Unknown input observer for a robot arm using TS fuzzy descriptor models

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***Abstract*— This paper presents an unknown input observer for a two-link robot arm. To handle the inherent nonlinearities, a Takagi-Sugeno fuzzy model in descriptor form is used. The design conditions are given as linear matrix inequalities, which can be efficiently solved. The observer is tested both in simulation and on experimental, measured data.**

I. INTRODUCTION

Robotics has a growing impact on our everyday life. Traditional applications in industry are complemented by an increasing integration of robots in the human environment, with domestic and assistive robotics being prime examples of this trend. However, many of the practical challenges associated to the real time control and monitoring of robotic systems are not yet solved. Assistive robots have to act safely and reliably [2], [3] in a partially unknown and dynamically changing environment.

Robotic arm applications typically require advanced model based control algorithms [4], [6]. The dynamic model is usually obtained from Newton-Euler equations. Once these equations are obtained, a state space representation naturally leads to a descriptor model [6], [7]. The resulting descriptor model is highly nonlinear. Linear approximations are very common; however, they only provide local conclusions [8].

In this paper, we address the estimation of the unknown inputs for two joints of a Cyton Gamma robot arm. This robot arm is designed to create an easy access for every user, and by default is controlled in position. However, depending on the payload, the default controller performances may degrade. In order to implement a high-performance model-based controller, an accurate model is necessary. To obtain such a model, both the parameters of the robot arm and the actual torque input applied need to be identified/estimated.

In general, systems are subject to known and unknown inputs (disturbances, measurement noise, modeling uncertainties, etc.). Designing observers for both the states of the system and unknown inputs is an important task in robust control, monitoring and fault-tolerant control [9], [10]. Estimating unknown inputs also reduces the number of sensors to be used. For instance, in biomechanics, the estimation of unknown inputs such as the joint torques and angular velocities avoids the use of sensors on the person under study [11].

In order to efficiently address the nonlinear dynamics and at the same time keep it in a natural form, in this paper Takagi-Sugeno fuzzy models [12] in descriptor form [13] will be used. TS models are nonlinear, convex combinations of local linear models, and are able to exactly represent large class of nonlinear systems in a compact set of the state-space [14]. TS descriptor models generalize the standard TS model, and allow obtaining a smaller number of conditions [15], [16] by keeping apart the nonlinearities on the two sides of the dynamic equation. For TS models, well-established methods and algorithms have already been developed to design observers. In general, Lyapunov synthesis is used, employing common quadratic, piecewise quadratic, or, recently, nonquadratic [15], [17] Lyapunov functions. The observer design conditions are generally in the form of linear matrix inequalities (LMIs), which can be solved using convex optimization methods [18]. For TS descriptor models, several new results have been obtained for discrete-time controller design [19], although the observer design problem is still solved based on a common quadratic Lyapunov function. In this paper, we generalize existing results to estimate the unknown inputs of the robotic arm.

The rest of the paper is organized as follows: Section II provides the description of the robot arm considered. Section III presents the TS model and the unknown input observer design. The observer is evaluated on simulated and experimental data in Section IV. Section V concludes the paper.

Notations: In order to develop our results we will use the following notations. Let $F = F^T \in R^{n_F \times n_F}$ be a symmetric matrix; $F > 0$ and $F < 0$ stands for positive and negative definiteness. I denotes the identity matrix, 0 is the zero matrix of appropriate dimensions. A $(*)$ in a matrix indicates a transposed quantity in the symmetric position. For instance $\begin{pmatrix} P & (*) \\ A & P \end{pmatrix}$ is equivalent to $\begin{pmatrix} P & A^T \\ A & P \end{pmatrix}$, and $A+(*)$ is equivalent to $A + A^T$.

