Observer-based controller design for Takagi-Sugeno fuzzy systems with local nonlinearities

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Motivation

Takagi-Sugeno fuzzy models

- Can exactly represent a nonlinear model in a compact set
- Convex combination of local linear models
- Computational complexity exponentially increases with the number of nonlinearities

Slope-restricted nonlinearities

- Approach to handle certain type of nonlinearities for observer design

This paper:

Combining the advantages of both TS fuzzy and slope-restricted nonlinearities for observer-based controller design

- Reducing the number of fuzzy rules
- Separately handling measured- and unmeasured-state nonlinearities

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Outline

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Takagi-Sugeno fuzzy model

- Model:

$$\dot{x} = \sum_{i=1}^{s} h_i(z)(A_i x + B_i u)$$
$$y = \sum_{i=1}^{s} h_i(z) C_i x,$$

- $x \in \mathbb{R}^{n_x}$ state vector, $u \in \mathbb{R}^{n_u}$ input vector
- $y \in \mathbb{R}^{n_u}$ output vector
- $h_i(z)$ - membership function
- z - premise variable (subset of the independent states x)
- convex sum: $h_i(z) \in [0, 1]$, $\sum_{i=1}^{s} h_i(z) = 1$
- A_i, B_i and C_i are local linear models
- Problem formulation - Linear Matrix Inequalities (LMI)

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Example

- Nonlinear model:

$$\dot{x}_1 = x_1 \sin(x_1)$$
$$\dot{x}_2 = 3x_1 + 10x_2 + u$$
$$\dot{x} = \begin{pmatrix} \sin(x_1) & 0 \\ 3 & 10 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u,$$

- Exact TS model:

$$\dot{x} = \left(h_1(x_1)A_1 + h_2(x_1)A_2\right) x + Bu$$
$$A_1 = \begin{pmatrix} -1 & 0 \\ 3 & 10 \end{pmatrix} \quad A_2 = \begin{pmatrix} 1 & 0 \\ 3 & 10 \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$
$$h_1(x_1) = \frac{1 - \sin(x_1)}{2} \quad h_2(x_1) = 1 - h_1(x_1)$$

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Slope-restricted nonlinearities

- Model:

$$\dot{x} = Ax + G\psi(Hx) + f(y,u)$$
$$y = Cx,$$

- Where $\psi(Hx)$ is a vector function, each entry a scalar
- $f(y,u)$ contains the "known" terms
- H scalar combination of the states

(Arcak and Kokotovic TAC 2001
Chong et al. Automatica 2014)

Observer design

- Assumption similar to Mean-value theorem

$$\psi_i(v) - \psi_i(w) = \delta_i(t)(v - w),$$
$$\forall\, v, w \in \mathbb{R},\ v \neq w, \quad \delta_i(t) \in [0,\ b_i]$$

- $\hat{x}$ estimate of x
- Form of the observer:

$$\dot{\hat{x}} = A\hat{x} + G\psi(H\hat{x}) + f(y, u) + L(y - \hat{y})$$

- $e := x - \hat{x}$, and the error dynamics:
  $\dot{e} = (A - LC)e + G(\psi(Hx) - \psi(H\hat{x}))$

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Outline

TS fuzzy systems with local nonlinearities

- Model:

$$\dot{x} = A_z x + B_z u + B_z G_z \psi(Hx)$$
$$y = C_z x$$

- Notation: $A_z \Leftrightarrow \sum_{i=1}^{s} h_i(z) A_i$
- Matching nonlinearities, motivation: mechanical systems

$$M(\theta)\ddot{\theta} = -F(\theta, \dot{\theta}) - G(\theta) + \tau$$

- Similar idea by Moodi and Farrokhi IJAMCS 2013

2 DOF robot arm

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Observer Design

Observer structure:

$$\dot{\hat{x}} = A_z\hat{x} + B_z u + B_z G_z \psi(H\hat{x} + L_\psi(y - \hat{y})) + L_z(y - \hat{y})$$
$$\hat{y} = C_z\hat{x}$$

- L_z observer gain
- L_ψ injection term, less conservative design (Arcak and Kokotovic TAC 2001)
- Assumption on $\psi(\cdot)$ leads to an LMI formulation

Controller design

Control law:

$$u = -K_z\hat{x} - G_z\psi(H\hat{x} + L_\psi(y - \hat{y}))$$

- Closed loop system dynamics:

$$\dot{e} = (A_z - L_z C_z)e + B_z G_z\left(\psi(Hx) - \psi(H\hat{x} + L_\psi(y - \hat{y}))\right)$$
$$\dot{\hat{x}} = (A_z - B_z K_z)\hat{x} + L_z C_z e$$

- Assumption

$$\psi_i(v) - \psi_i(w) = \delta_i(t)(v - w),$$
$$\forall\, v, w \in \mathbb{R},\ v \neq w, \quad \delta_i(t) \in [0,\ b_i]$$

