some preliminary steps were made towards the integration of performance indices in the controller and observer design and the real-time application of the developed control laws.

1 Analysis and design

1.1 First-principle models

1.1.1 Robotic arms

The second class of systems considered is robotic arms, specifically a Robai Cyton Gamma 1500 robot arm available at the Technical University of Cluj-Napoca. The mathematical models that characterize a robotic arm are structured into three categories:

1. The geometrical model - representing the position and the orientation of the joints and the gripper relative to a fixed coordinate system.

2. The kinematic model - representing the velocity and acceleration of the joints and the gripper relative to a fixed coordinate system.

3. The dynamic model - representing the effect on the position and orientation of the forces and the torques acting in the physical system.

For open-chain mechanical structures, such as the robot arm, the direct models are analytical and are represented by the equations which describe the position, orientation, velocity, acceleration and active forces, with respect to a fixed coordinate system and knowing all the motion parameters of the active joints relative to their coordinate system.

The design of the control algorithms for a robot with n degrees of freedom imposes that the mechanical structure to be geometrically modeled. The geometrical model of the robot can be obtained using different methods like: vectorial method, rotation matrices, general parameters, Denavit-Hartenberg method, etc. The Denavit-Hartenberg parameters define the relative position both for the kinematic axes of the element and for the neighbor elements connected on the same motion axis, and they define the homogenous transformation between two consecutive axes.

Figure 1: Cyton Gamma 1500 robot arm

From the mechanical point of view the Robai Cyton Gamma 1500 robot arm is an open chain kinematic structure, having seven active rotation joints. The equations which define the position and the orientation of the robot gripper relative to the fixed coordinate system placed in the robot base were determined using the Denavit-Hartenberg method.

The direct kinematic model of the robot Robai Cyton Gamma 1500 implied the determination of the joints' motion vectors, angular and linear velocities, angular and linear accelerations.

Keeping in mind the objective of designing controllers, rigid bodies have been assumed. Thus, the robot elements are rigid connected with class V joints (one degree of freedom) and considered perfect from the friction and elasticity point of view. The dynamic model of the robot has been determined using the iterative Newton-Euler method, based on the following known parameters:

1. the mass of the elements (joint and connection element)

2. the mass center point relative the joint coordinate system

3. the moment of inertia of an element relative to the frame placed in mass center point

Each element is acted upon by external forces and torques. Therefore, in order to determine the dynamic equations of the mechanical structure, the following parameters have been determined for each element:

1. the external force acting on the mass center point of the element;
2. the external torque generated by the force;
3. the connection force between two consecutive elements applied in the origin of the coordinate system of the second element;
4. the torque generated by the connection force for each element of the system.

In the connection links of the joints have been considered geometrically simple elements, with the mass center point being placed at a half of the element length. With these assumptions an extremely complex mathematical model has been obtained. For this model, the limits of all the variables and parameters involved have been determined.

1.1.2 Aerial vehicles

The first class of systems that has been modelled is quadrotors. Classic models exist for fixed wing planes and helicopters. However, since commonly flying vehicles are modelled as rigid bodies, multirotors can use the same modelling principles as for instance the ones used for helicopters. The differences between various architectures are reflected in the way the flight is achieved.

Flight dynamics can be modelled using one of the following two approaches: using the equation of energy conservation and the Euler-Lagrange formalism [7, 8, 9, 10], or describing the movement (rotation and translation) of a rigid body in an inertial space, based on the Newton-Euler approach [11, 12, 13, 14, 15, 16]. The former procedure determines the translational dynamics in a simple way, but in the existing models the rotational dynamics seems to remain at a higher level of description. On the other hand, the Newton-Euler approach provides a complete relationship between rotor speed inputs and the translational and rotational dynamics of the vehicle.