II. ROBOT ARM MODEL

The last two joints of Robai Cyton Gamma robot arm, and its schematic representation are presented in Figs. 1 and 2. The dynamic model of the robot arm in Fig. 2 is given by:

$$\begin{aligned} \dot{q} =& \dot{q} \\ M(q)\ddot{q} =& -D(q,\dot{q})\dot{q} + I\tau \end{aligned} \tag{1}$$

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Fig. 1. Robai Cyton Gamma robot arm, two joints

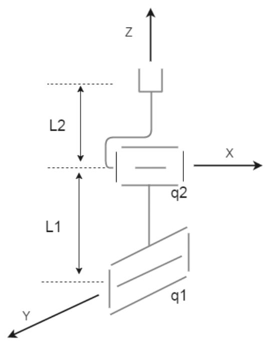

Fig. 2. Schematic representation of the 2DOF robot arm

where $q = \begin{bmatrix} q_1 & q_2 \end{bmatrix}^T$ are the angles of the two joints, $\dot{q} = \begin{bmatrix} \dot{q}_1 & \dot{q}_2 \end{bmatrix}^T$ are the angular velocities, $\tau = \begin{bmatrix} \tau_1 & \tau_2 \end{bmatrix}^T$ are the torques. M is the inertia matrix and D contains the Coriolis and centrifugal forces together with the damping. The parameters of the system are presented in Table I.

TABLE I

SYSTEM PARAMETERS

Notation	Value	Description
$L_1[m]$	0.075	length first-second joint
$L_2[m]$	0.16	length second joint end-effector
$M_1[kg]$	0.11	mass first joint
$M_2[kg]$	0.21448	mass second joint
$I_{1x}[kgm^2]$	$0.472 \cdot 10^{-4}$	moment of inertia
$I_{1y}[kgm^2]$	$0.3675 \cdot 10^{-4}$	moment of inertia
$I_{1z}[kgm^2]$	$0.1932 \cdot 10^{-5}$	moment of inertia
$I_{2x}[kgm^2]$	$0.280771 \cdot 10^{-3}$	moment of inertia
$I_{2y}[kgm^2]$	$0.2857 \cdot 10^{-2}$	moment of inertia
$I_{2z}[kgm^2]$	$0.64961 \cdot 10^{-3}$	moment of inertia
$b_1[-]$	0.094	friction coefficient, first joint
$b_2[-]$	0.028	friction coefficient, second joint

Since the effects of gravity depend only on the positions and they are known, they do not affect the estimation and are therefore not included in the model. Using (1) and the notation $x = \begin{bmatrix} q_1 & q_2 & \dot{q}_1 & \dot{q}_2 \end{bmatrix}^T$, the following descriptor form can be written:

$$E(x)\dot{x} = A(x)x + B\tau \tag{2}$$

with

$$E(x) = \begin{pmatrix} I & 0 \\ 0 & M(x) \end{pmatrix}, A(x) = \begin{pmatrix} 0 & I \\ 0 & -D(x) \end{pmatrix}, B = \begin{pmatrix} 0 \\ I \end{pmatrix} \tag{3}$$

where

$$\begin{aligned} M(x) &= \begin{pmatrix} M(1,1) & 0 \\ 0 & \frac{M_2 L_2^2}{4} + I_{2y} \end{pmatrix} \\ M(1,1) &= I_{1x} + I_{2z} + \cos(x_2)^2 (I_{2x} - I_{2z}) \\ &\quad + M_2(L_1 + \frac{L_2 \cos(x_2)}{2})^2 \end{aligned} \tag{4}$$

and

$$\begin{aligned} D(x) &= \begin{pmatrix} D(1,1) & 0 \\ D(2,1) & b_2 \end{pmatrix} \\ D(1,1) &= -\,(x_4(\sin(2x_2)(\frac{M_2 L_2^2}{4} + I_{2x} - I_{2z}) \\ &\quad + L_1 L_2 M_2 \sin(x_2)) - b_1) \\ D(2,1) &= x_3(\frac{\sin(2x_2)}{2}(I_{2x} - I_{2z} + \frac{L_2}{4}) \\ &\quad + \frac{L_2 M_2 L_1 \sin(x_2)}{2}) \end{aligned} \tag{5}$$