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Controller design cont'd

Augmented system dynamics:

$$\begin{bmatrix}\dot{\hat{x}}\\ \dot{e}\end{bmatrix}=\begin{bmatrix}A_z-B_zK_z & L_zC_z\\ 0 & A_z-L_zC_z\end{bmatrix}\begin{bmatrix}\hat{x}\\ e\end{bmatrix}+\begin{bmatrix}0\\ B_zG_z\end{bmatrix}\left(\psi(Hx)-\psi(H\hat{x}+L_\psi(y-\hat{y})\right)$$

- Stability of cascaded systems (Lendek et al. TFS 2009)
- L_z and L_ψ can be found so that the error dynamics is globally asymptotically stable (GAS)
- If K_z can be found so that

$$\dot{\hat{x}}=(A_z-B_zK_z)\hat{x},$$

is GAS, then also the augmented system is GAS.

Controller design cont'd

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Outline

Numerical example

- Nonlinear model of an inverted pendulum

$$
\begin{aligned}
\dot{x}_1 =& x_2 \\
\dot{x}_2 =& \frac{-dx_2 - a(mlx_2)^2 \sin(x_1)\cos(x_1) + mgl\sin(x_1)}{\alpha(x_1)} \\
&+ \frac{-aml\cos(x_1)}{\alpha(x_1)}\tilde{u} \\
y =& x_1
\end{aligned}
$$

- x_1 is the angle, and x_2 is the angular velocity
- Due to physical limits, we assume $x_1 \in [\frac{-\pi}{3}, \frac{\pi}{3}]$, $x_2 \in [-\sigma, \sigma]$
- $\sin(x_1)$, $\cos(x_1)$, $\alpha(x_1)$ nonlinearities which are handled with TS fuzzy modeling
- x_2^2 handled with slope-restricted

Numerical example

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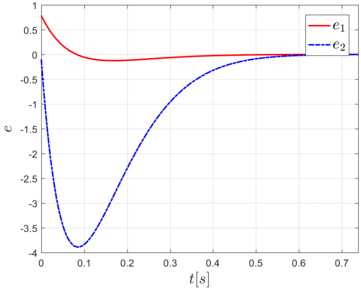
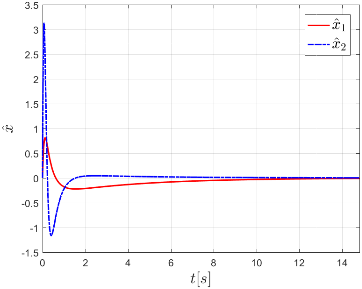


Estimation error



Estimated states

Outline

Conclusions

- Approach to handle unmeasured-state nonlinearities and reduce computational complexity

Future work

- Non-scalar inputs for the nonlinearity $\psi(Hx)$
- non-input matching nonlinearity

Thank you for your attention!

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Parameter table

Notation	Value	Description
g $[m^s/s]$	9.8	gravitational acceleration
m [kg]	0.3	mass of pendulum
M [kg]	15	mass of cart
d [N/rad/s]	0.0007	friction coefficient
l [m]	0.3	length of pendulum
J $[kg\ m^2]$	0.3	moment of inertia
σ [rad/s]	4	max angular velocity

Observer gains

$$L_\psi = 4.32 \cdot 10^{-5},\ L_1 = \begin{bmatrix} 27.48 \\ 182.13 \end{bmatrix},\ L_2 = \begin{bmatrix} 28.13 \\ 186.41 \end{bmatrix}$$

$$L_3 = \begin{bmatrix} 27.52 \\ 182.38 \end{bmatrix},\ L_4 = \begin{bmatrix} 28.1 \\ 186.22 \end{bmatrix},\ L_5 = \begin{bmatrix} 22.15 \\ 146.65 \end{bmatrix}$$

$$L_6 = \begin{bmatrix} 22.79 \\ 150.9 \end{bmatrix},\ L_7 = \begin{bmatrix} 22.19 \\ 146.92 \end{bmatrix},\ L_8 = \begin{bmatrix} 22.76 \\ 150.72 \end{bmatrix},$$

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Controller gains

$$K_1 = \begin{bmatrix} -3.81 & -11 \end{bmatrix}, \; K_2 = \begin{bmatrix} -3.81 & -11 \end{bmatrix}$$
$$K_3 = \begin{bmatrix} -6.52 & -21.48 \end{bmatrix}, \; K_4 = \begin{bmatrix} -6.52 & -21.47 \end{bmatrix}$$
$$K_5 = \begin{bmatrix} -3.74 & -20.16 \end{bmatrix}, \; K_6 = \begin{bmatrix} -3.74 & -20.16 \end{bmatrix}$$
$$K_7 = \begin{bmatrix} -6.73 & -32.71 \end{bmatrix}, \; K_8 = \begin{bmatrix} -6.73 & -32.71 \end{bmatrix}$$