Besides modelling as a rigid body, several other representation alternatives have been considered. A simpler approach is to look at the vehicle as a point mass, in situations when only the evolution of the position is relevant (e.g. in certain high-level path planning tasks). More complex models consider the vehicles as multi-rigid-bodies, or non-rigid-bodies (e.g. flapping-wing architectures). However, in case of multirotors, single-rigid-body models are sufficient. Additional dynamics specific to multirotors and helicopters, such as propeller blade flapping and induced drag [13, 17, 18] are not considered at this moment.

Quadrotors are commonly modelled using one of the two configurations presented in Figure 2: "plus" configuration, when the quadrotor arms are aligned with the x and y axes; and "cross" configuration, when the quadrotor is rotated with 45^o around the z axis compared to the previous configuration. Since an ahead-looking camera is mounted on the cross, the latter one is preferred. In this way the forward direction corresponds to the view direction of the camera.

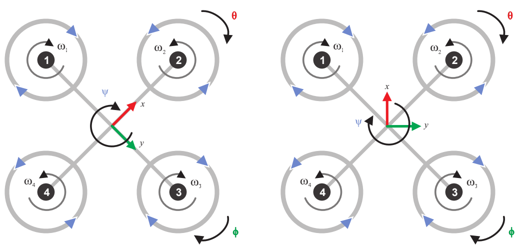

Figure 2: Quadrotor configurations: plus (left) and cross (right) configuration

We used both the Newton-Euler and Euler-Lagrange modelling approaches to build models for the AR Drone. It should be noted that, while commonly considered, we did not assume a near-hovering behaviour. The quadrotor is modelled as a single rigid body, having a symmetrical structure, with the origin of the body frame in the center of gravity. Additionally, the propellers are assumed to be rigid objects. For the Newton-Euler approach, we built upon the procedure from [15], whereas for the Euler-Lagrange formalism, we followed the steps presented in [19, 9].

Furthermore, we have identified the parameters of the AR.Drone 2.0 quadrotor [20]. Most parameters depend on the actual quadrotor that is used, e.g., the weight of the vehicle depends on the equipment used onboard. We have performed measurements for two AR Drone 2.0 quadrotors and obtained the results shown in Table 1.

Table 1: Parrot AR.Drone 2.0 parts weight measurements

first quadrotor	**mass (kg)**
fuselage	0.293
1500 mAh battery	0.120
indoor hull	0.057
outdoor hull	0.029
GPS module	0.033

second quadrotor	**mass (kg)**
fuselage	0.294
1000 mAh battery	0.101
indoor hull	0.076
outdoor hull	0.030

The different weights influence the other parameters as well, specifically the moments of inertia. Therefore, although the mass will be exactly known, we consider an uncertain model and robust controller design.

In the current configuration of the system the inputs are the desired velocities. In order to obtain a better control performance, the forces and torques acting on the rotors will be used as inputs. Based on the official product specifications [21], the minimum and maximum values of these inputs have been determined.

1.2 Analysis

In order to both efficiently address the nonlinear dynamics and keep the models in a natural form, once the first principle models have been developed, equivalent Takagi-Sugeno (TS) Takagi-Sugeno fuzzy models [22] in descriptor form [23] have been computed. TS models are nonlinear, convex combinations of local linear models, and are able to exactly represent large class of nonlinear systems in a compact set of the state-space [24]. TS descriptor models generalize the standard TS model, and allow obtaining a smaller number of conditions [25, 26] by keeping apart the nonlinearities on the two sides of the dynamic equation. For TS models, well-established methods and algorithms have already been developed to design observers for such models. In general, Lyapunov synthesis is used, employing common quadratic, piecewise quadratic, or, recently, nonquadratic [25, 27] Lyapunov functions. The analysis and design conditions are generally in the form of linear matrix inequalities (LMIs), which can be solved using convex optimization methods [28]. For TS descriptor models, several new results have been obtained for discrete-time controller design [29], although the observer design problem is still solved based on a common quadratic Lyapunov function.