In the $M(x)$ matrix one nonlinearity can be observed at $M(1,1)$ and in (5) two nonlinearities appear at $D(1,1)$ and $D(1,2)$. Since the measurements and the control are in discrete-time, a discrete-time model is developed:

$$E(x)x_{k+1} = (E(x) + T_s A(x))x_k + T_s B \tau_k \tag{6}$$

where $T_s = \frac{1}{140}$ is the sampling time. We denote $Ed(x_k) = E(x_k)$, $Ad(x_k) = (E(x_k) + T_s A(x_k))$ and $Bd = T_s B$. The bounds on the state variables are:

$$\begin{aligned} x_1, x_2 &\in \begin{bmatrix} \frac{-3\pi}{4}, & \frac{3\pi}{4} \end{bmatrix} \\ x_3, x_4 &\in \begin{bmatrix} -3, & 3 \end{bmatrix} \end{aligned} \tag{7}$$

With these limits and the sampling time $T_s = \frac{1}{140}$ the following bounds were found for the nonlinearities:

$$\begin{aligned} Ad_{3,3} &\in \begin{bmatrix} -2.7808e-04, & 0.0049 \end{bmatrix} \\ Ad_{3,4} &\in \begin{bmatrix} -3.8584e-05, & 3.8584e-05 \end{bmatrix} \\ Ed_{3,1} &\in \begin{bmatrix} 0.0004, & 0.0056 \end{bmatrix} \end{aligned} \tag{8}$$

Although in descriptor form the nonlinear parts in the system equation change in a small range, in the classical state space form the range of the nonlinearities is much larger, of the order of 10^3.

III. TS MODELS AND UNKNOWN INPUT OBSERVER

In order to use the nonlinear model (instead of a linearized one), we employ TS descriptor models. The classic TS fuzzy model is a convex combination of discrete-time linear models, having the form:

$$\begin{aligned} x_{k+1} &= \sum_{i=1}^{r} h_i(z_k)(A_i x_k + B_i u_k) \\ y_k &= \sum_{i=1}^{r} h_i(z_k) C_i x_k \end{aligned} \tag{9}$$

where x_k is the state vector, u_k is the control input, z_k is the premise vector and r is the number of rules. Matrices A_i, B_i and C_i represents the i-th local model. The membership functions h are nonlinear functions with the property: $h_i \in \begin{bmatrix} 0, & 1 \end{bmatrix}$, $\sum_{i=1}^{r} h_i = 1$. The TS fuzzy discrete-time descriptor model has the following form:

$$\begin{aligned} \sum_{i=1}^{r_e} v_i(z_k) E_i x_{k+1} &= \sum_{i=1}^{r_a} h_i(z_k)(A_i x_k + B_i u_k) \\ y_k &= \sum_{i=1}^{r_a} h_i(z_k) C_i x_k \end{aligned} \tag{10}$$

In this case the nonlinearities are in both side of the equation. v and h are the membership functions, and E_i is the descriptor matrix of the local model. A TS (descriptor) model can exactly represent a nonlinear (descriptor) system in a compact set of the state space.

The expression

$$\Gamma_{hh} = \sum_{i_1=1}^{r_a}\sum_{i_1=1}^{r_a} h_{i_1}(z_k)h_{i_2}(z_k)\Gamma_{i_1,i_2}$$

is called double-convex sum, where h_{i_1} and h_{i_2} are the membership functions, Γ_{i_1,i_2} is a matrix and Γ_{hh} is the notation of the convex sum.

Lemma 1: [20] The double convex-sum

$$\Gamma_{hh} = \sum_{i_1=1}^{r_a}\sum_{i_2=1}^{r_a} h_{i_1}(z_k)h_{i_2}(z_k)\Gamma_{i_1,i_2} < 0$$

is negative if the following set of LMIs hold:

$$\begin{gathered}\Gamma_{i,i} < 0, \quad \forall i \in 1,2,...,r_a\\ \frac{2}{r_a-1}\Gamma_{i_1,i_1} + \Gamma_{i_1,i_2} + \Gamma_{i_2,i_1} < 0\\ i_1, i_2 \in 1,2,...,r_a, \quad i_1 \neq i_2\end{gathered}$$

Lemma 2: [21] Consider a vector $\boldsymbol{x} \in \mathbb{R}^{n_x}$ and two matrices $Q = Q^T \in \mathbb{R}^{n_x \times n_x}$ and $\mathcal{R} \in \mathbb{R}^{m \times n_x}$ such that $\text{rank}(\mathcal{R}) < n_x$. The two following expressions are equivalent:

1) $\boldsymbol{x}^T Q \boldsymbol{x} < 0$, $\boldsymbol{x} \in \{\boldsymbol{x} \in \mathbb{R}^{n_x}, \boldsymbol{x} \neq 0, \mathcal{R}\boldsymbol{x} = 0\}$
2) $\exists \mathcal{M} \in \mathbb{R}^{m \times n_x}$ such that $Q + \mathcal{M}\mathcal{R} + \mathcal{R}^T\mathcal{M}^T < 0$

Based on the sector nonlinearity approach [22] the discrete-time nonlinear system (6) can be written in the fuzzy descriptor representation. Denoting $E_v = \sum_{i=1}^{r_e} v(z_k)E_i$, $A_h = \sum_{i=1}^{r_a} h(z_k)A_i$ and $C_h = \sum_{i=1}^{r_a} h(z_k)C_i$. The discrete-time fuzzy descriptor model (10) has the form:

$$\begin{aligned}E_v x_{k+1} =& A_h x_k + B u_k\\ y_k =& C_h x_k\end{aligned} \tag{11}$$

Our main goal is to obtain an unknown input discrete-time fuzzy observer for the descriptor model. In this case the input torque is the unknown input. Based on [23] we consider the unknown inputs as state variables, so the new state vector has the form $x_k^e = \begin{bmatrix} x_k^T & d_k^T \end{bmatrix}^T$, where d_k represents the unknown input vector. Based on the continuous case described [23] we use the relation $d_{k+1} = I d_k$, considering the input as piece-wise constant. With these notations we have the following system:

$$\begin{aligned}E_v x_{k+1} =& A_h x_k + B d_k\\ d_{k+1} =& I d_k\end{aligned} \tag{12}$$

From here we have

$$\begin{aligned}E_v^e x_{k+1}^e =& A_h^e x_k^e\\ y_k =& C_h^e x_k^e\end{aligned} \tag{13}$$

where

$$\begin{aligned}&E_v^e = \begin{pmatrix} E_v & 0\\ 0 & I\end{pmatrix}, \quad A_h^e = \begin{pmatrix} A_h & B_k\\ 0 & I\end{pmatrix},\\ &C_h^e = \begin{pmatrix} C_h & 0\end{pmatrix}\end{aligned} \tag{14}$$

Now with these matrices we design a discrete-time TS observer. The aim is to make the estimation error converge to zero as time tends to infinity. An observer is proposed by [23], but for our TS fuzzy system the corresponding LMIs are unfeasible; we need a less conservative design. In [23] the M matrix was chosen to be $M = \begin{bmatrix} 0 & P\end{bmatrix}^T$, it is dependent on the P matrix, for this system we used a more general form $M = \begin{bmatrix} 0 & K\end{bmatrix}^T$, where K is a free matrix.

The TS fuzzy observer has the form:

$$\begin{aligned}E_v^e \hat{x}_{k+1}^e =& A_h^e \hat{x}_k^e + L_{hv}(y - \hat{y})\\ \hat{y}_k =& C_h^e \hat{x}_k^e\end{aligned} \tag{15}$$

Based on (15) and (13) we have the error dynamics:

$$E_v^e e_{k+1} = (A_h^e - L_{hv}C_h^e)e_k \tag{16}$$

where $e_k = x_k^e - \hat{x}_k^e$; this can be written in the form:

$$\begin{bmatrix} A_h^e - L_{hv}C_h^e & -E_v^e\end{bmatrix}\begin{bmatrix} e_k\\ e_{k+1}\end{bmatrix} = 0 \tag{17}$$

The following results can be formulated:

Theorem 1: The estimation error dynamics in (17) is asymptotically stable if there exist matrices $P = P^T > 0$, K, and $N_{i_2 j}$ for $i_1, i_2 = 1, 2, \ldots, r_e$ and $j = 1, 2, \ldots, r_a$ so that for every i_1, i_2 and j

$$\begin{array}{rcl}\Gamma_{i_1,i_2}^j & < & 0\\ \frac{2}{m-1}\Gamma_{i_1,i_1}^j + \Gamma_{i_1,i_2}^j + \Gamma_{i_2,i_1}^j & < & 0\end{array} \tag{18}$$

with

$$\Gamma_{i_1,i_2}^j = \begin{bmatrix} -P & (*)\\ KA_{i_1}^e - N_{i_2,j}C_{i_1}^e & -KE_j^e + (*) + P\end{bmatrix} \tag{19}$$

