Fuzzy modeling and design for a 3D Crane

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Summary

Objective: Fuzzy control of a 3D crane.

- TS fuzzy model of the crane
- TS observer
- The observer is tested in simulation and on a lab setup
- The controller is tested in simulation
1. Introduction

2. 3D Crane system
   - The experimental setup
   - Mathematical model
   - The discrete time fuzzy model

3. Observer design
   - TS observer
   - Results

4. Controller design
   - TS controller
   - Results

5. Conclusion
Introduction

Approaches for the control of a 3D Crane

- time-optimal control
- robust control for the crane swing and an LQ controller for tracking reference
- combined feedforward, feedback control and disturbance estimation
- fuzzy-logic projection controller designed based on cart position and swing angle
- fuzzy-logic controller to reduce the load swing during positioning of the crane
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Experimental setup

- **States:**
  - $x_1$, $x_3$, $x_9$: positions on $y$, $x$ and $z$ axes
  - $x_2$, $x_4$, $x_{10}$: velocities
  - $x_5$, $x_7$: angles
  - $x_6$, $x_8$: angular velocities
- **Inputs:** acceleration
- **Model alterations:**
  - $x_9 \rightarrow x_9 + 0.1$
- **Physical constraints**
Mathematical model

Model equations

\[ \dot{x}_1 = x_2 \]
\[ \dot{x}_2 = u_1^* + \mu_1 x_5 u_3^* - \mu_1 g x_5 \]
\[ \dot{x}_3 = x_4 \]
\[ \dot{x}_4 = u_2^* - \mu_2 x_7 u_3^* + \mu_2 g x_7 \]
\[ \dot{x}_5 = x_6 \]
\[ \dot{x}_6 = (u_1^* + \mu_1 x_5 u_3^* - \mu_1 g x_5 - g x_5 - 2 x_6 x_{10}) \frac{1}{x_9 + 0.1} \]
\[ \dot{x}_7 = x_8 \]
\[ \dot{x}_8 = -(u_2^* - \mu_2 x_7 u_3^* + \mu_2 g x_7 + g x_7 + 2 x_8 x_{10}) \frac{1}{x_9 + 0.1} \]
\[ \dot{x}_9 = x_{10} \]
\[ \dot{x}_{10} = u_3^* \]
Euler discretization

Sampling time $T_e = 0.01 \text{ s}$ (system data acquisition rate)

Discrete time model:

$$\begin{align*}
x(k + 1) &= A_d(x(k))x(k) + B_d(x(k))u(k) \\
y(k) &= Cx(k)
\end{align*}$$

Four nonlinear terms $\rightarrow$ fuzzy model with 16 rules:

$$x(k + 1) = \sum_{j=1}^{16} h_j(z(k))(A_{dj}x(k) + B_{dj}u(k))$$
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TS observer

- The purpose is to apply advanced control techniques
- Five measurable states out of 10 → observer design
- TS system:

\[
x(k + 1) = \sum_{i=1}^{r} h_i(z)(A_i x(k) + B_i u(k))
\]

\[
y(k) = C x(k)
\]

- Fuzzy estimator:

\[
\hat{x}(k + 1) = \sum_{i=1}^{r} h_i(z(k))(A_i \hat{x}(k) + B_i u(k) + L_i(y(k) - \hat{y}(k)))
\]

\[
\hat{y}(k) = C \hat{x}(k)
\]

where \(L_i, \ i = 1, \ldots, r\), are the observer gains.
TS observer

- Design: calculate matrices $P$, $H$ and $M_i$, $i = 1\ldots16$, solving the LMIs:

$$
\begin{pmatrix}
-P & (H A_i - M_i C)^T \\
H A_i - M_i C & -H - H^T + P
\end{pmatrix} < 0
$$

- Observer gains:

$$L_i = H^{-1} M_i, \ i = 1, \ldots, r$$
Simulation results

- Inputs: randomly generated staircase signals
- Outputs: simulated states
Experimental results

- Inputs: PID controller command signals
- Outputs: measured data from the plant
Experimental results

- Speed computed by differentiation and filtering
- Two filters: a zero-phase filter (used as a benchmark) and a low-pass filter
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Controller design

- PDC controller

\[ u(k) = - \sum_{i=1}^{r} h_i(z) F_i x(k) \]

- \( F_i \) - the local feedback gains - are obtained by solving:

\[
G_{ii} < 0 \\
\left( \frac{G_{ij} + G_{ji}}{2} \right)^T + \left( \frac{G_{ij} + G_{ji}}{2} \right) < 0
\]

where:

\[
G_{ij} = \begin{pmatrix}
-H - H^T + P & (A_i H - B_i S_j)^T \\
A_i H - B_i S_j & -P
\end{pmatrix}
\]
State feedback control

- The controller has been tested in simulation.
- If all states are known, the closed-loop system is stabilized.
Observer-based control

- The controller was combined with the observer and tested in simulation
- Note on the separation principle
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- The observer has been tested in a real-time experiment
- Controller: tested in simulation
- Future work: implementation of the controller

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