The initial results, in particular those involving common quadratic Lyapunov functions, develop conditions that, when satisfied, imply the global stability of the TS model. This in fact means that any trajectory starting in the largest Lyapunov level set included in the considered compact set of the state-space will converge. In the case of the continuous-time TS models, with the introduction of nonquadratic Lyapunov functions, the developments involve the derivatives of the membership functions. Due to this, local stability results have been obtained, with the domain given by the bounds on the derivatives [30, 31, 32], usually being translated into bounds on the states.

In the discrete-time case, since the variation of the Lyapunov function does not involve any derivatives and thus further conditions, non-quadratic Lyapunov functions have shown a real improvement [25, 33, 34, 35, 36, 27] for developing global stability and design conditions. It has been proven that the solutions obtained by non-quadratic Lyapunov functions include and extend the set of solutions obtained using the quadratic framework. More recently, by using Polya's theorem [37, 38] asymptotically necessary and sufficient (ANS) LMI conditions have been obtained for stability in the sense of a chosen quadratic or nonquadratic Lyapunov function. [39] gave ANS stability conditions for both membership function-dependent model and membership function-dependent Lyapunov matrix. By increasing the complexity of the homogeneously polynomially parameter-dependent Lyapunov functions, in theory any sufficiently smooth Lyapunov function can be approximated. Unfortunately, the number of LMIs that have to be solved increase quickly, leading to numerical intractability [40]. However, all these results involve global stability, i.e., if an equilibrium point is not globally stable, no conclusion can be drawn.

Due to the physical constraints concerning the applications considered, not to mention the complexity of the models, we considered the problem of establishing local stability and estimating a domain of attraction of the equilibrium points, based on discrete-time TS models. For this, we have assumed that there exists a domain $\mathcal{D}_R$ where a condition on consecutive states is satisfied. Such an assumption can always be made, as the domain has to be verified a posteriori, and, in the worst case, may be reduced to zero. Our goal has been to establish the stability of the model in this domain and at the same time increase this domain. Stability analysis conditions have been developed both for quadratic and nonquadratic Lyapunov functions and they have been formulated as linear matrix inequalities, which can be efficiently solved. We have compared these conditions with classical ones in various simulation experiments and results have been obtained even when classic methods fail.

This research was developed in an international collaboration between the PI Zs. Lendek, and J. Lauber at the University of Valenciennes, France. Resulting publication:

- Zs. Lendek, J. Lauber, "Local stability of discrete-time TS fuzzy systems". In Proceedings of the 4th IFAC International Conference on Intelligent Control and Automation Sciences, pages 7–12, Reims, France, June 2016

1.3 Estimation and control methods

Although existing methods always assume that stabilization involves the convergence of the state variables to zero, it must be kept in mind that the TS model is actually a representation of a nonlinear system. The nonlinear system may have several equilibrium points due to which possibly only local stabilization can be achieved. Similarly, in case of observer design, the error dynamics may only be locally stabilizable, meaning that for some initial conditions, the unknown variables may not be observable. Therefore, we have considered local stabilization and local observer design.

Based on our results for local stability, design conditions have been developed for both control and estimation. These conditions have also been extended to the general case, the design involving delayed Lyapunov functions and delayed observer and controller gains. In this case, the controller and the observer take into account not just the current state and measurements, but also the past trajectory, thus reducing the conservativeness of the design.

It has to be noted, that, as expected, the observer design problem has not been the dual of the controller design problem. This is due to several reasons, among which we mention: the scheduling variables depend on unmeasured states; the estimation error dynamics depends both on the estimation error and the states – thus on "external" variables. Therefore, the results involve both the measurement error and the states of the system.

This research also involved an international collaboration between the project members, the University of Valenciennes, France, and the University of Lorraine, France.