Proof: Using the Lyapunov function:

$$V(e_k) = e_k^T P e_k \tag{20}$$

we have the difference:

$$\begin{gathered}\Delta V(e_k) = e_{k+1}^T P e_{k+1} - e_k^T P e_k < 0\\ \begin{bmatrix} e_k\\ e_{k+1}\end{bmatrix}^T \begin{bmatrix} -P & 0\\ 0 & P\end{bmatrix}\begin{bmatrix} e_k\\ e_{k+1}\end{bmatrix} < 0\end{gathered} \tag{21}$$

Using Lemma 2 together with the equality (17) and the inequality (21) we have:

$$M\begin{bmatrix} A_h^e - L_{hv}C_h^e & -E_v^e\end{bmatrix} + (*) + \begin{bmatrix} -P & 0\\ 0 & P\end{bmatrix} < 0 \tag{22}$$

Choosing $M = \begin{bmatrix} 0 & K\end{bmatrix}^T$ and applying Lemma 1, we obtain condition (18). ■

The observer gains are recovered as $L_{i_2,j} = K^{-1}N_{i_2,j}$. The LMIs in (18) can be efficiently solved using the Yalmip Matlab extension. We use the assumption that our input is a constant quantity, so the observer will work in a good manner in the case when we have a piece-wise constant input. But it does not take into consideration the case when the system is affected by noise, which happens usually in a real application. In order to get better performance we propose the H_∞ approach. This approach is usually used in controller design, but the idea can be applied also here. We can reformulate the problem in the following form:

$$\begin{aligned} E_v^e x_{k+1}^e &= A_h^e x_k^e + K_h w_k \\ y_k &= C_h^e x_k^e \end{aligned} \tag{23}$$

where w_k represents the noise; we can consider the difference between the ideal input and the estimated input as a noise on the system, and we want to find an observer which reduces this. The form of the observer is:

$$\begin{aligned} E_v^e \hat{x}_{k+1}^e &= A_h^e \hat{x}_k^e + H_{hv}^{-1} Q_{hv}(y_k - \hat{y}_k) \\ \hat{y}_k &= C_h^e \hat{x}_k^e \end{aligned} \tag{24}$$

The error dynamics has the form:

$$\begin{bmatrix} A_h^e - H_{hv}^{-1} Q_{hv} C_h^e & -E_v^e & K_h \end{bmatrix} \begin{bmatrix} e_k \\ e_{k+1} \\ w_k \end{bmatrix} = 0 \tag{25}$$

We have the following theorem:

Theorem 2: The estimation error dynamics in (25) is asymptotically stable and the noise is attenuated by at least γ, if there exist matrices $P = P^T > 0$, $H_{i,j}$, and $Q_{i,j}$, for $i = 1, 2, \ldots, r_e$ and $j = 1, 2, \ldots, r_a$, and $\gamma > 0$ such that

$$\Gamma_{hhvv} = \begin{bmatrix} -P + I & (*) & (*) \\ H_{hv} - Q_{hv} C_h^e & -H_{hv} E_v^e + (*) + P & (*) \\ \epsilon R(H_{hv} A_h^e - Q_{hv} C_h^e) & (H_{hv} K_h)^T - \epsilon R H_{hv} E_v^e & T \end{bmatrix} < 0 \tag{26}$$

where $T = \epsilon R H_{hv} K_h + (*) - \gamma^2 I$.

Proof: Using the Lyapunov function presented in (20); to achieve γ attenuation we have the inequality:

$$\Delta V + e_k^T e_k - \gamma^2 w_k^T w_k < 0$$

$$\begin{bmatrix} e_k \\ e_{k+1} \\ w_k \end{bmatrix}^T \begin{bmatrix} I - P & 0 & 0 \\ 0 & P & 0 \\ 0 & 0 & -\gamma^2 I \end{bmatrix} \begin{bmatrix} e_k \\ e_{k+1} \\ w_k \end{bmatrix} < 0 \tag{27}$$

Using Lemma 2 with (27) and (25), we have:

$$\begin{bmatrix} G_1 \\ G_2 \\ G_3 \end{bmatrix} \begin{bmatrix} A_h^e - H_{hv}^{-1} Q_{hv} C_h^e & -E_v^e & K_h \end{bmatrix} + (*) + \begin{bmatrix} I - P & 0 & 0 \\ 0 & P & 0 \\ 0 & 0 & -\gamma I \end{bmatrix} < 0 \tag{28}$$

For the sake of simplicity and to be able to recover results in the literature we choose $G_1 = 0$. To avoid bi-linear matrix inequalities we have $G_2 = H_{hv}$; for the same reason $G_3 = \epsilon R H_{hv}$, where the purpose of the R is to get the appropriate dimensions for the matrices. With these assumptions and using Lemma 1 we obtain (26). ■

IV. EXPERIMENTAL RESULTS

The TS fuzzy representation (13) of the robot arm contains the following matrices:

$$E_i^e = \begin{pmatrix} I & 0 & 0 \\ 0 & E(2,2) & 0 \\ 0 & 0 & I \end{pmatrix}$$

$$i = 1, 2 : E(2,2) = 10^{-2} \begin{pmatrix} 0.04 & 0 \\ 0 & 0.42 \end{pmatrix}, 10^{-2} \begin{pmatrix} 0.56 & 0 \\ 0 & 0.42 \end{pmatrix} \tag{29}$$

$$A_j^e = \begin{pmatrix} I & \frac{1}{140} I & 0 \\ 0 & A(2,2) & \frac{1}{140} I \\ 0 & 0 & I \end{pmatrix}$$

$$\begin{aligned} j = 1, 2 : A(2,2) &= 10^{-3} \begin{pmatrix} -0.27 & 0 \\ -0.38 & 4 \end{pmatrix}, 10^{-3} \begin{pmatrix} -0.27 & 0 \\ 0.38 & 4 \end{pmatrix} \\ j = 3, 4 : A(2,2) &= 10^{-3} \begin{pmatrix} 4.9 & 0 \\ -0.38 & 4 \end{pmatrix}, 10^{-3} \begin{pmatrix} 4.9 & 0 \\ 0.38 & 4 \end{pmatrix} \end{aligned} \tag{30}$$

As it can be seen we have 2 local matrices for E_v^e and 4 for A_h^e. For this system the state vector contains the unknown inputs d_k which are the torques acting on the joints. We measure the states, so we have the following C_h^e matrix:

$$C_h^e = \begin{pmatrix} I & 0 \end{pmatrix} \tag{31}$$

Because of their complicated form, the membership functions $h(z_k)$ and $v(z_k)$ are not presented here. For this system an observer was calculated based on Theorem 1, obtaining the observer gains (due to the lack of space only some of them are presented):

$$L_{i,j} = \begin{pmatrix} I & \frac{1}{140} I \\ 0 & L(2,2) \\ 0 & L(3,2) \end{pmatrix}$$

$$\begin{aligned} i = 1, j = 1 : \quad L(2,2) &= 10^{-2} \begin{pmatrix} 0.02 & 0 \\ 0 & 0.71 \end{pmatrix}, \\ L(3,2) &= 10^{-1} \begin{pmatrix} 0.73 & 0 \\ 0 & 4.4 \end{pmatrix} \\ i = 2, j = 4 : \quad L(2,2) &= 10^{-2} \begin{pmatrix} 0.56 & 0 \\ 0 & 0.71 \end{pmatrix}, \\ L(3,2) &= 10^{-1} \begin{pmatrix} 0.79 & 0 \\ 0 & 4.4 \end{pmatrix} \end{aligned} \tag{32}$$

The observer has first been tested in simulation. The input signals can be seen in Fig. 3. The differences between the estimated and the model input can be seen in Fig. 4. For this simulation the initial condition for the model was $x_0 = \begin{bmatrix} 0.5 & 1.3 & 0 & 0 \end{bmatrix}^T$ and for the observer $\hat{x}_0^e = 0$. As it can be seen in Fig. 4, the observer correctly estimates the input