Resulting publications:

- Zs. Lendek, J. Lauber, "Local quadratic and nonquadratic stabilization of discrete-time TS fuzzy systems". In Proceedings of the 2016 IEEE World Congress on Computational Intelligence, pages 1-6, Vancouver, Canada, July 2016.
- V. Estrada-Manzo, T. M. Guerra, Zs. Lendek, "Generalized observer design for discrete-time T-S descriptor models". Neurocomputing, vol. 182, pages 210-220, 2016
- V. Estrada-Manzo, Zs. Lendek, T. M. Guerra, "Observer Design for Robotic Systems via Takagi-Sugeno Models and Linear Matrix Inequalities". In Handling uncertainty and networked structure in robot control, series Studies in Systems, Decision and Control, L. Busoniu and L. Tamas, Editors, pages 103-128. Springer International Publishing, 2016.

Furthermore, two journal and one conference publications are currently being evaluated.

2 Applications

In this phase of the project we have applied the theoretical results on the Cyton Gamma robot arm and the A.R. drone available at the Technical University of Cluj-Napoca. For this, in the first phase, a driver has been necessary in order to directly control the applications from Matlab. Furthermore, a SimMechanics model has been built for a better visualization.

In the Cyton Gamma 1500 robot arm two types of servo motors are used: the motors in the shoulder joints are Dynamixel MX64 and the other actuators are Dynamixel MX28 motors. The MX64 servos can be controlled with torque, velocity and position commands, while the MX28 motors accept only velocity and position commands. All the physical parameters have been measured or identified from measured data. The original Dynamixel motor driver is a ROS interface. The controller package in the driver contains joint position and torque controllers for a single and a dual motor. Since the MX28 motors do not support torque control, the Dynamixel motor driver has been extended with a velocity controller for single and multiple motors. The driver is publishing the measurements of each motor on 20 [Hz] while the maximum frequency of the command signal is 7 [Hz].

The velocity controller is available at

- `https://bitbucket.org/ElodP/dynamixel-velocity-controller`

while the tutorials explaining the use of the controller and the SimMechanics model are available at:

- `https://sites.google.com/site/timecontroll/tutorials/dynamixle-velocity-control`
- `https://sites.google.com/site/timecontroll/tutorials/simmechanics---dynamixel-model`

Regarding the application of control and estimation methods, as a baseline, we have implemented a PID controller. However, since it has been experimentally tuned, no performance or stability guarantees are available for the whole workspace. Second, we have tested the developed observers and some of the controllers. Due to the complexity of the first-principle model, our aim has been to reduce the computational complexity of the design conditions. For this, we have used a descriptor model formulation, and in the first step, the designed observers and controllers were based on a quadratic Lyapunov function. Afterwards, the design was extended to nonquadratic Lyapunov functions and nonPDC observers and controllers, in which performance measures, such as disturbance attenuation and convergence speed has been included. An observer-based tracking controller has also been designed and tested in simulation and preliminary experiments.

Resulting publications:

- Nagy, Z., Lendek, Zs. "Takagi-Sugeno fuzzy modelling and control of a robot arm", Enelko-SzamOkt 2016, 17th International Conference on Energetics-Electrical Engineering, 26th International Conference on Computers and Education, 2016, pages 265-270
- E. Páll, L. Tamas, L. Busoniu, "Analysis and a home assistance application of online AEMS2 planning". 2016 IEEE/RSJ International Conference on Intelligent Robots and Systems., pages 5013-5019, Daejeon, Korea, October 2016.

Furthermore, one journal and two conference publications are currently being evaluated.

3 Preliminary and other work

Up until now the testing and validation of the developed observer and controller design methods have been focused on the robot arm application. We are currently working on demos for the AR drone and a journal publication on this application is being prepared.

A thorough experimental testing and validation of the control methods is currently underway for both applications.

We are also organizing an Invited Open track on this and related research at the 2017 IFAC World Congress, Toulouse, France. In our talks we will outline the research undertaken in this project.

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