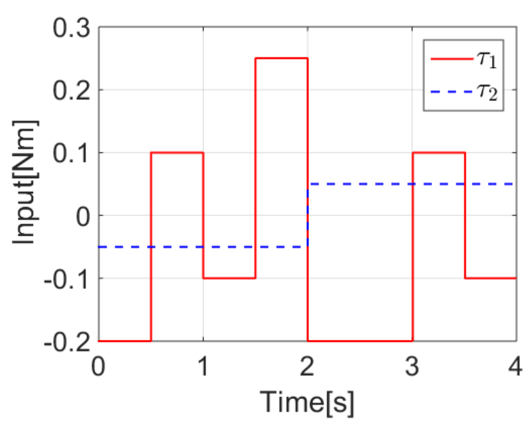

Fig. 3. Piece-wise constant input

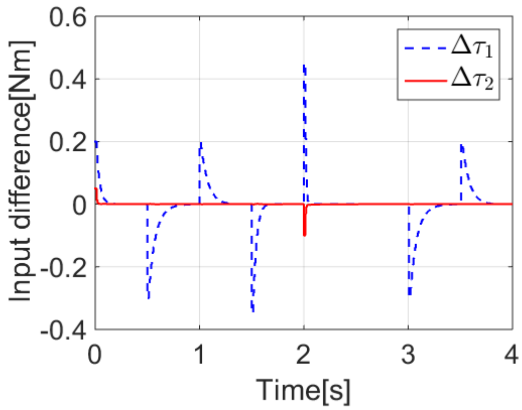

Fig. 4. Difference between the estimated and the true inputs

torque. The peak values appear when the input has a sudden change. For the second iteration we use the same input data, but now we apply a band limited white noise on it. The difference between the estimated noisy input and ideal input can be seen in Fig. 5.

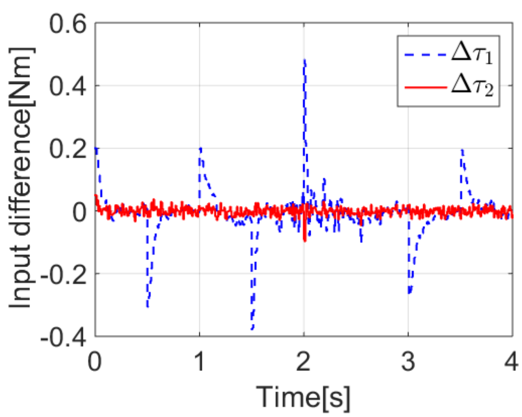

Fig. 5. Difference between the estimated and the ideal model values

Next we test the observer designed with conditions proposed in Theorem 2. The effect of the noise appears mostly on the unknown input so the size of w_k was chosen to be 2. For this reason the R matrix size is 2-by-6, to get the appropriate dimensions. For the sake of simplicity we used a constant matrix for K_h. The following matrices were chosen:

$$R = \begin{pmatrix} I & I & 0 \end{pmatrix}, \quad K_h = \begin{pmatrix} I & 0 & I \end{pmatrix}^T \tag{33}$$

Using the H_∞ observer with the same noisy input data we have the results in Fig. 6. Even though it is not so visible the peak values of the noise were reduced, the best position to see this is around 2 seconds. When the input changes there is still a large overshoot

In what follows we test the observers on real data. Our data source was a Robai Cyton Gamma 1500 robot arm. The model parameters presented in Table I were determined using this system. Because the angular velocities are actually

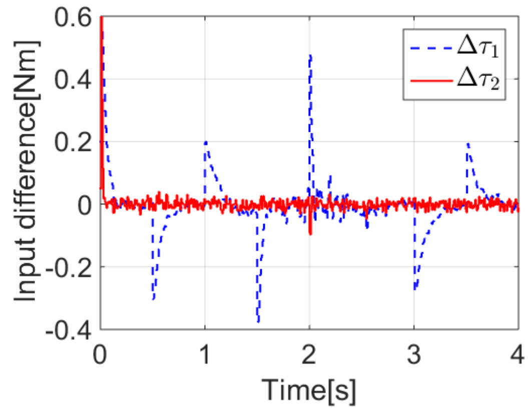

Fig. 6. Difference between the estimated and the model values with H_∞ observer

computed from the measured angles, the velocity measurements are noisy. An option to reduce the major noises is to use a filter. A $12th$ order Butterworth filter was applied on the angular velocity data with a normalized cut-off frequency 0.07. The filtered data can be seen in Fig. 7, where the initial condition was $x_0 = \begin{bmatrix} 0.08 & 0 & 0 & 0 \end{bmatrix}^T$. As it can be seen,

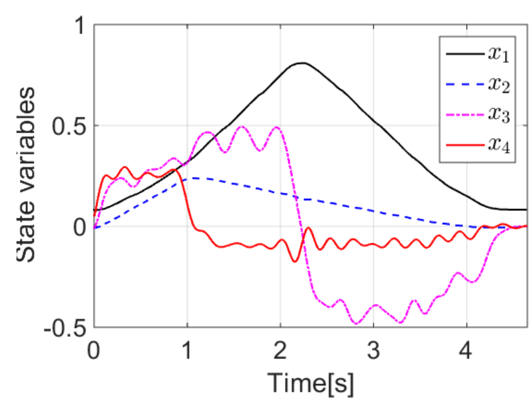

Fig. 7. State variables

both joints are moving. For the second joint we have a small input, which produces a slow response. From the form of the state variables on Fig. 7 we can assume that our input is close to a piece-wise constant one.

The estimated inputs using the observer based on Theorem 1 are presented in Fig. 8. Because of the system parameters, a small variation on the input may cause a relatively large variation on the output. In this case we have a situation where the output changes in a small range, so for the estimated inputs even smaller values appear. In order to compare the results we apply the observer designed based on Theorem 2 on the output data. The results can be seen in Fig. 9. In both cases the estimated inputs have similar form.

V. CONCLUSIONS AND FUTURE WORK

In this paper the estimation of unknown inputs was discussed for a robot arm. Theorem 1 presented an unknown input observer which was able to estimate piece-wise constant

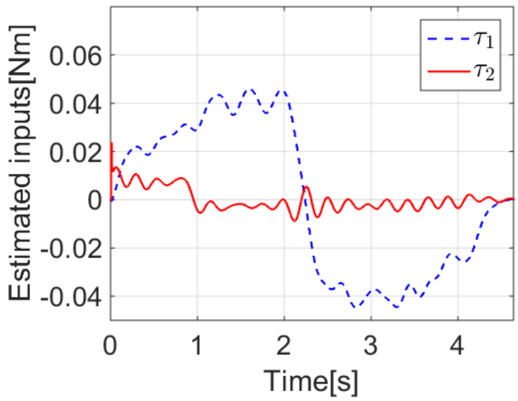

Fig. 8. Estimated input based on real data

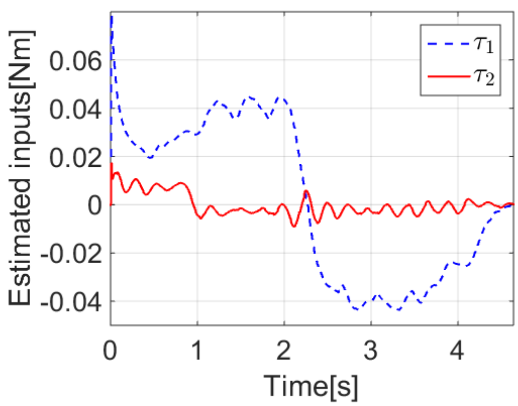

Fig. 9. Estimated input based on real data, H_∞ approach

inputs. In the case when the measured data was noisy, the observer was still able to estimate the input, but without any guarantee regarding the noise. In order to attenuate the noise an H_∞ observer was presented. Finally these two observers were tested on experimental data, situation where they had similar performances. Both observers perform well in the case when the input is piecewise constant.

In our future work we will deal with the non-constant inputs. Another option is to estimate the input based only on the measured angles. Furthermore, an observer-based-controller will be developed.

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